Supplementary material for 'How Portable Is Level-0 Behaviour? A Test of Level-k Theory in Games with Non-Neutral Frames' by Shaun Hargreaves Heap, David Rojo Arjona and Robert Sugden

This document provides additional information, data and analysis related to the paper. The material is divided as follows:

Section I: Additional details about the implementation of the experimental design

Section II: Screen shots of instructions, quizzes and decision tasks

Section III: Additional analysis of position effects

Section IV: Analysis restricted to games with neutral connotations

Section V: Patterns in Hide and Seek data consistent with 'plausible' level-k models

I. Additional details about the implementation of the experimental design

This section describes the methods by which games were assigned to pairs of subjects, and by which the order of games played by pairs was randomized.

In the HS sessions, subjects were randomized into four subgroups (HS1–HS4), each of 50 subjects. In the CD sessions, subjects were randomized into four subgroups (CD1–CD4), each of 20 subjects. Within each subgroup, and within each of the two parts of the experiment, each pair of subjects played two *blocks* of games. One block contained frames 1a–9a or 1b–9b; the other contained frames 10a–18a or 10b–18b. The order of these two blocks was counterbalanced. The order of games within each block was randomized, independently for each pair. Table A1 illustrates this procedure. For example, the top part of the first column of the table reports that in the first part of the experiment, each of the 50 subjects in subgroup HS1 played as hider in nine four-box games (frames 1a–9a) and in nine eight-box games (frames 10b–18b). It is evident from the third column that their co-players were the 50 subjects of subgroup HS3.

TABLE A1: ASSIGNMENT OF GAMES TO SUBJECTS

subgroup (and number of subjects)

	HS1	HS2	HS3	HS4	CD1	CD2	CD3	CD4
	(50)	(50)	(50)	(50)	(20)	(20)	(20)	(20)
Part 1								
frames1a-9a	Н	_	S	_	C	_	D	_
frames 1b–9b	_	Н	_	S	_	C	-	D
frames 10a-18a	_	Н	_	S	C	_	D	_
frames 10b-18b	Н	_	S	_	_	C	_	D
Part 2								
frames 1a-9a	S	_	Н	_	_	D	_	C
frames 1b–9b	_	S	_	Н	D	_	C	_
frames 10a-18a	_	S	_	Н	_	D	_	C
frames 10b–18b	S	_	Н	_	D	_	C	_

Letters denote roles played (H = hider, S = seeker, C = coordinator, D = discoordinator).

II. Screen shots of instructions, quizzes and decision tasks

This section presents screen shots in the order that subjects saw the screens. Screens 1–6b and 8a–8b are instructions given at the start of the experiment. Screens 10a–11b are instructions given at the start of part 2 of the experiment. Screens 7a and 7b are quizzes to test subjects' understanding of the instructions. Screens 9a – 9h are typical decision screens for the different games. Screens 12 and 13 inform subjects of the outcomes of the games for which they were to be paid, and of their resulting earnings. If a screen is specific to particular types of session, games or roles it is identified by a number and a letter (e.g. Screen 3a); otherwise it is identified only by a number.

Screen 1: General instructions: CD and HS sessions

Thanks for taking part in this experiment.

You are requested not to communicate during the experiment. If at any time you are not sure about what you are being asked to do, raise your hand and the experimenter will come to your desk to help you.

This experiment is divided into two parts. We will explain more about each part before these begin. In both parts, you will be presented with several tasks. In each part, one task will be for real.

What do we mean by real?

OK

Screen 2: Pairing of subjects: CD and HS sessions

You have been randomly paired with another person in this room. You will never be told who you have been paired with.

At the end of the experiment, the computer will randomly pick one task for each part of the experiment and you will be paid according to the choices that you and the other person made in those tasks. The tasks picked by the computer are the ones we call 'real'.

Because you will not know which tasks will be real until the end of the experiment, <u>you should treat each task</u> as if it was real. So, when thinking about each task, remember that it could be a real one and think about it in isolation from the others.

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Screen 3a: Tasks and incentives: CD sessions

Now, we will explain the kinds of task you will face. In each task, you and the other person will be shown a row with the same boxes in the same order.
You be asked to choose one (and only one) box. You do this by clicking on it.
Each real task is worth £5.
ОК

Screen 3b: Tasks and incentives: HS sessions

Now, we will explain the kinds of task you will face. In each task, you and the other person will be shown a row with the same boxes in the same order.

