SUPPLEMENT TO "GEOGRAPHY, TRANSPORTATION, AND ENDOGENOUS TRADE COSTS" (Econometrica, Vol. 88, No. 2, March 2020, 657–691)

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1. CONSTRUCTION OF SHIP TRAVEL HISTORIES AND SEARCHING SHIPS

HERE, we describe the construction of ships' travel histories. The first task is to identify stops that ships make using the EE data. A stop is defined as an interval of at least 24 hours, during which (i) the average speed of the ship is below 5 mph (the sailing speed is between 15 and 20 mph), and (ii) the ship is located within 250 miles from the coast. A trip is the travel between two stops.

The second task is to identify whether a trip is loaded or ballast. To do so, we use the ship's draft: high draft indicates that a larger portion of the hull is submerged and therefore the ship is loaded. The distribution of draft for a given vessel is roughly bimodal, since, as described in Section 2, a hired ship is usually fully loaded. Therefore, we can assign a "high" and a "low" draft level for each ship and consider a trip loaded if the draft is high (in practice, the low draft is equal to 70% of the high draft). As not all satellite signals contain the draft information, we consider a trip ballast (loaded) if we observe a signal of low (high) draft during the period that the ship is sailing. If we have no draft information during the sailing time, we consider the draft at adjacent stops. Finally, we exclude stops longer than six weeks, as such stops may be related to maintenance or repairs.

The third and final task is to refine the origin and destination information provided in the Clarksons contracts. Although the majority of Clarksons contracts provide some information on the origin and destination of the trip, this is often vague (e.g., "Far East," "Japan–S. Korea–Singapore"), especially in the destinations. We use the EE data to refine the contracted trips' origins and destinations by matching each Clarksons contract to the identified stop in EE that is closest in time and, when possible, location. In particular, we use the loading date annotated on each contract to find a stop in the ship's movement history that corresponds to the beginning of the contract. For destinations where information in Clarksons is noisy, we search the ship's history for a stop that we can classify as the end of the contract. In particular, we consider all stops within a three-month window (duration of the longest trip) since the beginning of the contract. Among these stops, we

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eliminate all those that (i) are in the same country in which the ship loaded the cargo and (ii) are in Panama, South Africa, Gibraltar, or at Suez and in which the draft of arrival is the same as the draft of departure (to exclude cases in which the ship is waiting to pass through a strait or a canal). To select the end of the contract among the remaining options, we consider the following possibilities:

1. If the contract reports a destination country and if there are stops in this country, select the first of these stops as the end of the trip;

2. If the destination country is "Japan–S. Korea–Singapore," and if there are stops in either Japan, China, Korea, Taiwan, or Singapore, we select the first among these as the end of the trip;

3. If the contract does not report a destination country and there are stops in which the ship arrives full and leaves empty, we select the first of these as the end of the trip.

We check the performance of the algorithm by comparing the duration of some frequent trips, with distances found online (at https://sea-distances.org), and find that durations are well matched.

Next, we turn to the construction of searching ships $s_t = [s_{t1}, \ldots, s_{tI}]$ and matches $m_t = [m_{t1}, \ldots, m_{tI}]$, where s_{it} denotes the number of ships in region *i* and week *t* that are available to transport a cargo and m_{it} denotes the realized matches in region *i* and week *t*. To construct s_{it} , we consider all ships that ended a trip (loaded or ballast) in region *i* and week t - 1. We exclude the first week post arrival in the region to account for loading/unloading times (on average, (un)loading takes 3–4 days but the variance is large; removing one week will tend to underestimate port wait times). To construct m_{it} , we consider the number of ships that began a loaded trip from region *i* in week *t*.

2. ADDITIONAL FIGURES AND TABLES



FIGURE S1.—Definition of regions. Each color depicts one of the 15 geographical regions.



