Econometrica Supplementary Material

SUPPLEMENT TO "MONOPOLISTIC COMPETITION: BEYOND THE CONSTANT ELASTICITY OF SUBSTITUTION"

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IN THIS APPENDIX, we prove the various statements made in our paper. In Appendix A, we study the impact of market size on the FEE. Appendix B is devoted to the multisector economy, while Appendix C shows that equilibrium under the translog behaves like equilibrium under the CARA.

APPENDIX A: THE IMPACT OF MARKET SIZE ON THE FEE

It is readily verified that (6) is equivalent to

$$r'_u x + (r_u - r_C)(1 - r_u) > 0. \tag{A.1}$$

This expression will be used below.

Output. Differentiating (10) leads to

$$\frac{[qV''(\bar{q}) + V'(\bar{q})]C(\bar{q}) - q[V'(\bar{q})]^2}{[C(\bar{q})]^2}\frac{d\bar{q}}{dL} = -r'_u\left(\frac{1}{L}\frac{d\bar{q}}{dL} - \frac{\bar{q}}{L^2}\right).$$

Using

$$V'(\bar{q})\bar{q} = (1 - r_u)C(\bar{q}),$$

we obtain

$$(r_u - r_C)(1 - r_u)\frac{L}{\bar{q}} \cdot \frac{d\bar{q}}{dL} = -r'_u\left(\frac{d\bar{q}}{dL} - \frac{\bar{q}}{L}\right),$$

which amounts to

$$(r_u - r_C)(1 - r_u)\mathcal{E}_{\bar{q}/L} = -r'_u\frac{\bar{q}}{L}(\mathcal{E}_{\bar{q}/L} - 1).$$

Thus, the elasticity of $\bar{q}$ with respect to (w.r.t.) to L is equal to

$$\mathcal{E}_{\bar{q}/L} = \frac{r'_u\bar{x}}{r'_u\bar{x} + (r_u - r_C)(1 - r_u)}.$$

It follows from (6) that the denominator is positive. Consequently, a firm's output increases (decreases) when the RLV is increasing (decreasing). Furthermore, the weak convexity of V implies that

$$\mathcal{E}_{\bar{q}/L} < 1. \tag{A.2}$$

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Consumption per capita. It is readily verified that the elasticity of $\bar{x}$ w.r.t. L can be derived from $\mathcal{E}_{\bar{q}/L}$ as

$$\mathcal{E}_{\bar{x}/L} = \mathcal{E}_{\bar{q}/L} - 1 = -\frac{(r_u - r_C)(1 - r_u)}{r'_u \bar{x} + (r_u - r_C)(1 - r_u)}.$$

Thus, $\bar{x}$ decreases with L when $r_u - r_V > 0$. Observe that this inequality holds when V is convex or not too concave.

Markup. From the comparative statics above, it is straightforward that markups decrease (increase) with L if and only if the RLV is increasing (decreasing).

Price. It follows from (9) that

$$\text{(A.3)} \qquad \frac{d\bar{p}}{dL} = \frac{V'(\bar{q})\bar{q} - C(\bar{q})}{\bar{q}^2} \cdot \frac{d\bar{q}}{dL}.$$

Then, firms' output and market price move in opposite directions with L:

$$\frac{d\bar{p}}{dL} = -r_u \frac{C(\bar{q})}{\bar{q}^2} \cdot \frac{d\bar{q}}{dL}.$$

Number of varieties. The number of varieties $\bar{N}$ is determined by labor market clearing:

$$\bar{N} C(\bar{q}) = L.$$

Thus, the elasticity of $\bar{N}$ w.r.t. L is

$$\mathcal{E}_{\bar{N}/L} + \mathcal{E}_C \cdot \mathcal{E}_{\bar{q}/L} = 1,$$

which amounts to

$$\mathcal{E}_{\bar{N}/L} = 1 - \mathcal{E}_C \cdot \frac{r'_u \bar{x}}{r'_u \bar{x} + (r_u - r_C)(1 - r_u)}.$$

Again, the denominator of the second term is strictly positive by (A.1). Furthermore, at the equilibrium, it must be that $0 < \mathcal{E}_C(L\bar{x}) = 1 - r_u(\bar{x}) < 1$ and, thus, the sign of $\mathcal{E}_{\bar{N}/L} - 1$ is determined by r'_u. Consequently, the elasticity of $\bar{N}$ w.r.t. L is smaller (larger) than 1 if the RLV is increasing (decreasing).