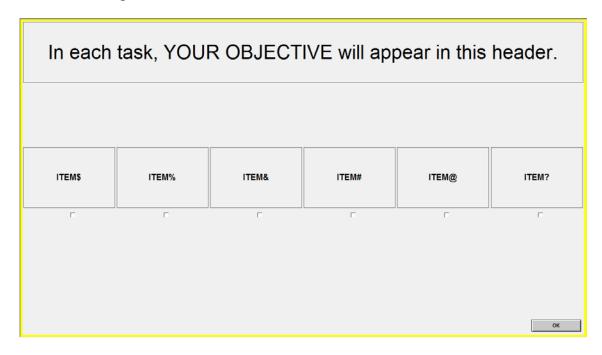
You will be assigned to one of two roles: you will be either HIDER or SEEKER. HIDER's task is to hide a treasure behind one of the boxes. SEEKER's task is to guess the location of the treasure. HIDER chooses one (and only one) box to hide the treasure behind. SEEKER chooses one (and only one) box as his/her guess about where the treasure is hidden.

If SEEKER's guess is correct, SEEKER has found the treasure and keeps it. If SEEKER's guess is not correct, HIDER keeps the treasure.

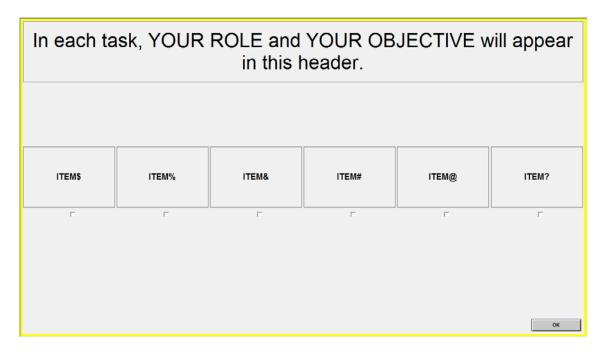
In each real task, the treasure is worth £10. The person who has kept the treasure will be paid £10 at the end of the experiment.

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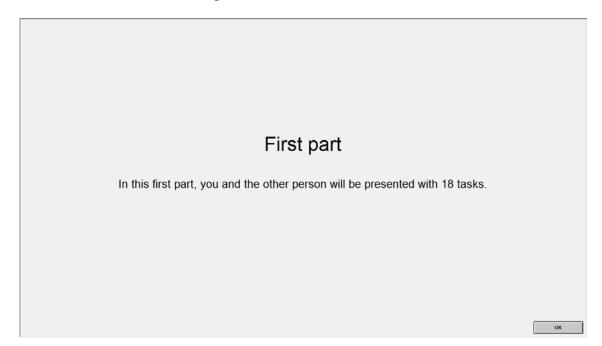
Screen 4a: Sample of decision screen (in instructions): CD sessions



Screen 4b: Sample of decision screen (in instructions): HS sessions



Screen 5: Introduction to first part: CD and HS sessions



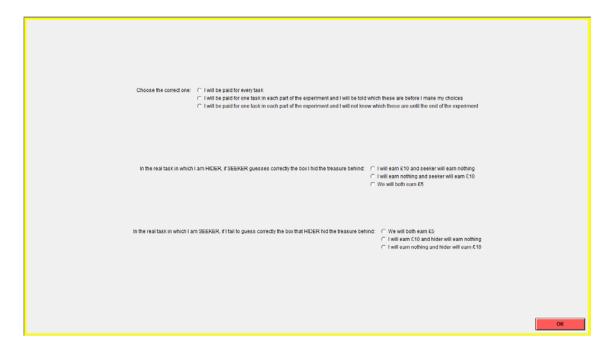
Screen 6a: Instructions for Coordination games

In each task, VOII and the other person have to change the SAME have You show which have you should	
In each task, YOU and the other person have to choose the SAME box. You show which box you choose b clicking on it. You must choose one (and only one) box.	у
If you and the other person choose the SAME box, each of you will earn £5 (if this is the real task). If you an the other person do NOT choose the same box, you will earn NOTHING.	d
Remember that the real task will be picked by the computer at the end of the experiment.	
ОК	
Screen 6b: Instructions for Discoordination games	
In each task, YOU and the other person have to choose DIFFERENT boxes. You show which box you choose by clicking on it. You must choose one (and only one) box.	se
If you and the other person choose DIFFERENT boxes, each of you will earn £5 (if this is the real task). If you and the other person do NOT choose different boxes, you will earn NOTHING.	u
Remember that the real task will be picked by the computer at the end of the experiment.	
ОК	

Screen 7a: Quiz: CD sessions



Screen 7b: Quiz: HS sessions



Screen 8a: Role assignment and reminder: hider in Hide and Seek games

For this part of the experiment you are HIDER.