Baseline
 Poisson

FIGURE S2.—Recovered exporters in our baseline specification and under a Poisson distributional assumption.

		log(price per day)			
	Ι	II	III	IV	
$I{\text{orig.} = \text{home country}}$			0.004		
			(0.019)		
$I{\text{dest.} = \text{home country}}$			-0.012		
			(0.015)		
log(Number employees)				0.008	
				(0.007)	
log(Operating revenues)				0.003	
				(0.005)	
Time FE	$Qtr \times Yr$	$Qtr \times Yr$	$Qtr \times Yr$	Qtr imes Yr	
Shipowner FE	No	Yes	No	No	
Ship characteristics	Yes	Yes	Yes	Yes	
Region FE	Orig.	Orig.	Orig.	Orig.	
	& Dest.	& Dest.	& Dest.	& Dest.	
Observations	7263	7263	7973	7973	
Adj. R^2	0.530	0.540	0.537	0.537	

TABLE SI

REGRESSION OF SHIPPING PRICES ON SHIPOWNER CHARACTERISTICS AND FIXED EFFECTS^a

^aShipping prices, ships' characteristics (age and size), and the identity of the shipowner are obtained from Clarksons. Information on shipowner characteristics is obtained from ORBIS. In particular, we match the shipowners in Clarksons to ORBIS; we do so for two reasons: (i) ORBIS allows us to have reliable firm identities, as shipowners may appear under different names in the contract data; (ii) ORBIS reports additional firm characteristics (e.g., number of employees, revenue, headquarters). Here, we identify the shipowner with the global ultimate owner (GUO); results are robust to controlling for the identity of the domestic owner (DUO) and the shipowner as reported in Clarksons. Finally, the data used span the period 2010–2016.



FIGURE S3.—Ballast discrete choice model fit. The left panel depicts the observed and predicted probabilities of staying at port (P_{ii}) for all regions *i*. The right panel depicts the observed and predicted probabilities of ballasting (P_{ij}) to all regions $i \neq j$.

3. STEADY-STATE EXISTENCE

PROPOSITION 1: Suppose that the matching function is continuous, ϵ and ϵ^e have full support, \mathcal{E}_i and S are finite, and $e_i \leq \mathcal{E}_i/(1-\delta)$. Then, a steady state exists, that is, there exist (s^*, e^*) that satisfy equations (11) through (13).

PROOF: We first derive equations (11) and (12). Consider the model's state transitions. Exporters in region i at time t transition as follows:

$$e_{it+1} = \delta(e_{it} - m_i(s_{it}, e_{it})) + \mathcal{E}_i(1 - P_{i0}^e),$$
(S1)

with $\mathcal{E}_i(1 - P_{i0}^e)$ the (endogenous) flow of new freights. Ships at location *i* transition as follows:

$$s_{it+1} = (s_{it} - m_i(s_{it}, e_{it}))P_{ii} + \sum_{j \neq i} d_{ji}s_{jit}.$$
(S2)

In words, out of s_{it} ships, m_{it} ships get matched and leave *i*, while out of the ships that did not find a match, fraction P_{ii} chooses to remain at *i* rather than ballast away; moreover, out of the ships traveling towards *i*, fraction d_{ji} arrive. Finally, ships that are traveling from *i* to *j*, s_{ijt} , evolve as follows:

$$s_{ijt+1} = (1 - d_{ij})s_{ijt} + P_{ij}(s_{it} - m_i(s_{it}, {}_{it})) + \frac{P_{ij}^e}{1 - P_{i0}^e}m_i(s_{it}, e_{it}).$$
(S3)

In words, fraction d_{ij} of the traveling ships arrive, fraction P_{ij} of ships that remained unmatched in location *i* chose to ballast to *j*, and finally, $P_{ij}^e/(1 - P_{i0}^e)$ of ships matched in *i* depart loaded to *j*.