APPENDIX B: THE MULTISECTOR ECONOMY

Properties of the Expenditure Function in the Two-Sector Economy

The following two lemmas provide a rationale for the following assumptions made in Section 4.1:

$$\text{(B.1)} \qquad 0 \le \frac{p}{E} \cdot \frac{\partial E}{\partial p} < 1, \qquad \frac{N}{E} \cdot \frac{\partial E}{\partial N} < 1.$$

Set

$$D \equiv U''_{11} \cdot \left(v'_E\right)^2 - 2U''_{12}v'_E + U''_{22} + U'_1 v''_{EE}.$$

LEMMA 1: *If $U''_{21} \geq 0$, then the elasticity of E w.r.t. N is such that*

$$\frac{\partial E}{\partial N} \cdot \frac{N}{E} - 1 = \frac{-U''_{11}v'_E v + U''_{21}(v + v'_E E) - U''_{22}E}{DE} \leq 0.$$

LEMMA 2: *If $U''_{21} \geq 0$ and the inequality*

$$\text{(B.2)} \qquad \frac{1 - r_u(x)}{\mathcal{E}_u(x)} \leq \frac{U''_{21}(X, Y)X}{U'_2(X, Y)} - \frac{U''_{11}(X, Y)X}{U'_1(X, Y)}$$

hold at a symmetric outcome, then the elasticity of E w.r.t. p is such that

$$\text{(B.3)} \qquad -1 \leq \frac{\partial E}{\partial p} \cdot \frac{p}{E} - 1 = \frac{U'_1 v'_E + U''_{21} E v'_E - E U''_{22}}{DE} \leq 0.$$

REMARK: Under $u(0) = 0$, the indirect utility function

$$v(p, E, N) = Nu\left(\frac{E}{pN}\right)$$

is homogeneous of degree 0 w.r.t. (p, E) and of degree 1 w.r.t. (E, N). Therefore, v'_E and v'_p are homogeneous of degree -1 w.r.t. (p, E) and of degree 0 w.r.t. (E, N). Finally, we have $v''_{EE} < 0$.

Let $E(p, N)$ be the unique solution to the first-order condition for the upper-tier utility maximization,

$$\text{(B.4)} \qquad U'_1\big(v(p, E, N), 1 - E\big)v'_E(p, E, N) - U'_2\big(v(p, E, N), 1 - E\big) = 0,$$

where the second-order condition is given by

$$D < 0.$$

Note that $U(v(p, E, N), 1 - E)$ is concave w.r.t. E because U is concave, while the concavity of u implies that of v.

PROOF OF LEMMA 1: Differentiating (B.4) w.r.t. N and solving for $\partial E/\partial N$, we get

$$\frac{\partial E}{\partial N} = -\frac{U''_{11}v'_E v'_N + U'_1 v''_{EN} - U''_{21}v'_N}{D} = -\frac{(U''_{11}v'_E - U''_{21})v'_N + U'_1 v''_{EN}}{D}.$$

Consequently,

$$\frac{\partial E}{\partial N}\cdot\frac{N}{E}-1=-N\frac{(U''_{11}v'_E-U''_{21})v'_N+U'_1v''_{EN}}{DE}-1$$
$$=\Big(-U''_{11}\big[v'_ENv'_N+E\big(v'_E\big)^2\big]+U''_{21}\big(Nv'_N+2v'_EE\big)$$
$$-U'_1\big(Nv''_{EN}+Ev''_{EE}\big)-EU''_{22}\Big)$$
$$/(DE).$$