In each task, YOUR aim is to hide the treasure behind one of the boxes. You show which box you want to hide the treasure behind by clicking on it. You must choose one (and only one) box.

The other person's aim is to guess where YOU hid the treasure. If he/she finds the treasure, he/she will keep it, and so YOU WILL NOT keep it. If he/she does not find the treasure, YOU WILL keep it, and so the other person will not keep it.

Remember that at the end of the experiment one of these tasks will be picked by the computer and the person keeping the treasure in that task will receive £10.

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Screen 8b: Role assignment and reminder: seeker in Hide and Seek games

For this part of the experiment you are SEEKER.

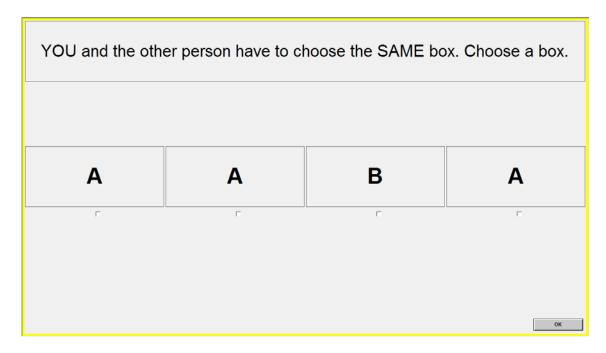
In each task, YOUR aim is to guess the location of the treasure. You show which box you guess the treasure is hidden behind by clicking on it. You must choose one (and only one) box.

The other person's aim is to hide the treasure. If YOU find the treasure, YOU WILL keep it, and so the other person will not keep it. If YOU do not find the treasure, the other person will keep it, and so YOU WILL NOT keep it.

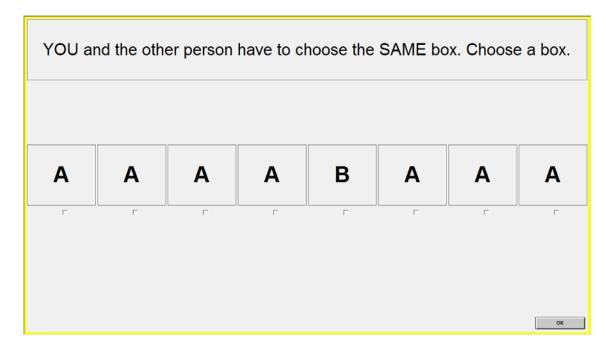
Remember that at the end of the experiment one of these tasks will be picked by the computer and the person keeping the treasure in that task will receive £10.

OK

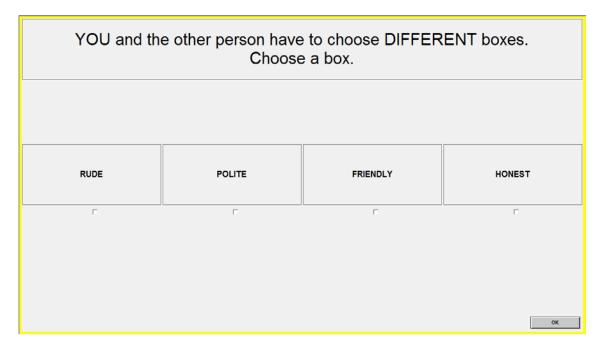
Screen 9a: Typical decision screen for four-box Coordination game (frame 4a)



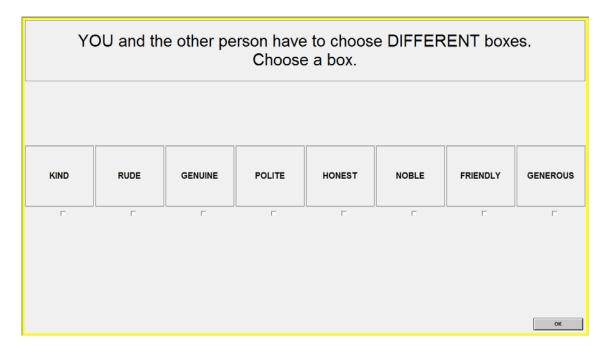
Screen 9b: Typical decision screen for eight-box Coordination game (frame 4b)



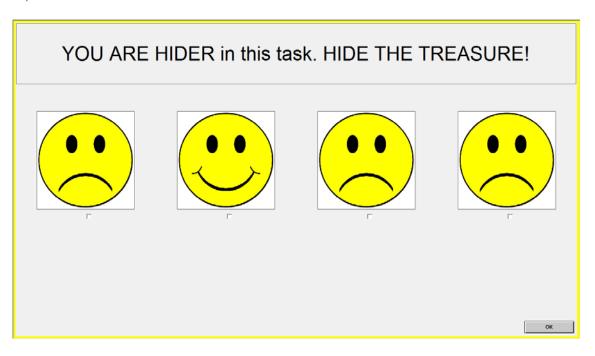
Screen 9c: Typical decision screen for four-box Discoordination game (frame 2b)



