

SUPPLEMENT TO “FINANCIAL INNOVATION AND THE
TRANSACTIONS DEMAND FOR CASH”: APPENDICES
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APPENDIX A: ON ATM CARDS AND ATM CASH WITHDRAWALS IN ITALY

This appendix provides some institutional information on the ATM card transactions technology in Italy. Cash intermediation (e.g., debiting or crediting cash on a deposit account) is reserved to banks: as a consequence, all ATM terminals are owned by banks. About 80% of the ATM terminals are located in the premises of a bank branch; the remaining 20% are not (e.g., located in airports, shopping malls, etc.). These figures do not change much over time in our sample period. Hence the number of bank branches is a good proxy for the number of ATM terminals. The time-series correlation between the two series is positive and very strong, and so is the cross-section correlation (the linear correlation across provinces is between 0.75 and 0.94 in each year of the sample). However, in the time series, ATMs grow faster than bank branches: the ratio of the total number of ATMs to bank branches was 0.6 in 1993 and 1.2 in 2004.

Most ATM cards can also be used as debit cards, and they can be used nationwide for cash withdrawals. The cash-back option (i.e., getting cash at a supermarket in exchange for a debit transactions) does not exist in Italy: only banks can act as cash intermediaries. Most ATM cards exact a fee of about 2 euros if cash is taken from a terminal that does not belong to the agent's bank; withdrawals at own bank are free.

Next, we compare data on average ATM withdrawals drawn from two sources: our households survey data (Survey of Household Income and Wealth (SHIW)) and the data drawn from banks' records as reported in the European

Central Bank (ECB) Blue Book (2006). Table 12.1a in the bluebook reports the total number of cash withdrawals at ATMs in a year. Table 13.1a gives the total value of cash withdrawals at ATMs in a year. The average withdrawal computed as the ratio of these two numbers for the years 2001, 2002, and 2004 is 162, 205, and 169 euros, respectively (these years are the closest to those of the SHIW survey years). In the household survey we compute the analogous statistics for the years 2000, 2002, and 2004, obtaining 177, 185, and 205 euros, respectively. For each year, the latter statistics were computed as the ratio between the sum across households of the amount of cash withdrawn from ATMs and the sum across households of the number of withdrawals from ATMs. For each household, the total amount of cash withdrawn from ATM was given by the average ATM withdrawal times the number of ATM withdrawals. These statistics differ from the statistics on W reported in Table I in the paper for three reasons. First, because even for households with an ATM card, W includes withdrawals done at the bank desk (which are larger on average). Second, W is measured in 2004 euros. Third, W reports the average withdrawal per household, so the weighting is different.

APPENDIX B: STATISTICS ON THE PROBABILITY OF CASH THEFT IN ITALY

The statistics on the probability of cash thefts (κ) for Italy are computed in three steps.

Step 1. We consider four crimes where cash is lost: bag-snatching (*scippi*), pickpocketing (*borseggi*), theft (*furti*), and robbery (*rapine*). Using survey data on victimization per person (aged 14 or older) for the whole of Italy (i.e., average of 103 provinces) in the year 2002 gives the following percentages for each of the crimes, respectively:¹ 0.4, 1.4, 2.2, and 0.3.

Step 2. Next, we adjust the statistic for each crime to take into account information on the percentage of crimes where cash is taken (source: Istituto nazionale di statistica (Istat) victimization survey). For instance, for bag-snatching cash is taken 49% of the time. The statistics that we are interested in for 2002 is

$$\kappa = (0.4 \cdot 0.49 + 1.4 \cdot 0.61 + 2.2 \cdot 0.37 + 0.3 \cdot 0.59) = 2.041.$$

Step 3. Finally, using data on bag-snatching (*scippo*) and pickpocketing (*borseggi*) across 103 Italian provinces, and using a time series for these two crimes at the country level across years (source: Istat), we construct values of κ that vary across provinces and years. A comparison between the statistics for Italy and those obtained for the United States following a similar procedure is

¹Victimization rates: Istat, Figure 1.1 on page 13 of “La Sicurezza dei Cittadini, Year 2002” (N. 18, published in 2004); Fraction of crimes where cash is taken: Istat Table 7.1 on page 67 of report “La Sicurezza dei cittadini.”

given in Technical Appendix B, available on our website. In summary, the figures on the probability of cash theft are not that different between the United States and Italy. Table III in the Technical Appendix shows that an overall similar picture emerges using three different data sources.

APPENDIX C: ON THE AVERAGE CASH BALANCE M WITH
A PRECAUTIONARY MOTIVE

PROPOSITION 1: *Assume that $\pi = 0$ and let λ denote the time elapsed between two consecutive withdrawals. Let $M(\lambda)$ be the average cash balance during this elapsed time, let $W(\lambda)$ be the withdrawal at the end of a period of length λ , and let $\underline{M}(\lambda)$ be the cash balance just prior to the withdrawal. Let M be the expected value of cash holdings under the invariant distribution and let $g(\lambda)$ be the density of the distribution of the lengths. We then have*

$$(S1) \quad M(\lambda) = \underline{M}(\lambda) + \frac{W(\lambda)}{2} = m^* - \frac{(c\lambda)}{2},$$

$$(S2) \quad M = \frac{\int_0^\infty M(\lambda)\lambda g(\lambda) d\lambda}{\int_0^\infty \lambda g(\lambda) d\lambda}.$$

PROOF: Let $t \in [0, \lambda]$ index the time elapsed in an interval of length λ . The law of motion of cash and the optimal policy imply that cash holdings obey $m(t) = m^* - c\lambda$ for $t \in [0, \lambda)$ and $m(\lambda) = m^*$; $W(\lambda) = m^+(\lambda) - m^-(\lambda)$ and $m^* = W(\lambda) + \underline{M}(\lambda)$ imply equation (S1). The ergodic theorem implies, using ω to index the sample space,

$$(S3) \quad M = \lim_{T \rightarrow \infty} (1/T) \int_0^T m(t, \omega) dt \quad \text{in probability,}$$

from which equation (S2) can be derived.

Q.E.D.

REMARK 1: If the distribution of the length λ is concentrated at a single value $\bar{\lambda}$, as in a deterministic model, then $M = M(\bar{\lambda})$. Then

$$M = M(\bar{\lambda}) = \underline{M}(\bar{\lambda}) + W(\bar{\lambda})/2.$$

REMARK 2: When the distribution of the length λ is not degenerate, then

$$M < \int_0^\infty M(\lambda)g(\lambda) d\lambda = \int_0^\infty \underline{M}(\lambda)g(\lambda) d\lambda + \frac{1}{2} \int_0^\infty W(\lambda)g(\lambda) d\lambda,$$

where the inequality follows because $M(\lambda)$ is decreasing in λ . Thus M , the duration weighted expected value of $M(\lambda)$, is smaller than the unweighted expected value in the right hand side of the inequality.