Suppose s_{ijt}, e_{it} approach s_{ij}, e_i as $t \to \infty$. Then (S2) becomes

$$s_i = (s_i - m_i(s_i, e_i))P_{ii} + \sum_{j \neq i} d_{ji}s_{ji},$$
 (S4)

while for ships traveling from j to i, (S3) becomes

$$s_{ji} = (1 - d_{ji})s_{ji} + P_{ji}s_j + \left(\frac{P_{ji}^e}{1 - P_{j0}^e} - P_{ji}\right)m_j(s_j, e_j)$$
(S5)

or

$$d_{ji}s_{ji} = P_{ji}s_j + \left(\frac{P_{ji}^e}{1 - P_{j0}^e} - P_{ji}\right)m_j = P_{ji}(s_j - m_j) + \frac{P_{ji}^e}{1 - P_{j0}^e}m_j,$$

where $m_i = m_i(s_i, e_i)$. Summing this with respect to $j \neq i$, we obtain

$$\sum_{j \neq i} d_{ji} s_{ji} = \sum_{j \neq i} P_{ji} (s_j - m_j) + \sum_{j \neq i} \frac{P_{ji}^e}{1 - P_{j0}^e} m_j,$$

and replacing in (S4), we get (11).

Equation (12) is a direct consequence of (S1).

The steady-state equations (11) and (12) have a fixed point over a properly defined subset of \mathbb{R}^{2I} , by the Leray–Schauder–Tychonoff theorem (Bertsekas and Tsitsiklis (2015)), which states that if X is a nonempty, convex, and compact subset of \mathbb{R}^{2I} and $h: X \to X$ is continuous, then h has a fixed point. Indeed, let $h: \mathbb{R}^{2I} \to \mathbb{R}^{2I}$, $h = (h^s, h^e)$ with

$$h_i^s(s, e) = \sum_{j=1}^{I} P_{ji}(s, e) (s_j - m_j(s_j, e_j)) + \sum_{j \neq i} \frac{P_{ji}^e}{1 - P_{j0}^e} m_j(s, e),$$

$$h_i^e(s, e) = \delta (e_i - m_i(s_i, e_i)) + \mathcal{E}_i \sum_{j \neq 0, i} P_{ij}^e(s, e),$$

for i = 1, ..., I. Let $X = \prod_{i=1}^{I} [0, \mathcal{E}_i/(1-\delta)] \times \Delta s$, where $\Delta s = \{s_i \ge 0 : \sum_{i=1}^{I} s_i \le S\}$. *X* is nonempty, convex, and compact, while *h* is continuous on *X*. We assume that the matching function is such that λ , λ^e are zero at the origin and continuous. It remains to show that $F(X) \subseteq X$. Let $(s, e) \in X$. Then, $e_i \le \mathcal{E}_i/(1-\delta)$ and $\sum_{i=1}^{I} s_i \le S$. Now,

$$h_{i}^{s}(s,e) = \sum_{j=1}^{I} P_{ji}(s,e) \left(s_{j} - \lambda_{j}(s_{j},e_{j})s_{j} \right) + \sum_{j \neq i} \frac{P_{ji}^{e}}{1 - P_{j0}^{e}} \lambda_{j}(s,e)s_{j}$$

or

$$h_{i}^{s}(s,e) = \sum_{j=1}^{I} s_{j} \bigg[P_{ji}(s,e) \big(1 - \lambda_{j}(s_{j},e_{j}) \big) + \frac{P_{ji}^{e}}{1 - P_{j0}^{e}} \lambda_{j}(s,e) \bigg],$$

where let $P_{ii}^e = 0$ (no inter-region trips). Summing over *i* gives

$$\sum_{i=1}^{I} h_i^s(s, e) = \sum_{j=1}^{I} s_j \left[\sum_{i=1}^{I} P_{ji}(s, e) \left(1 - \lambda_j(s_j, e_j) \right) + \sum_{i=1}^{I} \frac{P_{ji}^e}{1 - P_{j0}^e} \lambda_j(s, e) \right]$$

or

$$\sum_{i=1}^{I} h_i^s(s,e) = \sum_{j=1}^{I} s_j \big[1 - \lambda_j(s_j,e_j) + \lambda_j(s,e) \big] \le S.$$

Hence, $h_i^s(s, e) \in \Delta s$.