Applying the Euler theorem to v and v', we obtain the equalities

$$-U''_{11}\big[v'_ENv'_N+E\big(v'_E\big)^2\big]=-U''_{11}v'_E\big(Nv'_N+Ev'_E\big)=-U''_{11}v'_Ev,$$
$$U''_{21}\big(Nv'_N+2Ev'_E\big)=U''_{21}\big(v+Ev'_E\big),$$
$$-U'_1\big(Nv''_{EN}+Ev''_{EE}\big)=0.$$

As a result, we have

$$\frac{\partial E}{\partial N}\cdot\frac{N}{E}-1=\frac{-U''_{11}v'_Ev+U''_{21}(v+Ev'_E)-EU''_{22}}{DE}.$$

Since $U''_{21}\geq 0$, the numerator of this expression is positive. Since $D<0$, we have

$$\frac{\partial E}{\partial N}\cdot\frac{N}{E}-1\leq 0.$$

Q.E.D.

PROOF OF LEMMA 2: Differentiating (B.4) w.r.t. p and solving for $\partial E/\partial p$, we get

$$\frac{\partial E}{\partial p}=\frac{-U''_{11}v'_pv'_E-U'_1v''_{Ep}+U''_{21}v'_p}{D}, \tag{B.5}$$

which implies

$$\frac{\partial E}{\partial p}\cdot\frac{p}{E}-1=p\frac{-U''_{11}v'_pv'_E-U'_1v''_{Ep}+U''_{21}v'_p}{DE}-1$$
$$=\Big(-U''_{11}\big[pv'_pv'_E+E\big(v'_E\big)^2\big]-U'_1\big(pv''_{Ep}+Ev''_{EE}\big)$$
$$+U''_{21}\big(pv'_p+2Ev'_E\big)-EU''_{22}\Big)$$
$$/(DE).$$

Applying the Euler theorem to v and v' yields

$$-U''_{11}\big[pv'_p v'_E + E(v'_E)^2\big] = -U''_{11}v'_E(pv'_p + Ev'_E) = 0$$

and

$$-U'_1(pv''_{Ep} + Ev''_{EE}) = U'_1 v'_E > 0.$$

Therefore,

$$\frac{\partial E}{\partial p}\cdot\frac{p}{E} - 1 = \frac{U'_1 v'_E + U''_{21}Ev'_E - EU''_{22}}{DE} \leq 0$$

since $U''_{21} \geq 0$. Consequently, the right inequality of (B.3) is proven.

To show that $\partial E/\partial p > 0$, we rewrite (B.4) as

$$\frac{\partial E}{\partial p} = \frac{v'_p}{D}\left(-U''_{11}v'_E - U'_1\frac{v''_{Ep}}{v'_p} + U''_{21}\right).$$

By definition of v, we have

$$v'_p = -\frac{Eu'}{p^2} < 0, \quad v'_E = \frac{u'}{p}, \quad v''_{Ep} = -\frac{u'}{p^2} - \frac{Eu''}{Np^3}.$$

Since $v'_p/D > 0$, the sign of $\partial E/\partial p$ is the same as that of the bracketed term of (B.5). Substituting these three expressions into (B.5) leads to

$$\begin{aligned}
&-U''_{11}v'_E - U'_1\frac{v''_{Ep}}{v'_p} + U''_{21}\\
&\quad = -U''_{11}\frac{u'}{p} - U'_1\frac{-\dfrac{u'}{p^2} - \dfrac{Eu''}{Np^3}}{-\dfrac{Eu'}{p^2}} + U''_{21}\\
&\quad = -\frac{U'_1}{E}\left[\left(\frac{U''_{11}Nu}{U'_1} - \frac{U''_{21}Nu}{U'_2}\right)\frac{Eu'}{Npu} + 1 + \frac{Eu''}{Npu'}\right].
\end{aligned}$$