APPENDIX D: CASH-FLOW IDENTITY: THEORY AND EVIDENCE

We derive the relationship

$$(S4) \quad c = nW - \pi M$$

between the average (real) cash balances M , average (real) withdrawal amount W , average (real) consumption flow c , average number of withdrawals n per unit of time, and the inflation rate π for a (large) class of cash management policies. In what follows we fixed a particular path and denote the real cash balances at time t by $m(t)$, let τ_i be the times at which there are withdrawals for this sample path, and let w_i be the corresponding withdrawal amounts. In between withdrawals, cash balances satisfy

$$\frac{dm(t)}{dt} = -c - m(t)\pi.$$

At times $t = \tau_i$, a withdrawal of size w_i , defined as an upward jump on m , occurs:

$$w_i \equiv \lim_{t \downarrow \tau_i} m(t) - \lim_{t \uparrow \tau_i} m(t) > 0.$$

Thus we have that

$$m(t) = m(0) - \int_0^T (c + \pi m(s)) ds + \sum_{i=1}^{N(T)} w_i,$$

where $N(T)$ denotes the number of (upward) jumps up to time T in the path:

$$N(T) \equiv \{N : \tau_N \leq T \leq \tau_{N+1}\}.$$

Dividing by T and rearranging gives

$$\frac{m(t) - m(0)}{T} = -c - \pi \frac{1}{T} \int_0^T m(s) ds + \left[\frac{N(T)}{T} \right] \left[\frac{1}{N(T)} \sum_{i=1}^{N(T)} w_i \right].$$

Then define

$$M \equiv \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T m(s) ds, \quad n \equiv \lim_{T \rightarrow \infty} \frac{N(T)}{T}, \quad \text{and}$$

$$W \equiv \lim_{T \rightarrow \infty} \frac{1}{N(T)} \sum_{i=1}^{N(T)} w_i,$$

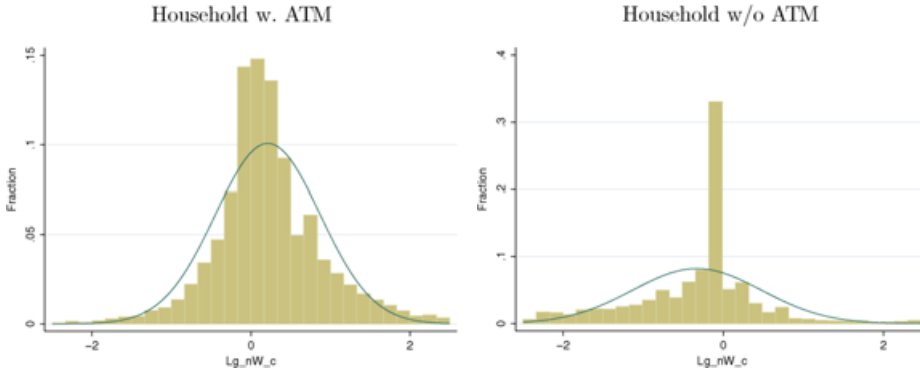


FIGURE S.1.—Measurement error: deviation from the cash-flow identity.

where M , n , and W are the average money balances, average number of withdrawals per unit of time, and average amount of withdrawal. Assuming that, for almost all paths, the limits M , n , and W are well defined, and that the process is ergodic, so that these time averages converge to the unconditional expectations for almost all paths, we obtain equation (S4). In all the models we analyze, these limits exist and coincide for all paths as a consequence of basic results on renewal theory, but of course their validity is much more general.

An illustration of the extent of the measurement error can be derived by assuming that the data satisfy the identity in (S4). Figure S.1 reports a histogram of the logarithm of $n(W/c) - \pi(M/c)$ for each type of household. In the absence of measurement error, all the mass should be located at zero. It is clear that the data deviate from this value for many households.² At least for households with an ATM card, we view the histogram to be well approximated by a normal distribution (in log scale).

APPENDIX E: WEIGHTS USED IN THE ESTIMATION

Table S.I displays the average weights N_j/σ_j^2 used in estimation, the average N_j (across provinces and years), and the estimated value of σ_j^2 . The latter are estimated as the variance of the residual of a regression of each of the j variables at the household level against dummies for each province–year combination (separate regressions are used for households with and without ATM cards).

²Besides measurement error in reporting, which is important in this type of survey, there is also the issue of whether households have an alternative source of cash. An example of such a source occurs if households are paid in cash. This will imply that they require fewer withdrawals to finance the same flow of consumption or, alternatively, that they effectively have more trips per periods.

TABLE S.I
WEIGHTS USED IN ESTIMATION^a

	$\log(M/c)$	$\log(W/M)$	$\log(n)$	$\log(\underline{M}/M)$
Households with ATM				
Average weight (N_j/σ_j^2)	30	17	22	14
Variance (σ_j^2)	0.46	0.42	0.53	0.82
Average no. of households in province–year–consumption cell (N_j)	13.5	6.3	12	9.5
Households without ATM				
Average weight (N_j/σ_j^2)	26	14	12	11
Variance (σ_j^2)	0.41	0.51	0.62	0.82
Mean no. of households in province–year–consumption cell (N_j)	10.7	7.4	7.6	7.6

^aThere is a total of 3189 estimation cells (the available observations of the Cartesian product of 6 years, 103 provinces, ATM ownership, and 3 consumption groups).

APPENDIX F: ESTIMATION UNDER ALTERNATIVE CELL DEFINITIONS

This appendix reports the estimation results of the model with random free withdrawals obtained under five alternative aggregations and selection of the raw data.

The baseline aggregation used in the estimates of Section 5 includes all households with a deposit account for whom the survey data are available (see the paper for details). The elementary household data were aggregated at the province–year–household type (ATM/no ATM and three consumption groups), providing us with a total of about 1800 observations per type of withdrawal technology (ATM/no ATM) to be fitted (103 provinces \times 6 years \times 3 consumption groups), each one based on approximately 13 elementary household observations. Four additional aggregations of the data were explored. Table S.II provides a quick synopsis that is helpful for comparing the results obtained from our benchmark specification (reported in the fifth column for ease of comparison) with the ones produced by those alternatives.