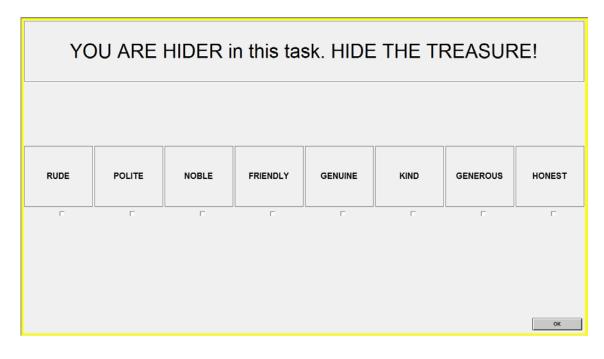
Screen 9d: Typical decision screen for eight-box Discoordination game (frame 2b)



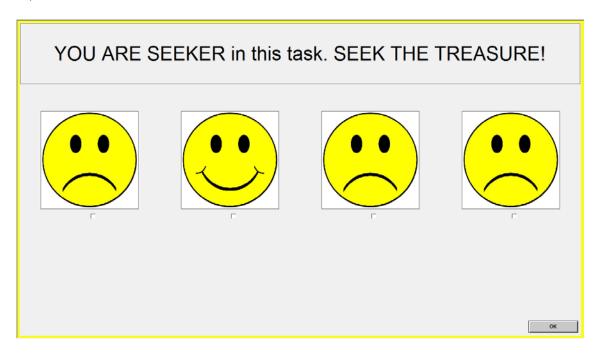
Screen 9e: Typical decision screen for hider in four-box Hide and Seek game (frame 1a)



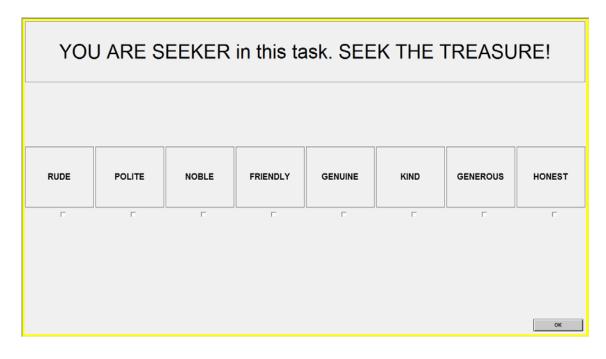
Screen 9f: Typical decision screen for hider in eight-box Hide and Seek game (frame 2b)

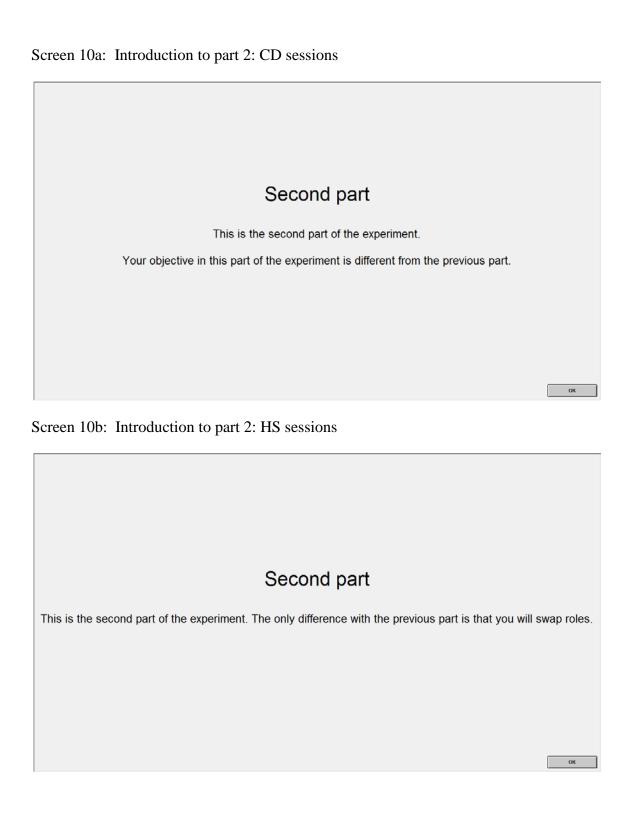


Screen 9g: Typical decision screen for seeker in four-box Hide and Seek game (frame 1a)



Screen 9h: Typical decision screen for seeker in eight-box Hide and Seek game (frame 2b)





Screen 11a: Reminder of instruction in the new role: hider

In the previous part of the experiment, you were hider. NOW you are SEEKER.