Finally, consider h^e ; since $m_i \ge 0$, we have

$$h_{i}^{e} \leq \delta e_{i} + \mathcal{E}_{i} \sum_{j \neq 0, i} P_{ij}^{e}(s, e) \leq \delta e_{i} + \mathcal{E}_{i} \leq \delta \frac{\mathcal{E}_{i}}{1 - \delta} + \mathcal{E}_{i} = \frac{\mathcal{E}_{i}}{1 - \delta}.$$

$$(i) \in [0, \mathcal{E}_{i}/(1 - \delta)].$$

$$O.E.D.$$

Hence, $h_i^e(s, e) \in [0, \mathcal{E}_i/(1 - \delta)].$

4. ESTIMATION OF SHIP COSTS

Since our model features a number of inter-related value functions (V, U), it does not fall strictly into the standard Bellman formulation. Hence, we provide Lemma S1, which proves that our problem is characterized by a contraction map and thus the value functions are well defined.

LEMMA S1: For each value of the parameter vector $\theta \equiv \{c_{ij}^s, c_i^w, \sigma\}$ all i, j, the map T_{θ} : $\mathbb{R}^I \to \mathbb{R}^I, V \to T_{\theta}(V)$ with

$$T_{\theta}(V)_{i} = -c_{i}^{w} + \lambda_{i} \sum_{j \neq i} G_{ij} \tau_{ij} + \lambda_{i} \sum_{j \neq i} G_{ij} \left[-\frac{c_{ij}^{s}}{1 - \beta(1 - d_{ij})} + \beta d_{ij} \frac{V_{j}}{1 - \beta(1 - d_{ij})} \right] + (1 - \lambda_{i}) U_{i}(\theta, V),$$

where $\tau_{ij} \equiv E_r \tau_{ijr}$ is the mean price from *i* to *j* and $G_{ij} = \frac{P_{ij}^e}{1 - P_{i0}^e}$, is a contraction and $V(\theta)$ is the unique fixed point.

PROOF: Fix θ . Let $\phi_{ij} = \frac{1}{1-\beta(1-d_{ij})}$. The map $T_{\theta}(V)$ is differentiable with respect to $V \in \mathbb{R}^{I}$ with Jacobian

$$\frac{\partial T_{\theta}(V)}{\partial V} = \beta \left(DG + (I - D)P \right) \odot Z, \tag{S6}$$

where *D* is a diagonal matrix with λ_i its *i* diagonal entry; *P* is the matrix of choice probabilities, *G* is the matrix of matched trips, *Z* is an $L \times L$ matrix whose (i, j) element is $\phi_{ij}d_{ij}$, and \odot denotes the pointwise product. We next drop θ for notational simplicity; the (i, j) entry of $\frac{\partial T}{\partial V}$ is

$$\left(\frac{\partial T}{\partial V}\right)_{ij} = 1\{i=j\} - \beta \lambda_i G_{ij} d_{ij} \phi_{ij} - (1-\lambda_i) \frac{\partial U_i}{\partial V_j}.$$

Now,

$$\frac{\partial U_i}{\partial V_j} = \frac{1}{e^{\frac{\beta V_i}{\sigma}} + \sum_k e^{\frac{V_{ij}}{\sigma}}} e^{\frac{V_{ij}}{\sigma}} \frac{\partial V_{ij}}{\partial V_j} = \beta P_{ij} d_{ij} \phi_{ij},$$

and thus

$$\left(\frac{\partial T}{\partial V}\right)_{ij} = 1\{i=j\} - \beta \left(\lambda_i G_{ij} + (1-\lambda_i) P_{ij}\right) d_{ij} \phi_{ij},$$

which in matrix form becomes (S6) (as a convention, set $d_{ii} = 1$). Let $H = (DG + (I - D)P) \odot Z$. Take $||H|| = \max_i \sum_j |H_{ij}|$. Note that G, P are stochastic matrices and the diagonal matrix D is positive with entries smaller than 1. Thus, DG + (I - D)P is stochastic. It is also true that $0 < d_{ii}\phi_{ii} \le 1$. Thus,

$$\sum_{j} |H_{ij}| = \sum_{j} \left(\lambda_i G_{ij} + (1 - \lambda_i) P_{ij} \right) d_{ij} \phi_{ij} \le \sum_{j} \left(\lambda_i G_{ij} + (1 - \lambda_i) P_{ij} \right) \le 1$$

and therefore $||H|| \le 1$. We deduce that $\left\|\frac{\partial T_{\theta}(V)}{\partial V}\right\| \le \beta < 1$.