The first alternative aggregation of the data, reported in the second column of Table S.II, differs from the baseline case in that it does not split households according to their consumption level. This increases by about three times the number of elementary household observations used for the estimate of (p , b/c) in a given province–year–household type. The value of the point estimates is close to the one obtained in the baseline exercise, though the greater number of underlying observation increases the statistical significance of the estimates.

Two alternative aggregations of the data exclude households that receive more than 50% of their income in cash or violate the cash-flow identity of equation (S4) by more than 200%. This choice removes households for whom cash inflows are an important source of replenishment (as this channel is ignored

TABLE S.II
ESTIMATION OUTCOMES OVER FIVE DIFFERENT DATA SETS^a

	Data Set ^b				
	Province-Year		Region-Year-Consumption (Filtered)	Province-Year-Consumption	
	Raw	Filtered		Raw	Filtered
Households with ATM					
No. of estimates	576	563	532	1654	1454
Mean no. of HH per est.	39	17	16	14	6
% of estimates where					
Hypothesis $f = 0$ rejected	40	33	42	19	17
$F(\theta, x) < 4.6$	42	47	38	57	60
Mean estimate of p	22	29	27	22	29
Mean t -statistic	4.9	4.4	4.4	3.1	3.0
Corr. w. bank branches ^c	0.1	0.0	0.0	0.1	0.1
Mean estimate of $b/c \cdot 100$	2.6	2.5	2.5	4.0	3.8
Mean t -statistic	4.5	3.3	3.5	2.8	2.3
Corr. w. bank branches ^c	-0.2	-0.2	-0.3	-0.2	-0.2
Households without ATM					
No. of estimates	550	538	535	1539	1411
Mean no. of HH per est.	30	14	13	11	5
% of estimates where					
Hypothesis $f = 0$ rejected	9	6	3	2	1
$F(\theta, x) < 4.6$	49	66	70	64	74
Mean estimate of p	7	7	7	8	8
Mean t -statistic	3.7	3.1	3.0	2.4	2.1
Corr. w. bank branches ^c	0.0	0.0	-0.1	0.0	0.1
Mean estimate of $b/c \cdot 100$	6.7	6.2	5.8	7.7	7.4
Mean t -statistic	4.2	3.3	3.3	2.7	2.3
Corr. w. bank branches ^c	-0.3	-0.2	-0.3	-0.3	-0.2

^aSample statistics computed on the distribution of the estimates after trimming the ($p, b/c$) distribution tails of the highest and lowest percentiles (1% from each tail). The variable b/c is measured as a percentage of the daily cash expenditure.

^bThe labels that appear below indicate the type of aggregation that was performed on the elementary household data in each year: 'Province' or 'Region' refers to the geographical level of aggregation; 'Consumption' indicates that households were clustered within the relevant observation unit, e.g., in a province-year, on the basis of their cash expenditure level (3 bins were considered for the province-year dataset, 5 bins for the region-year dataset); 'Raw' or 'Filtered' indicates whether the aggregation was based on the raw data or on a filtered dataset which excludes households who receive more than 50% of income in cash and/or violate the cash-holdings identity by more than 200%.

^cCorrelation coefficient between the estimated values of ($p, b/c$) and the number of bank branches per capita measured at the province level. All variables are measured in logs.

by our baseline model) and observations affected by large measurement error. This selection criterion roughly halves the number of elementary observations. The estimation results obtained from these data when one or three consump-

tion groups are considered (third and last columns of Table S.II, respectively) are extremely similar to the ones of the baseline case (fifth column).

The last experiment that we report involves aggregation of the household data at the regional, rather than province, level (a region is a geographical unit which contains several provinces; there are 103 provinces and 20 regions in Italy). This allows us to consider a finer grid of consumption classes, namely five for the instance reported in the fourth column of the table, thus increasing the mean number of elementary observations used in each estimation cell. Again, as the table shows, the results are similar to the ones produced by the other approaches.

APPENDIX G: HOUSEHOLD-LEVEL ESTIMATES WITH UNOBSERVED HETEROGENEITY

The estimation strategy pursued in Section 5.2 of the paper is based on two assumptions: (i) the parameters b/c and p are the same for all households in a given cell; (ii) the variables $(M/c, W/M, n, \underline{M}/M)$ are observed with a classical measurement error (in this section we refer to these estimates as cell estimates). An alternative estimation strategy, developed in Section 5.3 of the paper, also assumes that the household variables are observed with classical measurement error, but posits that the parameters b/c and p differ for each households, and are given by a simple function of household level variables. We refer to this case as the one with observed household level heterogeneity.

This section considers yet another strategy where the estimation incorporates unobserved household level heterogeneity and NO measurement error. We estimate the distribution of the household level values for $(p, b/c)$ non-parametrically. In this case we assume that each household has a pair of parameters b/c and p , and assume that we observe $(M/c, W/M, n, \underline{M}/M)$ with no error. With no need to assume a functional form for the distribution of the parameters $(p, b/c)$, these assumptions allow us to estimate the model for each household separately. Note that unless the four observables $(M/c, W/M, n, \underline{M}/M)$ for a given household can be rationalized by the two parameters $(p, b/c)$, the observations will be inconsistent for this household. To address this stochastic singularity (that comes from having four observables, no measurement error, and only two parameters), we estimate the model using only two observables: $(M/c, n)$, the two variables for which we have more observations.

To be concrete we apply this estimation strategy to all households in four large provinces (Turin, Milan, Florence, Rome), using data for all the years (from 1993 to 2004), for households (HH) with or without ATM cards, and for each third-tile of the cash consumption distribution. We label these estimates as HH Unobs. Heterogeneity estimates and compare them with the cell estimates obtained using observation on $(M/c, n)$ and assuming that these are measured with error. Since there are two parameters and two observables, the Cell estimates produce one $(p, b/c)$ parameter vector for the whole cell, while

TABLE S.III
ESTIMATES OF p, b OVER 1993–2004 USING M/c AND n^a

Province	No. of Obs.	Cash Expenditure Group					
		Low	Medium	High	Low	Medium	High
		HH Unobs. Heterogeneity			Cell Estimates		
Household with ATM							
				Estimates of p			
Turin	1215	25	25	32	26	28	28
Milan	1355	31	32	35	37	32	32
Florence	632	35	39	32	29	37	27
Rome	752	36	32	38	30	28	29
				Estimates of $b/c \cdot 100$			
Turin	1215	12	4	2	2	2	2
Milan	1355	18	7	4	5	4	4
Florence	632	27	15	4	5	8	5
Rome	752	19	8	5	6	5	5
Household without ATM							
				Estimates of p			
Turin	563	8	8	10	0	0	0
Milan	358	8	8	8	0	5	0
Florence	292	16	13	9	0	0	0
Rome	344	9	8	8	0	0	0
				Estimates of $b/c \cdot 100$			
Turin	563	16	7	5	5	4	5
Milan	358	38	23	8	9	12	12
Florence	292	31	19	19	12	19	14
Rome	344	36	27	23	20	14	13