In each task, YOUR aim is to guess the location of the treasure. You show which box you guess the treasure is hidden behind by clicking on it. You must choose one (and only one) box.

The other person's aim is to hide the treasure. If YOU find the treasure, YOU WILL keep it, and so the other person will not keep it. If YOU do not find the treasure, the other person will keep it, and so YOU WILL NOT keep it.

Remember that at the end of the experiment one of these tasks will be picked by the computer and the person keeping the treasure in that task will receive £10.

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Screen 11b: Reminder of instruction in the new role: seeker

In the previous part of the experiment, you were seeker. NOW you are HIDER.

In each task, YOUR aim is to hide the treasure behind one of the boxes. You show which box you want to hide the treasure behind by clicking on it. You must choose one (and only one) box.

The other person's aim is to guess where YOU hid the treasure. If he/she finds the treasure, he/she will keep it, and so YOU WILL NOT keep it. If he/she does not find the treasure, YOU WILL keep it, and so the other person will not keep it.

Remember that at the end of the experiment one of these tasks will be picked by the computer and the person keeping the treasure in that task will receive £10.

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Note: instruction screens for Coordination and Discoordination games in the second part of CD sessions are the same as screens 6a and 6b respectively.

Screen 12: Typical outcome screen: CD and HS sessions

RESULTS AND PAYMENTS							
In the first randomly picked question, you were a hider and your choice is shown below.	The other person's choice is shown below.						
In the second randomly picked question, you were a seeker and your choice is shown below.	The other person's choice is shown below.						
FRIENDLY	RUDE						
ОК							

Note: the example shown above is for an HS session. In CD sessions, the screen makes no reference to roles.

Screen 13: typical payment screen: CD and HS sessions



III. Additional analysis of position effects

In Section 3.1 of the main paper, we claimed that position was not a major determinant of players' choices. In support of this claim, we reported the frequencies with which subjects' chosen boxes were in each of the four or eight possible positions (Table 2 in the main paper). As a further robustness check, we consider whether the position of the oddity influenced the likelihood of its being chosen. Table A2 reports the relative frequency with which the oddity was chosen, conditional on its position. Under the null hypothesis that choices are not influenced by position, the expected relative frequency of oddity choices is the same for all oddity positions. For each combination of player role and type of game, Table 4 in the main paper reports 95 per cent confidence intervals for the probability of oddity choices, derived from the aggregate data. Cases in which, for a specific position, the observed relative frequency of oddity choices lies outside these confidence intervals are shown in bold in Table A2. As with the data reported in Table 2, deviations from randomness are relatively small and show no obvious pattern.

TABLE A2: RELATIVE FREQUENCY OF ODDITY CHOICES BY POSITION OF ODDITY

	Percentage of oddity choices when oddity is in position:								
role	1	2	3	4	5	6	7	8	Avg.
four- box games									
coordinators $(n = 40)$	76.4	73.5	77.7	76.0					76.0
discoordinators ($n = 40$)	23.2	19.1	23.0	28.3					23.3
hiders (part 1: $n = 100$)	10.1	15.3	17.7	15.5					14.6
hiders (part 2: $n = 100$)	17.9	18.1	20.7	15.8					18.2
seekers (part 1: $n = 100$)	24.1	30.7	27.9	21.0					25.8
seekers (part 2: $n = 100$)	22.3	20.5	22.0	18.1					20.8
eight- box games									
coordinators ($n = 40$)	73.0	81.7	72.4	80.6	81.2	66.7	78.3	73.1	76.0
discoordinators ($n = 40$)	11.5	15.3	24.4	18.1	9.4	11.0	19.3	12.1	15.4
hiders (part 1: $n = 100$)	8.3	8.8	9.3	9.3	5.7	6.9	7.6	12.8	8.6
hiders (part 2: $n = 100$)	14.2	11.3	8.2	7.9	11.2	11.1	8.5	6.5	9.8
seekers (part 1: $n = 100$)	25.7	31.0	34.6	22.9	22.0	24.5	19.3	23.9	25.3
seekers (part 2: $n = 100$)	17.0	16.1	23.5	17.5	20.8	16.2	8.5	11.4	16.1

IV. Analysis restricted to games with neutral connotations

One difference between our analysis and CI's is that CI restrict their analysis to Hide and Seek games in which the oddity had neutral connotations, while our experiment also included games in which those connotations were positive or negative. In this Section, we consider whether our results would be affected if we restricted our analysis to neutrally framed games (i.e. frames 4a, 4b, 5a, 5b, 6a, 6b, 13a, 13b, 14a, 14b, 15a and 15b). Tables A3 and A4 reproduce the analyses in Tables 2 and 4 respectively of the main paper, but using only data from neutral frames.

