Econometrica Supplementary Material

SUPPLEMENT TO "IDENTIFYING LONG-RUN RISKS: A BAYESIAN MIXED-FREQUENCY APPROACH"

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THIS APPENDIX contains supplementary material and consists of the following sections:

A. Data Sources
B. The Measurement Error Model for Consumption
C. Solving the Long-Run Risks Model
D. State-Space Representations of the Empirical Models
E. Posterior Inference
F. Supplementary Figures and Tables

A. DATA SOURCE

A.1. *Nominal PCE*

We download seasonally adjusted data for nominal PCE from NIPA Tables 2.3.5 and 2.8.5. We then compute within-quarter averages of monthly observations and within-year averages of quarterly observations.

A.2. *Real PCE*

We use Table 2.3.3, Real Personal Consumption Expenditures by Major Type of Product, Quantity Indexes (A:1929–2014) (Q:1947:Q1–2014:Q4) to extend Table 2.3.6, Real Personal Consumption Expenditures by Major Type of Product, Chained Dollars (A:1995–2014) (Q:1995:Q1–2014:Q4). Monthly data are constructed analogously using Table 2.8.3 and Table 2.8.6.

A.3. *Real Per Capita PCE: ND + S*

The LRR model defines consumption as per capita consumer expenditures on nondurables and services. We download mid-month population data from NIPA Table 7.1 (A:1929–2014) (Q:1947:Q1–2014:Q4) and from Federal Reserve Bank of St. Louis's FRED database (M:1959:M1–2014:M12). We convert consumption to per capita terms.

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A.4. *Dividend and Market Returns Data*

Data are from the Center for Research in Security Prices (CRSP). The three monthly series from CRSP are the value-weighted with-, RN_t, and without-dividend nominal returns, RX_t, of CRSP stock market indexes (NYSE/AMEX/NASDAQ/ARCA), and the CPI inflation rates, π_t. The sample period is from 1929:M1 to 2014:M12. The monthly real dividend series are constructed as in Hodrick (1992):

1. A normalized nominal value-weighted price series is produced by initializing $P_0 = 1$ and recursively setting $P_t = (1 + RX_t)P_{t-1}$.
2. A normalized nominal divided series, D_t^{Raw}, is obtained by recognizing that $D_t^{\text{Raw}} = (RN_t - RX_t)P_{t-1}$.
3. Following Robert Shiller, we smooth out dividend series by aggregating three months' values of the raw nominal dividend series $D_t = \sum_{i=0}^{2} D_{t-i}^{\text{Raw}}$ and apply the following quarterly interpolation. Here, $D_t, D_{t-3}, \dots$ is the last month of the quarter:

$$D_{t-m} = D_t - \frac{m}{3}(D_t - D_{t-3}), \quad m \in \{0, 1, 2\}. \tag{A.1}$$

4. We then compute the real dividend growth $g_{d,t}$ by subtracting the actual inflation from the interpolated nominal dividend growth

$$g_{d,t} = \log(D_t) - \log(D_{t-1}) - \pi_t. \tag{A.2}$$

Here inflation rates are computed using the log differences of the consumer price index (CPI) from the Bureau of Labor Statistics.

Market returns, RN_{t+1}, are also converted from nominal to real terms using the CPI inflation rates and denoted by $r_{m,t+1}$.

A.5. *Ex Ante Risk-Free Rate*

The ex ante risk-free rate is constructed as in the online appendix of Beeler and Campbell (2012). Nominal yields to calculate risk-free rates are the CRSP Fama Risk Free Rates. Even though our model runs in monthly frequencies, we use the three-month yield because of the larger volume and higher reliability. We subtract annualized three-month inflation, $\pi_{t,t+3}$, from the nominal yield, $i_{f,t}$, to form a measure of the ex post (annualized) real three-month interest rate. The ex ante real risk-free rate, $r_{f,t}$, is constructed as a fitted value from a projection of the ex post real rate on the current nominal yield, $i_{f,t}$, and inflation over the previous year, $\pi_{t-12,t}$:

$$\begin{aligned} i_{f,t} - \pi_{t,t+3} &= \beta_0 + \beta_1 i_{f,t} + \beta_2 \pi_{t-12,t} + \varepsilon_{t+3}, \\ r_{f,t} &= \hat{\beta}_0 + \hat{\beta}_1 i_{f,t} + \hat{\beta}_2 \pi_{t-12,t}. \end{aligned}$$