Using $-U'_1/E < 0$ and $U'_1 v'_E(p, E, N) = pU'_2/u'$, it follows from (B.2) that

$$\left(\frac{U''_{11}Nu}{U'_1} - \frac{U''_{21}Nu}{U'_2}\right)\frac{Eu'}{Npu} + 1 + \frac{Eu''}{Npu'} < 0 \quad\Longrightarrow\quad \frac{\partial E}{\partial p} > 0,$$

which implies the left inequality of (B.3). *Q.E.D.*

The Impact of Market Size on the Mass of Firms in the Two-Sector Economy

We now show that the equilibrium mass of firms decreases with market size. Using the budget constraint and the zero-profit condition yields

$$N\big[F + V\big(\bar{q}(L)\big)\big] = LE\big(\bar{p}(L), N\big).$$

Rewriting this expression in elasticity terms w.r.t. L, we get

$$\mathcal{E}_N + \frac{\bar{q}V'(\bar{q})}{F + V(\bar{q})}\mathcal{E}_q = 1 + \frac{\partial E}{\partial p}\frac{\bar{p}}{E} \cdot \mathcal{E}_p + \frac{\partial E}{\partial N}\frac{N}{E} \cdot \mathcal{E}_N,$$

which can be rewritten as

$$\text{(B.6)} \qquad \mathcal{E}_N\left(1 - \frac{\partial E}{\partial N}\frac{N}{E}\right) = 1 + \frac{\partial E}{\partial p}\frac{\bar{p}}{E} \cdot \mathcal{E}_p - \frac{\bar{q}V'(\bar{q})}{F + V(\bar{q})}\mathcal{E}_q.$$

The expression (A.3) is equivalent to

$$\text{(B.7)} \qquad \mathcal{E}_p = -r_u\mathcal{E}_q.$$

Using (10) and (B.7), (B.6) implies

$$\begin{aligned}\mathcal{E}_N\left(1 - \frac{\partial E}{\partial N}\frac{N}{E}\right) &= 1 + \frac{\partial E}{\partial p}\frac{\bar{p}}{E} \cdot \mathcal{E}_p - (1 - r_u)\mathcal{E}_q \\ &= 1 + \frac{\partial E}{\partial p}\frac{\bar{p}}{E} \cdot \mathcal{E}_p + \frac{1 - r_u}{r_u}\mathcal{E}_p \\ &> 1 - \left(\frac{\partial E}{\partial p}\frac{\bar{p}}{E} + \frac{1 - r_u}{r_u}\right)r_u = \left(1 - \frac{\partial E}{\partial p}\frac{\bar{p}}{E}\right)r_u,\end{aligned}$$

where we have used (A.2) for the inequality. Since the elasticity of E w.r.t. p is smaller than 1 by assumption, the last term in the above expression is positive. Since the elasticity of E w.r.t. N in the first term is also smaller than 1, it must be that

$$\mathcal{E}_N = \frac{dN}{dL} \cdot \frac{L}{N} > 0.$$

APPENDIX C: RELATIONSHIP BETWEEN THE TRANSLOG AND CARA MODELS

Under the translog, the profit is given by

$$\text{(C.1)} \qquad \pi(p_i; \Lambda_{\text{trans}}, L) - F = (p_i - c)\frac{L}{p_i}(\Lambda_{\text{trans}} - \beta \ln p_i) - F.$$

Differentiating this expression w.r.t. p_i yields

$$\frac{c}{p_i^2}(\Lambda_{\text{trans}} - \beta \ln p_i) - \beta \frac{p_i - c}{p_i^2} = 0.$$

Solving for

$$\Lambda_{\text{trans}} - \beta \ln p_i = \beta \frac{p_i - c}{c},$$

plugging this expression into (C.1), and rearranging terms leads to the equilibrium condition

$$\beta(p - c)^2/(cp) = F/L.$$

Applying the same argument to the CARA model yields the desired expression:

$$\beta(p - c)^2/p = F/L.$$

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