In brief, our estimation algorithm proceeds in the following steps:

1. Guess an initial set of parameters $\{c_{ii}^s, c_i^w, \sigma\}$.

2. Solve for the ship value functions via a fixed point. Set an initial value V^0 . Then at each iteration l and until convergence:

(a) Solve for V_{ij}^l from

$$V_{ij}^{l} = \frac{-c_{ij}^{s} + d_{ij}\beta V_{j}^{l}}{1 - \beta(1 - d_{ij})}.$$

(b) Update U^l from

$$U_i^l = \sigma \log \left(\exp \frac{\beta V_i^l}{\sigma} + \sum_{j \neq i} \exp \frac{V_{ij}^l}{\sigma} \right) + \sigma \gamma^{\text{euler}},$$

where γ^{euler} is the Euler constant.¹ (c) Update V_i^{l+1} from

$$V_{i}^{l+1} = -c_{i}^{w} + \lambda_{i}E_{j,r}\tau_{ijr} + \lambda_{i}\sum_{j\neq i}\frac{P_{ij}^{e}}{1 - P_{i0}^{e}}V_{ij}^{l} + (1 - \lambda_{i})U_{i}^{l},$$

where we use the actual average prices from *i* to *j*, that is, $E_{j,r}\tau_{ijr} = \sum_{j \neq i} \frac{P_{ij}^e}{1-P_{i0}^e} \tau_{ij}$. Note that λ_i is known (it is simply the average ratio $\frac{1}{T} \sum m_{it}/s_{it}$). Similarly, $\frac{P_{ij}^e}{1-P_{ij}^e}$, the probability that an exporter ships from i to j (conditional on exporting), is obtained directly from the observed trade flows (see Section 5.2).

3. Form the likelihood using the choice probabilities:

$$\mathcal{L} = \sum_{i} \sum_{j} \sum_{k} \sum_{t} y_{ijkt} \log P_{ij}(c_{ij}^{s}, c_{i}^{w}, \sigma) = \sum_{i} \sum_{j} \log P_{ij}(c_{ij}^{s}, c_{i}^{w}, \sigma)^{n_{ij}},$$

where y_{ijkt} is an indicator equal to 1 if ship k chose to go from i to j in week t, n_{ij} is the number of observations (ship-weeks) that we observe a ship in i choosing j, and $P_{ij}(c_{ii}^s, c_i^w, \sigma)$ are given by (7) and (8).

Q.E.D.

¹This formula for the ex ante value function $U_i \equiv E_{\epsilon} U_i(\epsilon)$ is the closed form expression for the expectation of the maximum over multiple choices, and is obtained by integrating $U_i(\epsilon)$ over the distribution of ϵ .

5. IDENTIFICATION OF SHIP PORT AND SAILING COSTS

PROPOSITION 2: Given the choice probabilities $P_{ij}(\theta)$, the parameters $\theta = \{\frac{c_{ij}^s}{\sigma}, \frac{c_i^w}{\sigma}, \frac{1}{\sigma}\}$ satisfy a $(I^2 - I) \times (I^2 + 1)$ linear system of equations of full rank $I^2 - I$. Hence, I + 1 additional restrictions are required for identification.

PROOF: Let $\phi_{ij} = \frac{1}{1-\beta(1-d_{ij})}$. The Hotz and Miller (1993) inversion states

$$\sigma \log \frac{P_{ij}}{P_{ii}} = V_{ij}(\theta) - \beta V_i(\theta).$$

Substituting from (1), we obtain

$$\sigma \log \frac{P_{ij}}{P_{ii}} = -\phi_{ij}c_{ij}^s + \beta d_{ij}\phi_{ij}V_j(\theta) - \beta V_i(\theta).$$
(S7)