^aThe entries for the HH level unobserved heterogeneity are the median of the distribution of estimated p, b values. All are based on the two observables M/c and n .

the HH unobserved heterogeneity estimates produce a distribution of the parameters ($p, b/c$) for each cell. Table S.III reports the median of the ($p, b/c$) estimates for four major Italian provinces and, for comparison, we report the values obtained using cell estimates for the case in which two observable variables were used.

Table S.III shows that the cell estimates and the HH estimates with unobserved heterogeneity display similar patterns: Households with an ATM card have higher values of p and smaller values of b/c compared to households without an ATM. Moreover, the value of b/c is decreasing in the level of cash expenditure, while the value of p is roughly independent of c .

G.1. Mean vs. Medians and Two vs. Four Variables

This subsection analyzes means, medians, and standard deviations obtained from the HH unobserved heterogeneity estimates for the province of Turin,

TABLE S.IV
ESTIMATES OF (p, b) FOR HOUSEHOLD WITH ATM IN TURIN OVER 1993–2004^a

	Cash Expenditure Group					
	Low	Medium	High	Low	Medium	High
	Parameter p			Parameter $b/c_{\text{day}} \cdot 100$		
HH estimates						
Mean	33	30	37	53	19	9
Median	25	25	32	12	4	2
Std. dev.	28	27	29	141	102	37
Cell mean estimates						
Two variables	26	28	28	2	2	2
Four variables (as in paper)	14	17	23	6	2	2

^a The means and the standard deviations for the HH level unobserved heterogeneity estimates are computed by trimming 1% of observation from each tail of the distribution estimates.

and compares them with two types of cell estimates obtained using two and four observables, respectively.

The first three rows of Table S.IV display the HH unobserved heterogeneity estimates, the next two rows report the cell estimates. There is a large dispersion across households on the values of the HH unobserved heterogeneity estimates for $(p, b/c)$, reflecting the large variability of M/c and n in a given cell. As noted above, the median values of the HH unobserved heterogeneity estimates for $(p, b/c)$ are similar to the corresponding values of the cell estimates using two observables. Instead, the mean values of the HH unobserved heterogeneity estimates for $(p, b/c)$ are larger than the corresponding values of the cell estimates using two observables, especially for b/c , even after trimming the top and bottom 1% of the estimate distribution. This difference is due, mechanically, to the effect of a large right tail in the distribution of the estimated values of $(b/c, p)$.

The large right tail of the estimates using unobserved heterogeneity reflects two facts: the large variability of the observables $(M/c, n)$ across households and a particular nonlinearity of our model. Below we illustrate the nature of this nonlinearity and conduct a Monte Carlo experiment to analyze its effect on the estimates. In the model there is a relationship between M/c and n , due to the nature of the optimal policy, which has to hold for any value of b/c and R , and that we derived in the Section 5.1 in the paper for the case of zero inflation:

$$\frac{M}{c} = \xi(n, p) = \frac{1}{p} \left(-\frac{n}{p} \log \left(1 - \frac{p}{n} \right) - 1 \right).$$

Without measurement error, this relationship has to hold exactly for each household. This means that given the observable values of $(M/c, n)$ for this

household, the function ξ implicitly identifies this household estimate of p . Furthermore, since the model requires $n \geq p$, fixing p , the function ξ has a vertical asymptote as $n \downarrow p$ in the $(n, M/c)$ coordinates. Moreover, fixing p , the different points in the curve defined by $(n, M/c) = (n, \xi(n, p))$ correspond to different values of $b/(cR)$. In other words, the value of $b/(cR)$ is identified by the location of the pair $(n, M/c)$ on a given ξ curve indexed by p . Thus, since the $\xi(\cdot, p)$ function becomes vertical as $n \downarrow p$, observations with large M/c relative to n correspond to extremely large values of $b/(cR)$.

We conducted the following Monte Carlo experiment. We assume that the data generating mechanism is one where all households in a cell have the same values of $(p, b/c)$, that is, we assume that there is NO unobserved heterogeneity. In particular, we set the value of $b/c = 0.02$ and $p = 28$, corresponding to some of the typical values estimated above (see Turin two-variable cell estimates for the median cash expenditure group). We create a sample of size $N = 10,000$ of values of $(M/c, n)$ obtained by adding independent measurement error, additive in logs, with variance $\sigma_{M/c}^2 = 0.46$ and $\sigma_n^2 = 0.53$ (as from our estimates; see Table S.I). We treat this artificial sample as if it were a sample of households characterized by unobserved household level heterogeneity and estimate the distribution of $(p, b/c)$. The medians of the estimated values of p and b/c are, respectively, 37 and 3. These values are close to the true ones (28 and 2, respectively). The means of the estimated values of p and b/c are, respectively, 41 and 16. It appears that, consistent with our explanation of the nonlinearity presented above, the mean value of b/c is much higher than its true value.

We conclude by analyzing the effect of the number of observables used on the estimates of p and b/c . The cell estimates that appear in the last line of Table S.IV use four observables $(M/c, W/M, n, \underline{M}/M)$, as in the benchmark case considered in the paper; the previous line instead, used only two observables: M/c and n . Using four variables reduces the estimated values for p with minor effects on the value of b/c . Clearly the observed values of W/M and \underline{M}/M are not in line with the values predicted by the model when the estimates with two variables are used. In particular, at these values the model has a larger precautionary component than is observed (i.e., smaller values of W/M and higher values of \underline{M}/M). Another clear illustration of the difference produced by using more variables comes from observing that the estimates for p for households without an ATM presented in Table S.III are often equal to zero, while this is not true in the paper. The reason, again, is that if $p = 0$, the model predicts values for $W/M = 2$ and $\underline{M}/M = 0$ which are far from those observed in the data. That is why, when estimated using four variables, the value of p is closer to 7.