The ex ante real risk-free rates are available from 1929:M1 to 2014:M12.

B. THE MEASUREMENT ERROR MODEL FOR CONSUMPTION

For expositional purposes, we assume that the accurately measured low-frequency observations are available at quarterly frequency (instead of annual frequency as in the main text). Correspondingly, we define the time subscript $t = 3(j - 1) + m$, where month $m = 1, 2, 3$ and quarter $j = 1, \dots$. We use uppercase C to denote the level of consumption and lowercase c to denote percentage deviations from some log-linearization point.

Growth rates are approximated as log differences and we use a superscript o to distinguish observed from "true" values.

The measurement error model presented in the main text can be justified by assuming that the statistical agency uses a high-frequency proxy series to determine monthly consumption growth rates. We use $Z_{3(j-1)+m}$ to denote the monthly value of the proxy series and $Z^q_{(j)}$ the quarterly aggregate. Suppose the proxy variable provides a noisy measure of monthly consumption. More specifically, we consider a multiplicative error model of the form

$$Z_{3(j-1)+m} = C_{3(j-1)+m} \exp(\epsilon_{3(j-1)+m}). \tag{A.3}$$

The interpolation is executed in two steps. In the first step we construct a series $\tilde{C}^o_{3(j-1)+m}$, and in the second step we rescale the series to ensure that the reported monthly consumption data add up to the reported quarterly consumption data within the period. In Step 1, we start from the level of consumption in quarter $j-1$, $C^q_{(j-1)}$, and define

$$\begin{aligned}
\tilde{C}^o_{3(j-1)+1} &= C^{q,o}_{(j-1)}\left(\frac{Z_{3(j-1)+1}}{Z^q_{(j-1)}}\right), \\
\tilde{C}^o_{3(j-1)+2} &= C^{q,o}_{(j-1)}\left(\frac{Z_{3(j-1)+1}}{Z^q_{(j-1)}}\right)\left(\frac{Z_{3(j-1)+2}}{Z_{3(j-1)+1}}\right) = C^{q,o}_{(j-1)}\left(\frac{Z_{3(j-1)+2}}{Z^q_{(j-1)}}\right), \\
\tilde{C}^o_{3(j-1)+3} &= C^{q,o}_{(j-1)}\left(\frac{Z_{3(j-1)+1}}{Z^q_{(j-1)}}\right)\left(\frac{Z_{3(j-1)+2}}{Z_{3(j-1)+1}}\right)\left(\frac{Z_{3(j-1)+3}}{Z_{3(j-1)+2}}\right) = C^{q,o}_{(j-1)}\left(\frac{Z_{3(j-1)+3}}{Z^q_{(j-1)}}\right).
\end{aligned} \tag{A.4}$$

Thus, the growth rates of the proxy series are used to generate monthly consumption data for quarter q. Summing over the quarter yields

$$\begin{aligned}
\tilde{C}^{q,o}_{(j)} &= \sum_{m=1}^{3} \tilde{C}^o_{3(j-1)+m} \\
&= C^{q,o}_{(j-1)}\left[\frac{Z_{3(j-1)+1}}{Z^q_{(j-1)}} + \frac{Z_{3(j-1)+2}}{Z^q_{(j-1)}} + \frac{Z_{3(j-1)+3}}{Z^q_{(j-1)}}\right] \\
&= C^{q,o}_{(j-1)} \frac{Z^q_{(j)}}{Z^q_{(j-1)}}.
\end{aligned} \tag{A.5}$$