It also holds that (see Kalouptsidi, Scott, and Souza-Rodrigues (2018))

$$\log P_{ij} = \frac{V_{ij}}{\sigma} - \frac{U_i}{\sigma} + \gamma^{\text{euler}}$$

or

$$\sigma \log P_{ij} = -\phi_{ij}c_{ij}^{s} + \beta d_{ij}\phi_{ij}V_{j}(\theta) - U_{i} + \sigma\gamma^{\text{euler}}$$
(S8)

and

$$\tau \log P_{ii} = \beta V_i(\theta) - U_i + \sigma \gamma^{\text{euler}}.$$
(S9)

Now, replace V_{ij} from (S8) into the definition of V, (2), to get

$$V_i(\theta) = -c_i^w + \lambda_i \tau_i + \sigma \lambda_i \sum_{j \neq i} G_{ij} \log P_{ij} - \sigma \lambda_i \gamma^{\text{euler}} + U_i$$

where $G_{ij} = \frac{P_{ij}^e}{1 - P_{i0}^e}$ and $\tau_i \equiv E_{j,r} \tau_{ijr} = \sum_{j \neq i} G_{ij} \tau_{ij}$. Substitute U_i from (S9):

$$V_i(\theta) = -\frac{1}{1-\beta}c_i^w + \frac{\sigma}{1-\beta}\left((1-\lambda_i)\gamma^{\text{culer}} + \lambda_i\sum_{j\neq i}G_{ij}\log P_{ij} - \log P_{ii}\right) + \frac{1}{1-\beta}\lambda_i\tau_i,$$

so that, given the CCP's, V_i is an affine function of c_i^w and σ . Next, we replace this into the Hotz and Miller (1993) inversion (S7) to obtain

$$c_{ij}^{s} = \frac{\beta}{\phi_{ij}(1-\beta)}c_{i}^{w} - \frac{\beta}{1-\beta}d_{ij}c_{j}^{w}$$

+ $\sigma \left(\frac{\beta}{1-\beta}\left(d_{ij}\left[(1-\lambda_{j})\gamma^{\text{euler}} + \lambda_{j}\sum_{l\neq j}G_{jl}\log P_{jl} - \log P_{jj}\right]\right)$
- $\frac{1}{\phi_{ij}}\left[(1-\lambda_{i})\gamma^{\text{euler}} + \lambda_{i}\sum_{l\neq i}G_{il}\log P_{il} - \log P_{ii}\right]\right)\right)$
- $\frac{\sigma}{\phi_{ij}}\log \frac{P_{ij}}{P_{ii}} + \frac{\beta}{1-\beta}d_{ij}\lambda_{j}\tau_{j} - \frac{\beta}{(1-\beta)\phi_{ij}}\lambda_{i}\tau_{i}.$

Note that

$$\frac{1}{\phi_{ij}(1-\beta)} = \frac{1-\beta(1-d_{ij})}{1-\beta} = 1 + \frac{\beta d_{ij}}{1-\beta}$$

and set $\rho_{ij} = \frac{\beta d_{ij}}{1-\beta}$; then $\frac{1}{(1-\beta)\phi_{ij}} = 1 + \rho_{ij}$. We divide by σ :

$$\begin{aligned} \frac{c_{ij}^s}{\sigma} &= (1+\rho_{ij})\frac{c_i^w}{\sigma} - \rho_{ij}\frac{c_j^w}{\sigma} - \left[\beta(1+\rho_{ij})\lambda_i\tau_i - \rho_{ij}\lambda_j\tau_j\right]\frac{1}{\sigma} \\ &+ \rho_{ij}\left[(1-\lambda_j)\gamma^{\text{euler}} + \lambda_j\sum_{l\neq j}G_{jl}\log P_{jl} - \log P_{jj}\right] \\ &- \beta(1+\rho_{ij})\left[(1-\lambda_i)\gamma^{\text{euler}} + \lambda_i\sum_{i\neq j}G_{il}\log P_{il} - \log P_{ii}\right] \\ &- \frac{1}{\phi_{ij}}\log \frac{P_{ij}}{P_{ii}}.\end{aligned}$$

This is a linear system of full rank in the parameters $\{\frac{c_{ij}^s}{\sigma}, \frac{c_i^w}{\sigma}, \frac{1}{\sigma}\}$, since $\frac{c_{ij}^s}{\sigma}$ can be expressed with respect to $\{\frac{c_i^w}{\sigma}, \frac{1}{\sigma}\}$.