APPENDIX H: A MODEL WITH COSTLY RANDOM WITHDRAWALS

The dynamic model discussed in the paper has the unrealistic feature that agents withdraw every time a match with a financial intermediary occurs, thus

making as many withdrawals as contacts with the financial intermediary; many of the withdrawals are of a very small size. In this section we extend the model to the case where the withdrawals (deposits) done upon the random contacts with the financial intermediary are subject to a fixed cost f , assuming $0 < f < b$. The model produces a more realistic depiction of the distribution of withdrawals by limiting the minimum withdrawal size. In particular, we show that the minimum withdrawal size is determined by the fixed cost relative to the interest cost (i.e., f/R) and that it is independent of p . On the other hand, if f is large relative to b , the predictions get closer to those of the Baumol and Tobin (BT) model. Indeed, as f goes to b , then there is no advantage to a chance meeting with the intermediary, and hence the model is identical to the one of the previous section, but with $p = 0$.

In this section we formulate the dynamic programming problem for $f > 0$, solve its Bellman equation, and characterize its optimal decision rule. We also derive the corresponding invariant distribution and the expressions for n , M , W , and \underline{M} . As several features of this case are similar to the previous one we streamline the presentation and do not report results on comparative statics or welfare.

We skip the formulation of the total cost problem, which is exactly parallel to the one for the case of $f = 0$. Using notation that is analogous to the one that was used above, the Bellman equation for this problem when the agent is not matched with a financial intermediary is given by

$$(S5) \quad rV(m) = Rm + p \min\{V^* + f - V(m), 0\} + V'(m)(-c - m\pi),$$

where $V^* \equiv \min_{\hat{m}} V(\hat{m})$ and $\min\{V^* + f - V(m), 0\}$ takes into account that it may not be optimal to withdraw/deposit for all contacts with a financial intermediary. Indeed, whether the agent chooses to do so will depend on her level of cash balances.

We will guess, and later verify, a shape for $V(\cdot)$ that implies a simple threshold rule for the optimal policy. Our guess is that $V(\cdot)$ is strictly decreasing at $m = 0$ and single peaked, attaining a minimum at a finite value of m . Then we guess that there will be two thresholds, \underline{m} and \bar{m} , that satisfy

$$(S6) \quad V^* + f = V(\underline{m}) = V(\bar{m}).$$

Thus solving the Bellman equation is equivalent to finding five numbers m^* , m^{**} , \underline{m} , \bar{m} , and V^* , and a function $V(\cdot)$ such that

$$(S7) \quad V^* = V(m^*), \quad 0 = V'(m^*),$$

$$(S8) \quad V(m) = \begin{cases} \frac{Rm + p(V^* + f) - V'(m)(c + m\pi)}{r + p}, & \text{if } m \in (0, \underline{m}), \\ \frac{Rm - V'(m)(c + m\pi)}{r}, & \text{if } m \in (\underline{m}, \bar{m}), \\ \frac{Rm + p(V^* + f) - V'(m)(c + m\pi)}{r + p}, & \text{if } m \in (\bar{m}, m^{**}), \end{cases}$$

and the boundary conditions

$$(S9) \quad V(0) = V^* + b, \quad V(m) = V^* + b \quad \text{for } m > m^{**}.$$

Hence the optimal policy in this model is to pay the fixed cost f and withdraw cash if contact with the financial intermediary occurs when cash balances are in the $(0, \underline{m})$ range, or to deposit if cash balances are larger than \bar{m} . In either case, the withdrawal or deposit is such that the posttransfer cash balances are equal to m^* . If the agent contacts a financial intermediary when her cash balances are in (\underline{m}, \bar{m}) , then no action is taken. If the agent's cash balances get to zero, then the fixed cost b is paid, and after the withdrawal the cash balances are set to m^* . Notice that $m^* \in (\underline{m}, \bar{m})$. Hence in this model withdrawals have a minimum size given by $m^* - \underline{m}$. This is a more realistic depiction of actual cash management.

Now we turn to the characterization and solution of the Bellman equation.

PROPOSITION 2: For a given V^* , \underline{m} , \bar{m} , and m^{**} satisfying $0 < \underline{m} < \bar{m} < m^{**}$, the solution of (S8) for $m \in (\underline{m}, \bar{m})$ is given by

$$(S10) \quad V(m) = \varphi(m, A_\varphi) \\ \equiv \frac{-Rc/(r + \pi)}{r} + \frac{Rm}{r + \pi} + \left(\frac{c}{r}\right)^2 A_\varphi \left[1 + \pi \frac{m}{c}\right]^{-r/\pi}$$

for an arbitrary constant A_φ .

Likewise, the solution of (S8) for $m \in (0, \underline{m})$ or $m \in (\bar{m}, m^{**})$ is given by

$$(S11) \quad V(m) = \eta(m, V^*, A_\eta) \\ \equiv \frac{p(V^* + f) - \frac{Rc}{r + p + \pi}}{r + p} + \frac{Rm}{r + p + \pi} \\ + \left(\frac{c}{r + p}\right)^2 A_\eta \left[1 + \pi \frac{m}{c}\right]^{-(r+p)/\pi}$$

for an arbitrary constant A_η .

PROOF: The proposition is readily verified by differentiating (S10) and (S11) in their respective domains. Q.E.D.

Next we are going to list a system of five equations in five unknowns that describes a C^1 solution of $V(m)$ on the range $[0, m^*]$. The unknowns in the system are V^* , A_η , A_φ , \underline{m} , and m^* . Using Proposition 2, and the boundary conditions (S6), (S8), and (S9), the system is given by the equations

$$(S12) \quad \varphi_m(m^*, A_\varphi) = 0,$$

$$(S13) \quad \varphi(m^*, A_\varphi) = V^*,$$

$$(S14) \quad \eta(\underline{m}, V^*, A_\eta) = V^* + f,$$

$$(S15) \quad \eta(0, V^*, A_\eta) = V^* + b,$$

$$(S16) \quad \varphi(\underline{m}, A_\varphi) = V^* + f.$$

In the proof of Proposition 3 we show that the solution of this system can be found by solving one nonlinear equation in one unknown, namely \underline{m} . Once the system is solved, it is straightforward to extend the solution to the range (m^*, ∞) .

PROPOSITION 3: *There is a solution for the system (S12)–(S16). The solution characterizes a C^1 function that is strictly decreasing on $(0, m^*)$, convex on $(0, \bar{m})$, and strictly increasing on (m^*, m^{**}) . This function solves the Bellman equations described above. The value function satisfies*

$$(S17) \quad V(0) = \frac{R}{r} m^* + b.$$

See Appendix H.1 for the proof.