In Step 2, we adjust the monthly estimates $\tilde{C}^o_{3(j-1)+m}$ by the factor $C^{q,o}_{(j)}/\tilde{C}^{q,o}_{(j)}$, which leads to

$$\begin{aligned}
C^o_{3(j-1)+1} &= \tilde{C}^o_{3(j-1)+1}\left(\frac{C^{q,o}_{(j)}}{\tilde{C}^{q,o}_{(j)}}\right) = C^{q,o}_{(j)} \frac{Z_{3(j-1)+1}}{Z^q_{(j)}}, \\
C^o_{3(j-1)+2} &= \tilde{C}^o_{3(j-1)+2}\left(\frac{C^{q,o}_{(j)}}{\tilde{C}^{q,o}_{(j)}}\right) = C^{q,o}_{(j)} \frac{Z_{3(j-1)+2}}{Z^q_{(j)}}, \\
C^o_{3(j-1)+3} &= \tilde{C}^o_{3(j-1)+3}\left(\frac{C^{q,o}_{(j)}}{\tilde{C}^{q,o}_{(j)}}\right) = C^{q,o}_{(j)} \frac{Z_{3(j-1)+3}}{Z^q_{(j)}},
\end{aligned} \tag{A.6}$$

and guarantees that

$$C_{(j)}^{q,o} = \sum_{m=1}^{3} C_{3(j-1)+m}^{o}.$$

We now define the growth rates $g_{c,t}^{o} = \log C_t^o - \log C_{t-1}^o$ and $g_{c,t} = \log C_t - \log C_{t-1}$. By taking logarithmic transformation of (A.3) and (A.6) and combining the resulting equations, we can deduce that the growth rates for the second and third month of quarter q are given by

$$\begin{aligned} g_{c,3(j-1)+2}^{o} &= g_{c,3(j-1)+2} + \epsilon_{3(j-1)+2} - \epsilon_{3(j-1)+1}, \\ g_{c,3(j-1)+3}^{o} &= g_{c,3(j-1)+3} + \epsilon_{3(j-1)+3} - \epsilon_{3(j-1)+2}. \end{aligned} \tag{A.7}$$

The derivation of the growth rate between the third month of quarter $j-1$ and the first month of quarter j is a bit more cumbersome. Using (A.6), we can write the growth rate as

$$\begin{aligned} g_{c,3(j-1)+1}^{o} = {} & \log C_{(j)}^{q,o} + \log Z_{3(j-1)+1} - \log Z_{(j)}^{q} \\ & - \log C_{(j-1)}^{q,o} - \log Z_{3(j-2)+3} + \log Z_{(j-1)}^{q}. \end{aligned} \tag{A.8}$$

To simplify (A.8) further, we are using a log-linear approximation. Suppose we log-linearize an equation of the form

$$X_{(j)}^{q} = X_{3(j-1)+1} + X_{3(j-1)+2} + X_{3(j-1)+3}$$

around X_*^q and $X_* = X_*^q/3$, using lowercase variables to denote percentage deviations from the log-linearization point. Then,

$$x_{(j)}^{q} \approx \frac{1}{3}(x_{3(j-1)+1} + x_{3(j-1)+2} + x_{3(j-1)+3}).$$

Using (A.3) and the definition of quarterly variables as sums of monthly variables, we can apply the log-linearization as follows:

$$\begin{aligned} & \log C_{(j)}^{q,o} - \log Z_{(j)}^{q} \\ & \quad = \log\left(C_*^q / Z_*^q\right) + \epsilon_{(j)}^{q} - \frac{1}{3}(\epsilon_{3(j-1)+1} + \epsilon_{3(j-1)+2} + \epsilon_{3(j-1)+3}). \end{aligned} \tag{A.9}$$