6. ALGORITHM FOR COMPUTING THE STEADY-STATE EQUILIBRIUM

Here, we describe the algorithm employed to compute the steady state of our model to obtain the counterfactuals of Sections 7 and 8.

1. Make an initial guess for $\{s_i^0, e_i^0, V_i^0\}$ all *i*.

2. At each iteration l, inherit $\{s_i^l, e_i^l, V_i^l\}$ all i

(a) Update the ship's and exporter's optimal policies by repeating the following steps K times.²

i. Solve for V_{ii}^{l+1} from

$$V_{ij}^{l+1} = \frac{-c_{ij}^s + d_{ij}\beta V_j^l}{1 - \beta(1 - d_{ij})}.$$

ii. Update U_i^{l+1} from

$$U_i^{l+1} = \sigma \log \left(\exp \frac{\beta V_i^l}{\sigma} + \sum_{j \neq i} \exp \frac{V_{ij}^l}{\sigma} \right) + \sigma \gamma^{\text{euler}}.$$

iii. Compute the equilibrium prices using

$$\tau_{ijr}^{l} = \frac{\gamma \left(1 - \beta \delta \left(1 - \lambda_{i}^{e,l}\right)\right)}{1 - \beta \delta \left(1 - \gamma \lambda_{i}^{e,l}\right)} \left(U_{i}^{l+1} - V_{ij}^{l+1}\right) + \frac{(1 - \gamma)(1 - \beta \delta)}{1 - \beta \delta \left(1 - \gamma \lambda_{i}^{e,l}\right)} \bar{r}_{ij}.$$

 $^{^{2}}K$ is chosen to accelerate convergence in the spirit of standard modified policy iteration methods.

iv. Update P_{ij}^{el+1} :

$$P_{ij}^{el+1} = \frac{\exp\left(\frac{\beta\delta\lambda_i^e(\bar{r}_{ij}-\tau_{ij}^l)}{1-\beta\delta(1-\lambda_i^{e,l})}-\kappa_{ij}\right)}{1+\sum_{l\neq i}\exp\left(\frac{\beta\delta\lambda_i^e(\bar{r}_{ij}-\tau_{ij}^l)}{1-\beta\delta(1-\lambda_i^{e,l})}-\kappa_{il}\right)}$$

v. Update V_i^{l+1} :

$$V_i^{l+1} = -c_i^w + \lambda_i E_{j,r} \tau_{ijr} + \lambda_i \sum_{j \neq i} \left(\frac{P_{ij}^{e,l+1}}{1 - P_{i0}^{e,l+1}} \right) V_{ij}^{l+1} + (1 - \lambda_i) U_i^{l+1}.$$

- vi. Obtain the ships ballast choices P_{ij}^{l+1} , all i, j.
- 3. Update to $\{\tilde{s}^{l+1}, \tilde{e}^{l+1}\}$ from

$$\tilde{e}_{i}^{l+1} = \delta_{i} \left(e_{i}^{l} - m_{i}^{l} \right) + \mathcal{E}_{i} \left(1 - P_{i0}^{e,l+1} \right)$$

and

$$\tilde{s}_{i}^{l+1} = \sum_{j} P_{ji}^{l+1} (s_{j}^{l} - m_{j}^{l}) + \sum_{j} \frac{P_{ji}^{e,l+1}}{1 - P_{j0}^{e,l+1}} m_{j}^{l}.$$

4. If $\|\tilde{s}^{l+1} - s^l\| < \epsilon$, $\|\tilde{e}^{l+1} - e^l\| < \epsilon$, and $\|V^{l+1} - V^l\| < \epsilon$, stop, otherwise update freights and ships as follows:

$$s^{l+1} = \alpha s^{l} + (1 - \alpha) \tilde{s}^{l+1},$$

 $e^{l+1} = \alpha e^{l} + (1 - \alpha) \tilde{e}^{l+1},$

where α is a smoothing parameter.

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