Next we present a proposition about the determinants of the range of inaction $m^* - \underline{m}$ or, equivalently, the size of the minimum withdrawal.

PROPOSITION 4: *The scaled range of inaction $(m^* - \underline{m})/(c + m^* \pi)$ solves*

$$(S18) \quad \frac{f}{R(c + m^* \pi)} = \left(\frac{m^* - \underline{m}}{c + m^* \pi} \right)^2 \left[\frac{1}{2} + \sum_{k=1} \frac{1}{(k+2)!} \left(\frac{m^* - \underline{m}}{c + m^* \pi} \right)^k \prod_{j=2}^{k+1} (r + j\pi) \right];$$

hence it can be written as

$$(S19) \quad \frac{m^* - \underline{m}}{c + m^* \pi} = \sqrt{\frac{2f}{R(c + \pi m^*)}} + o\left(\left(\frac{f}{R(c + \pi m^*)}\right)^2\right)$$

and, for $\pi = 0$, it is increasing in f/R with elasticity smaller than $1/2$.

For the proof, see Appendix H.1.

The quantity $c + m^*\pi$ is a measure of the use of cash per period when $m = m^*$. The quantity $m^* - \underline{m}$ also measures the size of the smallest withdrawal. Hence $(m^* - \underline{m})/(c + m^*\pi)$ is a normalized measure of the minimum withdrawal. The proposition shows that, for $\pi = 0$, the minimum withdrawal does not depend on p and b , and that, as the approximation above makes clear, it is analogous to the withdrawal of the BT model facing a fixed cost f and an interest rate R . Quantitatively, these properties continue to hold for $\pi > 0$.

The next proposition examines the expected number of withdrawals n .

PROPOSITION 5: *The expected number of cash withdrawals per unit of time, $n(m^*/c, \underline{m}/c, \pi, p)$, is*

$$(S20) \quad n = \frac{p}{(p/\pi) \log(1 + (m^* - \underline{m})\pi/c) + 1 - (1 + \underline{m}\pi/c)^{-p/\pi}}$$

and the fraction of agents with cash balances below \underline{m} is given by

$$(S21) \quad H(\underline{m}) = \frac{1 - (1 + \underline{m}\pi/c)^{-p/\pi}}{(p/\pi) \log(1 + (m^* - \underline{m})\pi/c) + 1 - (1 + \underline{m}\pi/c)^{-p/\pi}}.$$

See Appendix H.1 for the proof.

Inspection of equation (S20) confirms that when $m^* > \underline{m}$, the expected number of withdrawals (n) is no longer bounded below by p . Indeed, as $p \rightarrow \infty$, then $n \rightarrow [(1/\pi) \log(1 + (m^* - \underline{m})\pi/c)]^{-1}$, which is the reciprocal of the time that it takes for an agent who starts with money holdings m^* (and consuming at rate c when the inflation rate is π) to reach real money holdings \underline{m} .

As in the case of $f = 0$, for any $m \in [0, \underline{m}]$, the density $h(m)$ solves the ordinary differential equation (ODE) given by

$$(S22) \quad \frac{\partial h(m)}{\partial m} = \frac{(p - \pi)}{(\pi m + c)} h(m).$$

The reason is that in this interval the behavior of the system is the same as for $f = 0$. On the interval $m \in [\underline{m}, m^*]$, the density $h(m)$ solves the ODE

$$(S23) \quad \frac{\partial h(m)}{\partial m} = \frac{-\pi}{(\pi m + c)} h(m).$$

In this interval the chance meetings with the intermediary do not trigger a withdrawal; hence it is as if $p = 0$.

PROPOSITION 6: For $H(\underline{m})$ as given in (S21), the cumulative distribution function (CDF) $H(m)$ for $m \in [0, \underline{m}]$ is

$$(S24) \quad H(m) = H(\underline{m}) \frac{\left(1 + \frac{\pi}{c} m\right)^{p/\pi} - 1}{\left(1 + \frac{\pi}{c} \underline{m}\right)^{p/\pi} - 1};$$

for $m \in [\underline{m}, m^*]$, it is

$$(S25) \quad H(m) = [1 - H(\underline{m})] \frac{\log\left(1 + \frac{\pi}{c} m\right) - \log\left(1 + \frac{\pi}{c} m^*\right)}{\log\left(1 + \frac{\pi}{c} m^*\right) - \log\left(1 + \frac{\pi}{c} \underline{m}\right)} + 1.$$

For the proof see Appendix H.1.

Using the previous density, the average money holding $M(\frac{m^*}{c}, \frac{\underline{m}}{c}, \pi, p)$ is

$$M = \int_0^{\underline{m}} m h(m) dm + \int_{\underline{m}}^{m^*} m h(m) dm,$$

a closed form expression of which can be found in Technical Appendix F, available on our websites.

The average withdrawal $W(\frac{m^*}{c}, \frac{\underline{m}}{c}, \pi, p)$ is given by

$$(S26) \quad W = m^* \left[1 - \frac{p}{n} H(\underline{m})\right] + \left[\frac{p}{n} H(\underline{m})\right] \frac{\int_0^{\underline{m}} (m^* - m) h(m) dm}{H(\underline{m})},$$

a closed form expression of which can be found in Technical Appendix G, available on our websites. To understand this expression notice that $n - pH(\underline{m})$ is the number of withdrawals in a unit of time that occur because agents reach zero balances, so if we divide it by the total number of withdrawals per unit of time, n , we obtain the fraction of withdrawals that occur when agents reach zero balances. Each of these withdrawals is of size m^* . The complementary fraction gives the withdrawals that occur due to a chance meeting with the intermediary. Conditional on having money balances in $(0, \underline{m})$, then a withdrawal of size $(m^* - m)$ happens with frequency $h(m)/H(\underline{m})$.