Table A3 shows that, for each class of games and for each role, each position was chosen with approximately the same frequency; there are a few instances of statistically significant differences from the random-choice benchmark, but these do not seem to follow any obvious pattern. Thus, the conclusion we drew from Table 2 – that position effects are relatively unimportant – is not an artifact of the pooling of data across games in which oddities have different connotations.

Table A4 reports the frequency of oddity choices by players in each role. The observed frequencies are generally very similar to those in Table 2, but because they are derived from fewer game, the confidence intervals are wider. The restricted data, just like the unrestricted, clearly disconfirm Implication 1 (for each of the three roles of discoordinator, hider and seeker, and for both four- and eight-box games, the upper bound of the 95 per cent confidence interval of the frequency of oddity choices is less than 0.33). As with the unrestricted data, the observed proportion of oddity choices by hiders is significantly higher in four-box games than in eight-box games, contrary to Implication 2, but there are no significant differences for coordinators (as in the unrestricted data), for seekers (as in the unrestricted data), or for discoordinators. In the light of these results, we see no reason to think that the main conclusions of our paper are attributable to our use of frames with positive and negative connotations.

TABLE A3: FREQUENCY OF CHOICES BY POSITION (NEUTRAL FRAMES ONLY)

percentage of choices that are of position:

			C					
role	1	2	3	4	5	6	7	8
four- box games								
coordinators $(n = 40)$	31.2 *	23.7	25.8	19.2				
discoordinators $(n = 40)$	26.7 **	21.7	28.0	23.7				
hiders (part 1: $n = 100$)	24.3	20.0	25.3	30.3**				
hiders (part 2: $n = 100$)	25.0	23.7*	26.0	25.3				
seekers (part 1: $n = 100$)	21.3	30.3	27.0	21.3				
seekers (part 2: $n = 100$)	23.3	24.0	33.7***	19.0*				
eight- box games								
coordinators $(n = 40)$	12.1	15.0	5.8***	11.2	18.3**	9.6	17.9*	10.0*
discoordinators $(n = 40)$	9.6	12.9	13.7	13.3	14.6	12.1	12.5	11.2
hiders (part 1: $n = 100$)	10.7	16.7 *	11.7	10.3	12.0	14.0	14.7	10.0
hiders (part 2: $n = 100$)	12.7	14.0	13.0	12.0	12.3	17.3**	10.3	8.3*
seekers (part 1: $n = 100$)	9.3	12.7	14.7	14.3	15.0	12.0	10.3	11.7
seekers (part 2: $n = 100$)	11.3	10.7	15.0	11.3	18.0**	11.3	12.0***	10.3

Note: n denotes the number of subjects who faced games of the relevant type. Each coordinator and discoordinator faced 6 games of that type; each hider and seeker faced 3 games of that type. Relative frequencies greater than the random-choice benchmark (25 per cent for four-box games, 12.5 per cent for eight-box games) are shown in bold. For each type of game and each position j, we find the number of choices made in position j by each subject i, and then run a chi-squared test of whether the distribution of these n numbers is different from the binomial distribution implied by random choice. Significant differences are shown by asterisks (*, ** and *** denoting significance at the 10, 5 and 1 per cent levels).

TABLE A4: FREQUENCY OF ODDITY CHOICES BY PLAYERS IN ALL ROLES (NEUTRAL FRAMES ONLY)

percentage of all choices that are of oddity:

role	lower bound	observed	upper bound
four-box games			
coordinators $(n = 40)$	72.0	80.8	89.7
discoordinators ($n = 40$)	13.2	20.8	28.4
hiders (part 1: $n = 100$)	10.3	15.0	19.7
hiders (part 2: $n = 100$)	13.7	19.0	24.3
seekers (part 1: $n = 100$)	17.1	22.7	28.2
seekers (part 2: $n = 100$)	14.4	19.7	24.9
eight-box games			
coordinators ($n = 40$)	76.6	84.2	91.7
discoordinators ($n = 40$)	11.1	18.7	26.4
hiders (part 1: $n = 100$)	3.9	8.0	12.0
hiders (part 2: $n = 100$)	3.6	7.3	11.0
seekers (part 1: $n = 100$)	17.3	22.3	27.4
seekers (part 2: <i>n</i> = 100)	10.2	15.3	20.4

Note: *n* denotes the number of subjects who faced games of the relevant type. Each coordinator and discoordinator faced 6 games of that type. Each hider and seeker faced 3 games of that type.