Substituting (A.9) into (A.8) yields

$$\begin{aligned} g_{c,3(j-1)+1}^{o} = {} & g_{c,3(j-1)+1} + \epsilon_{3(j-1)+1} - \epsilon_{3(j-2)+3} + \epsilon_{(j)}^{q} - \epsilon_{(j-1)}^{q} \\ & - \frac{1}{3}(\epsilon_{3(j-1)+1} + \epsilon_{3(j-1)+2} + \epsilon_{3(j-1)+3}) \\ & + \frac{1}{3}(\epsilon_{3(j-2)+1} + \epsilon_{3(j-2)+2} + \epsilon_{3(j-2)+3}). \end{aligned} \tag{A.10}$$

An “annual” version of this equation appears in the main text.

C. SOLVING THE LONG-RUN RISKS MODEL

This section provides solutions for the consumption and dividend claims for the endowment process:

$$\begin{aligned}
g_{c,t+1} &= \mu_c + x_t + \sigma_{c,t}\eta_{c,t+1}, \\
g_{d,t+1} &= \mu_d + \phi x_t + \pi\sigma_{c,t}\eta_{c,t+1} + \sigma_{d,t}\eta_{d,t+1}, \\
x_{t+1} &= \rho x_t + \sigma_{x,t}\eta_{x,t+1}, \\
x_{\lambda,t+1} &= \rho_\lambda x_{\lambda,t} + \sigma_\lambda \eta_{\lambda,t+1}, \\
\sigma^2_{c,t+1} &= (1-\nu_c)(\varphi_c\bar{\sigma})^2 + \nu_c\sigma^2_{c,t} + \sigma_{w_c}w_{c,t+1}, \\
\sigma^2_{x,t+1} &= (1-\nu_x)(\varphi_x\bar{\sigma})^2 + \nu_x\sigma^2_{x,t} + \sigma_{w_x}w_{x,t+1}, \\
\sigma^2_{d,t+1} &= (1-\nu_d)(\varphi_d\bar{\sigma})^2 + \nu_d\sigma^2_{d,t} + \sigma_{w_d}w_{d,t+1}, \\
&\eta_{i,t+1}, \eta_{\lambda,t+1}, w_{i,t+1} \sim N(0,1),\ i \in \{c, x, d\}.
\end{aligned} \tag{A.11}$$

The Euler equation for the economy is

$$\mathbb{E}_t\big[\exp(m_{t+1} + r_{i,t+1})\big] = 1, \quad i \in \{c, m\}, \tag{A.12}$$

where

$$m_{t+1} = \theta\log\delta + \theta x_{\lambda,t+1} - \frac{\theta}{\psi}g_{c,t+1} + (\theta-1)r_{c,t+1} \tag{A.13}$$

is the log of the real stochastic discount factor (SDF), $r_{c,t+1}$ is the log return on the consumption claim, and $r_{m,t+1}$ is the log market return. Equation (A.13) is derived in Section C.5 below. Returns are given by the approximation of Campbell and Shiller (1988):

$$\begin{aligned}
r_{c,t+1} &= \kappa_0 + \kappa_1 pc_{t+1} - pc_t + g_{c,t+1}, \\
r_{m,t+1} &= \kappa_{0,m} + \kappa_{1,m} pd_{t+1} - pd_t + g_{d,t+1}.
\end{aligned} \tag{A.14}$$

The risk premium on any asset is

$$\mathbb{E}_t(r_{i,t+1} - r_{f,t}) + \frac{1}{2}\operatorname{Var}_t(r_{i,t+1}) = -\operatorname{Cov}_t(m_{t+1}, r_{i,t+1}). \tag{A.15}$$

In Section C.1, we solve for the law of motion for the return on the consumption claim, $r_{c,t+1}$. In Section C.2, we solve for the law of motion for the market return, $r_{m,t+1}$. The risk-free rate is derived in Section C.3. All three solutions depend on linearization parameters that are derived in Section C.4. Finally, as mentioned above, the SDF is derived in Section C.5.