By the same reasoning as in the $f = 0$ case, the average amount of money that an agent has at the time of withdrawal, \underline{M} , satisfies

$$\underline{M} = 0 \left[1 - \frac{p}{n} H(\underline{m})\right] + \left[\frac{p}{n} H(\underline{m})\right] \frac{\int_0^{\underline{m}} m h(m) dm}{H(\underline{m})}.$$

As in the $f = 0$ model, the relation $\underline{M} = m^* - W$ holds. Inserting the definition of \underline{M} into the expression for M we obtain

$$\underline{M} = \frac{p}{n} M \left[1 - \frac{\int_{\underline{m}}^{m^*} mh(m) dm}{M} \right].$$

H.1. Proofs for the Model With Costly Withdrawals

PROOF OF PROPOSITION 3: Recall the five equation system (S12)–(S16). We use repeated substitution to arrive at one nonlinear equation in one unknown, namely \underline{m} . Equations (S12) and (S13) yield $V^* = R/rm^*$. Replacing V^* by this expression yields (S13), so we have a system of four equations in four unknowns. We use (S12) to define $A_\varphi(m^*)$ as its solution, that is, $\varphi_m(m^*, A_\varphi(m^*)) = 0$, which yields

$$(S27) \quad A_\varphi(m^*) = \frac{rR}{c(r+\pi)} \left[1 + \pi \frac{m^*}{c} \right]^{1+r/\pi}.$$

To solve for $A_\eta(m^*)$, we use (S14) and $rV^* = Rm^*$ to get

$$(S28) \quad A_\eta(m^*) = \frac{r+p}{c^2} \left(Rm^* + br + p(b-f) + \frac{Rc}{r+p+\pi} \right).$$

Next we replace A_η and A_φ in (S14) and (S16) so we get two nonlinear equations:

$$\begin{aligned} \eta(\underline{m}, (m^*R/r), A_\eta(m^*)) &= (m^*R/r) + f, \\ \varphi(\underline{m}, A_\varphi(m^*)) &= (m^*R/r) + f. \end{aligned}$$

The first equation, using (S28) to substitute for $A_\eta(m^*)$, yields

$$(S29) \quad m_1^*(\underline{m}) = \left(\frac{r+p}{R} \right) \left[\frac{c}{r+p} \left(\frac{pf}{c} - \frac{R}{(r+p+\pi)} \right) + \frac{\left(\frac{R}{r+p+\pi} \right) \underline{m} + b \left(1 + \frac{\pi}{c} \underline{m} \right)^{-(r+p)/\pi} - f}{1 - \left(1 + \frac{\pi}{c} \underline{m} \right)^{-(r+p)/\pi}} \right].$$

Notice that for $\pi > 0$, $m_1^*(\underline{m})$ is continuous in $(0, \infty)$ and that

$$\lim_{\underline{m} \rightarrow 0} m_1^*(\underline{m}) = +\infty \quad \text{and} \quad \lim_{\underline{m} \rightarrow \infty} \frac{m_1^*(\underline{m})}{\underline{m}} = \left(\frac{r+p}{r+p+\pi} \right) < 1.$$

The second equation, using (S27) to substitute for $A_\varphi(m^*)$, yields

$$(S30) \quad m^* = \sigma(m^*, \underline{m}) \\ \equiv \left[\frac{r}{r+\pi} \right] \underline{m} + \frac{c}{(r+\pi)} \left(\frac{\left[1 + \pi \frac{m^*}{c} \right]^{1+r/\pi}}{\left[1 + \frac{\pi}{c} \underline{m} \right]^{r/\pi}} - 1 \right) - f \frac{r}{R}.$$

We define $m_2^*(\underline{m})$ as the solution to $m_2^*(\underline{m}) = \sigma(m_2^*(\underline{m}), \underline{m})$. Notice that σ is increasing in m^* with

$$\frac{\partial \sigma(\underline{m}, \underline{m})}{\partial m^*} = 1, \quad \frac{\partial \sigma(m^*, \underline{m})}{\partial m^*} > 1 \quad \text{for} \quad m^* > \underline{m},$$

and

$$\sigma(\underline{m}, \underline{m}) = \underline{m} - f \frac{r}{R}$$

so that $m_2^*(\underline{m})$ is well defined and continuous on $[0, \infty)$, that $m_2^*(0) < \infty$, and that $m_2^*(\underline{m}) > \underline{m}$ for all \underline{m} . Using the properties of $m_1^*(\cdot)$ and $m_2^*(\cdot)$, the intermediate value theorem implies that there is an $\hat{m} \in (0, \infty)$ such that $m_1^*(\hat{m}) = m_2^*(\hat{m})$.

For $\pi < 0$, the range of the functions defined above is $[0, -\pi/c]$. By a straightforward adaptation of the arguments above one can show the existence of the solution of the two equations in this case.

Next we verify the guesses that the value function $V(m)$ is decreasing in a neighborhood of $m = 0$ and single peaked. The convexity of $V(m)$ is equivalent to showing that $A_\varphi > 0$ and $A_\eta > 0$, which can be readily established from (S27) and (S28) provided $b > f$. Moreover, since $A_\varphi > 0$ and $A_\eta > 0$, then $V(m)$ is strictly decreasing on $(0, m^*)$.

We extend the value function to the range (m^*, ∞) . Given the values already found for V^* and A_φ , we find \bar{m} as the solution to $\varphi(\bar{m}, A_\varphi) = V^* + f$, that is, \bar{m} solves

$$\left(\frac{R}{r+\pi} \right) \bar{m} + \left(\frac{c}{r} \right)^2 A_\varphi \left[1 + \frac{\pi}{c} \bar{m} \right]^{-r/\pi} = V^* + f + \frac{Rc/(r+\pi)}{r}.$$

Now given V^* and \bar{m} , we find the constant \bar{A}_η by solving $\eta(\bar{m}, V^*, \bar{A}_\eta) = V^* + f$:

$$\bar{A}_\eta = \left(\frac{r+p}{c}\right)^2 \left(1 + \frac{\pi}{c}\bar{m}\right)^{(r+p)/\pi} \cdot \left(V^* + f - \frac{p(V^* + f) - Rc/(r+p+\pi)}{r+p} - \frac{R}{r+p+\pi}\bar{m}\right).$$

Given V^* and \bar{A}_η , we find m^{**} as the solution of $\eta(m^{**}, V^*, \bar{A}_\eta) = V^* + b$.