V. Patterns in Hide and Seek data consistent with 'plausible' level-k models

This Section uses the same notation and assumptions as are used in Sections 1 and 4 of the main paper.

Consider a large population of potential players of the ABAA game. Let ρ_B^H and ρ_C^H be the probabilities with which hiders choose B and central A respectively. We define $\rho_E^H = (1 - \rho_B^H - \rho_C^H)/2$, i.e. the average probability with which hiders choose end labels. Ignoring ties for simplicity, we define the *modal choice* of hiders m^H so that it takes the value B, C or E according to which of ρ_B^H , ρ_C^H and ρ_E^H is greatest. The modal choice of seekers m^S is defined analogously. We define a *pattern* as a triple (m^H , m^S , z) where z takes the value H if $m^H > m^S$ and S if $m^S > m^H$. There are eighteen possible patterns. One of these, namely (C, C, S), is the fatal attraction pattern.

Our representation of CI's concept of a *plausible* level-*k* model is explained in Section 4 of the main paper. We treat the following assumptions about the population distribution of types as necessary properties of a plausible model:

- (1) $\pi_1, \pi_2, \pi_3, \pi_4 \geq 0$
- (2) $\pi_1 + \pi_2 + \pi_3 + \pi_4 = 1$
- (3) $\pi_2 > \pi_1$
- (4) $\pi_3 > \pi_4$.

Two alternative assumptions about salience are treated as plausible: *either* B is most salient (Specification 1), *or* the end As are most salient (Specification 2). We consider these in turn.

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Specification 1: B is most salient

Level-*k* theory implies the following choice probabilities:

(5)
$$\rho_B^H = \pi_3/3 + \pi_4$$

(6)
$$\rho_C^H = \pi_1 + \pi_2/3$$

(7)
$$\rho_E^H = \pi_2/3 + \pi_3/3$$

(8)
$$\rho_B^{\ S} = \pi_1 + \pi_4/3$$

(9)
$$\rho_C^{\ S} = \pi_2 + \pi_3/3$$

(10)
$$\rho_E^{S} = \pi_3/3 + \pi_4/3$$

Expressions (1) to (10) imply the following results:

Result 1(i): Not [$m_S = B$]. *Proof*: Suppose $m_S = B$. Then $\rho_B{}^S > \rho_C{}^S$ which, by (8) and (9), implies $\pi_1 + \pi_4/3 > \pi_2 + \pi_3/3$. But by (3) and (4), $\pi_1 + \pi_4/3 < \pi_2 + \pi_3/3$, a contradiction.

Result I(ii): Not $[m_H = C \text{ and } m_S = E]$. Proof: Suppose $m_H = C \text{ and } m_S = E$. Then $\rho_C^H > \rho_E^H$ which, by (6) and (7) implies $\pi_3/3 < \pi_1$. But also $\rho_E^S > \rho_B^S$ which, by (8) and (10), implies $\pi_1 < \pi_3/3$, a contradiction.

Result I(iii): Not $[m_H = E \text{ and } m_S = E]$. Proof: Suppose $m_H = E \text{ and } m_S = E$. Then $\rho_E^H > \rho_B^H$ which, by (5) and (7), implies $\pi_2/3 > \pi_4$. But also $\rho_E^S > \rho_C^S$ which, by (9) and (10), implies $\pi_4/3 > \pi_2$. Adding these inequalities gives $(\pi_2 + \pi_4)/3 > \pi_2 + \pi_4$. Given (1), this is a contradiction.

Result I(iv): Not $[m_S = E \text{ and } z = S]$. Proof: suppose $[m_S = E \text{ and } z = S]$. Then $\rho_E^S > \rho_B^H$ which, by (5) and (10), implies $\pi_4/3 > \pi_4$. Given (1), this is a contradiction.

Result I(v): Not $[m_H = E \text{ and } z = H]$. Proof: suppose $[m_H = E \text{ and } z = H]$. Then $\rho_E^H > \rho_C^S$ which, by (7) and (9), this implies $\pi_2/3 > \pi_2$. Given (1), this is a contradiction.

Of the eighteen possible patterns, only six are consistent with Results 1(i) to 1(v). These are listed below. For each pattern, we give an example of a 'plausible' distribution of types that induces that pattern.

Pattern 1: (B, C, H). Example: $\pi_1 = 0.05$, $\pi_2 = 0.15$, $\pi_3 = 0.51$, $\pi_4 = 0.29$.

Pattern 2: (B, C, S). Example: $\pi_1 = 0.05$, $\pi_2 = 0.36$, $\pi_3 = 0.30$, $\pi_4 = 0.29$.

Pattern 3: (B, E, H). Example: $\pi_1 = 0.04$, $\pi_2 = 0.06$, $\pi_3 = 0.50$, $\pi_4 = 0.40$.

Pattern 4: (*C*, *C*, *H*). Example: $\pi_1 = 0.35$, $\pi_2 = 0.40$, $\pi_3 = 0.15$, $\pi_4 = 0.10$.

Pattern 5: (*C*, *C*, *S*). Example: $\pi_1 = 0.20$, $\pi_2 = 0.40$, $\pi_3 = 0.30$, $\pi_4 = 0.10$.

Pattern 6: (*E*, *C*, *S*). Example: $\pi_1 = 0.10$, $\pi_2 = 0.13$, $\pi_3 = 0.73$, $\pi_4 = 0.04$.

Specification 2: End As are most salient

Level-*k* theory implies the following choice probabilities:

(11)
$$\rho_B^H = \pi_2/2 + \pi_3/3$$

(12)
$$\rho_C^H = \pi_1 + \pi_2/2$$

(13)
$$\rho_E^H = \pi_3/3 + \pi_4/2$$

(14)
$$\rho_B^S = \pi_3/2 + \pi_4/3$$

(15)
$$\rho_C^S = \pi_2 + \pi_3/2$$

(16)
$$\rho_E^S = \pi_1/2 + \pi_4/3$$

Expressions (1) to (4) and (11) to (16) imply the following results:

Result 2(i): Not $[m_S = E]$. Proof: suppose $m_S = E$. Then $\rho_E^S > \rho_C^S$ which, by (15) and (16), implies $\pi_1/2 + \pi_4/3 > \pi_2 + \pi_3/2$. But by (1), (3) and (4), $0 \le \pi_1 < \pi_2$ and $0 \le \pi_4 < \pi_3$. Thus $\pi_1/2 < \pi_2$ and $\pi_4/3 < \pi_3/2$. Summing these inequalities gives $\pi_1/2 + \pi_4/3 < \pi_2 + \pi_3/2$, a contradiction.

Result 2(ii): Not $[m_H = B \text{ and } m_S = B]$. Proof: Suppose $m_H = B \text{ and } m_S = B$. Then $\rho_B^H > \rho_E^H$ which, by (11) and (13), implies $\pi_2 > \pi_4$. But also $\rho_B^S > \rho_C^S$ which, by (14) and (15), implies $\pi_4/3 > \pi_2$. Given (1), this implies $\pi_4 > \pi_2$, a contradiction.

Result 2(iii): Not $[m_H = C \text{ and } m_S = B]$. Proof: Suppose $m_H = C \text{ and } m_S = B$. Then $\rho_C^H > \rho_B^H$ which, by (11) and (12), implies $\pi_1 > \pi_3/3$. But also $\rho_B^S > \rho_C^S$ which, by (14) and (15), implies $\pi_4/3 > \pi_2$. Adding these inequalities gives $\pi_1 + \pi_4/3 > \pi_3/3 + \pi_2$. But by (3) and (4), $\pi_1 < \pi_2$ and $\pi_4 < \pi_3$, a contradiction.

Result 2(iv): Not $[m_H = E \text{ and } z = H]$. Proof: suppose $[m_H = E \text{ and } z = H]$. Then $\rho_E^H > \rho_B^S$ which, by (13) and (14), implies $\pi_4 > \pi_3$. Given (4), this is a contradiction.

Result 2(v): Not $[m_H = B \text{ and } m_S = C \text{ and } z = H]$. Proof: suppose $m_H = B \text{ and } m_S = C$ and z = B. Then $B_H > C_S$ which, by (11) and (15), implies $\pi_2/2 + \pi_3/6 < 0$, contrary to (1): a contradiction.

Of the eighteen possible patterns, only five are consistent with Results 2(i) to 2(v). These are listed below. For the one pattern that is not also consistent with Case 1, we give an example of a 'plausible' distribution of types that induces that pattern:

Pattern 2: (B, C, S). Also consistent with Case 1.

Pattern 4: (C, C, H). Also consistent with Case 1.

Pattern 5: (*C*, *C*, *S*). Also consistent with Case 1.

Pattern 6: (E, C, S). Also consistent with Case 1.

Pattern 7: (E, B, S). Example: $\pi_1 = 0.04$, $\pi_2 = 0.06$, $\pi_3 = 0.50$, $\pi_4 = 0.40$.

We have now established that exactly seven of the eighteen possible patterns are consistent with 'plausible' level-*k* models.