Now we establish that V is strictly increasing in (m^*, m^{**}) . For this, notice that since $\eta(\bar{m}, V^*, \bar{A}_\eta) = \varphi(\bar{m}, A_\varphi)$, then by inspecting the Bellman equation (S8) it follows that they have the same derivative with respect to m at \bar{m} . Since $\varphi(\bar{m}, A_\varphi)$ is convex, this derivative is strictly positive. There are two cases. If \bar{A}_η is positive, then $\eta(\bar{m}, V^*, \bar{A}_\eta)$ is convex in this range and hence V is increasing. If \bar{A}_η is negative, then $\eta(\bar{m}, V^*, \bar{A}_\eta)$ is concave but it is increasing since it cannot achieve a maximum since it is the sum of a linear increasing and a bounded concave function. *Q.E.D.*

PROOF OF PROPOSITION 4: In Proposition 2 we show that $V(m)$ is analytical in the interval $[\underline{m}, m^*]$. Using $V^i(\cdot)$ to denote the i th derivative of $V(\cdot)$, we can write

$$V(m) = V(m^*) + \sum_{i=1}^{\infty} \frac{1}{i!} V^i(m^*) (m - m^*)^i.$$

Using $f = V(\underline{m}) - V(m^*)$, we write $f = \sum_{i=1}^{\infty} (1/i!) V^i(m^*) (\underline{m} - m^*)^i$. Next we find an expression for $V^i(m^*)$. Differentiating the Bellman equation (S5) with respect to m in a neighborhood of m^* yields

$$(S31) \quad R - [r + \pi]V^1(m) = V^2(m)[c + \pi m];$$

evaluating at m^* , using that $V^1(m^*) = 0$, we obtain $V^2(m^*) = R/c + \pi m^*$. Differentiating (S31) repeatedly and using induction yields

$$(S32) \quad [r + (1+i)\pi]V^{i+1}(m) = -V^{i+2}(m)[c + \pi m] \quad \text{for } i \geq 1.$$

Solving the difference equation in (S32) evaluated at m^* gives

$$(S33) \quad V^{i+1}(m^*) = (-1)^{i-1} \frac{R}{(c + m^*\pi)^i} \prod_{j=2}^i [r + j\pi] \quad \text{for } i \geq 2.$$

Using $V^1(m^*) = 0$, $V^2(m^*) = R/c + \pi m^*$ and (S33) for higher order derivatives into $f = \sum_{i=1}^{\infty} (1/i!) V^i(m^*) (\underline{m} - m^*)^i$, and rearranging yields equation (S18).

For $\pi = 0$, $z = (m^* - \underline{m})/c$ solves $f/(Rc) = z^2\psi(z)$ where $\psi(z) = 1/2 + \sum_{k=1}^{\infty} (r^k z^k / (k+2)!)$. Since $\psi > 0$ and increasing in z , then $(m^* - \underline{m})/c$ is increasing in $f/(Rc)$ with elasticity smaller than $1/2$. *Q.E.D.*

PROOF OF PROPOSITION 5: The proof for n is analogous to that of the baseline model with $f = 0$. Let \underline{t} be the time to deplete balances from m^* to \underline{m} : it solves $(m^* - \underline{m}) = c \int_0^{\underline{t}} e^{\pi s} ds$ or $\underline{t} = (1/\pi) \log(1 + (m^* - \underline{m})\pi/c)$. The distribution of the time between withdrawals for this model has density equal to zero over $(0, \underline{t})$ with the right truncation denoted by \bar{t} , which solves $\underline{m} = c \int_0^{\bar{t}} \exp(\pi s) ds$ or $\bar{t} = (1/\pi) \log(1 + \underline{m}\pi/c)$. Thus, the expected time between withdrawals is given by $\underline{t} + (1 - e^{-p\underline{t}})/p$. Substituting the above expressions into this formula and taking the reciprocal value yields equation (S20) in the paper.

Now we turn to the derivation of $H(\underline{m})$. After each withdrawal, the agent spends \underline{t} units of time with $m \in (\underline{m}, m^*)$. The fundamental theorem of renewal theory implies that the expected time that an agent spends with $m \in (\underline{m}, m^*)$ in a period of length T converges to $n\underline{t}$ as $T \rightarrow \infty$. By the ergodic theorem, $n\underline{t} = H(m^*) - H(\underline{m}) = 1 - H(\underline{m})$. Replacing the expressions for n and \underline{t} yields the desired result. *Q.E.D.*

PROOF OF PROPOSITION 6: By repeated differentiation of (S24) (respectively (S25)) it is readily verified that (S22) is satisfied on the domain $(0, \underline{m})$ (respectively S23 on the domain (\underline{m}, m^*)). The proof is completed by verifying that the piecewise definition of H satisfies the boundary conditions that $H(0) = 0$, $H(m^*) = 1$, and that both (S24) and (S25) evaluated at \underline{m} equal $H(\underline{m})$. *Q.E.D.*

APPENDIX I: TESTING THE $f = 0$ MODEL VS. THE $f > 0$ MODEL

We examined the extent to which imposing the constraint that $f = 0$ diminishes the ability of the model to fit the data. To do so, we reestimated the model letting f/c vary across province–year–household type, and compared the fit of the restricted ($f = 0$) with the unrestricted model using a likelihood ratio test.

Table S.V reports the percentage of province–year–consumption cells where the null hypothesis of $f = 0$ is rejected at a 5% confidence level. It appears that only for a small fraction of cases (19% for those cells that correspond to households with ATM cards and 2% for those without cards) there may be some improvement in the fit of the model by letting $f > 0$. We explored two approaches to estimate the $f > 0$ model. In one case we let f/c vary across province–year–household type; in the other case we fixed f/c to a common, nonzero value for all province–year–types (aggregating all the cash consumption levels). We argue that while there is an improvement in the fit for a relatively small fraction of province–years by letting $f > 0$, as documented in Table S.V, the variables

TABLE S.V
 TESTING THE $f > 0$ VS. $f = 0$ MODEL

	Household w/o ATM	Household w. ATM
Hypothesis $f = 0$ is rejected ^a	2%	19%

^aPercentage of estimates where the null hypothesis of $f = 0$ is rejected by a likelihood ratio test at the 5% confidence level. Based on a comparison between the likelihood for the restricted model ($f = 0$) with the likelihood for a model where f/c is allowed to vary across province-year type.

in our data set do not provide us with the type of information that would allow the parameter f to be identified. Indeed, our findings (not reported) show that when we let $f > 0$ and estimate the model for each province-year-type, the average as well as median t -statistic of the parameters ($p, b/c, f/c$) are very low, and the average correlation between the estimates is extremely high. Additionally, there is an extremely high variability in the estimated parameters across province-years.³ We conclude that the information in our data set does not allow us to estimate $p, b/c,$ and f/c with a reasonable degree of precision. As we explained when we introduced the model with $f > 0$, the reason to consider that model is to eliminate the extremely small withdrawals that the model with $f = 0$ implies. Hence, what would be helpful to estimate f is information on the minimum size of withdrawals or some other feature of the withdrawal distribution.

³The results are available upon request. In the case where f/c is fixed at the same value for all province-years, the average t -statistics are higher, but the estimated parameters still vary considerably across province-years.