C.1. *Consumption Claim*

In order to derive the dynamics of asset prices, we rely on approximate analytical solutions. Specifically, we conjecture that the price-consumption ratio follows

$$pc_t = A_0 + A_1 x_t + A_{1,\lambda}x_{\lambda,t} + A_{2,c}\sigma^2_{c,t} + A_{2,x}\sigma^2_{x,t} \tag{A.16}$$

and solve for A's using (A.11), (A.12), (A.14), and (A.16).

From (A.11), (A.14), and (A.16),

$$\begin{aligned} r_{c,t+1} = {} & \big\{\kappa_0 + A_0(\kappa_1 - 1) + \mu_c + \kappa_1 A_{2,x}(1-\nu_x)(\varphi_x\bar{\sigma})^2 \\ & + \kappa_1 A_{2,c}(1-\nu_c)(\varphi_c\bar{\sigma})^2\big\} \\ & + \frac{1}{\psi}x_t + A_{1,\lambda}(\kappa_1\rho_\lambda - 1)x_{\lambda,t} \\ & + A_{2,x}(\kappa_1\nu_x - 1)\sigma^2_{x,t} + A_{2,c}(\kappa_1\nu_c - 1)\sigma^2_{c,t} \\ & + \sigma_{c,t}\eta_{c,t+1} + \kappa_1 A_1\sigma_{x,t}\eta_{x,t+1} + \kappa_1 A_{1,\lambda}\sigma_\lambda\eta_{\lambda,t+1} \\ & + \kappa_1 A_{2,x}\sigma_{w_x}w_{x,t+1} + \kappa_1 A_{2,c}\sigma_{w_c}w_{c,t+1}, \end{aligned} \tag{A.17}$$

and from (A.11), (A.12), (A.14), and (A.16),

$$\begin{aligned} m_{t+1} = {} & (\theta-1)\big\{\kappa_0 + A_0(\kappa_1 - 1) + \kappa_1 A_{2,x}(1-\nu_x)(\varphi_x\bar{\sigma})^2 \\ & + \kappa_1 A_{2,c}(1-\nu_c)(\varphi_c\bar{\sigma})^2\big\} \\ & - \gamma\mu + \theta\log\delta - \frac{1}{\psi}x_t + \rho_\lambda x_{\lambda,t} \\ & + (\theta-1)A_{2,x}(\kappa_1\nu_x - 1)\sigma^2_{x,t} + (\theta-1)A_{2,c}(\kappa_1\nu_c - 1)\sigma^2_{c,t} \\ & - \gamma\sigma_{c,t}\eta_{c,t+1} + (\theta-1)\kappa_1 A_1\sigma_{x,t}\eta_{x,t+1} + \big\{(\theta-1)\kappa_1 A_{1,\lambda} + \theta\big\}\sigma_\lambda\eta_{\lambda,t+1} \\ & + (\theta-1)\kappa_1 A_{2,x}\sigma_{w_x}w_{x,t+1} + (\theta-1)\kappa_1 A_{2,c}\sigma_{w_c}w_{c,t+1}. \end{aligned} \tag{A.18}$$

The solutions for A's that describe the dynamics of the price-consumption ratio are determined from

$$\mathbb{E}_t[m_{t+1} + r_{c,t+1}] + \frac{1}{2}\operatorname{Var}_t[m_{t+1} + r_{c,t+1}] = 0,$$

and they are

$$\begin{aligned} A_1 &= \frac{1 - \frac{1}{\psi}}{1 - \kappa_1\rho}, \\ A_{1,\lambda} &= \frac{\rho_\lambda}{1 - \kappa_1\rho_\lambda}, \\ A_{2,x} &= \frac{\frac{\theta}{2}(\kappa_1 A_1)^2}{1 - \kappa_1\nu_x}, \\ A_{2,c} &= \frac{\frac{\theta}{2}\left(1 - \frac{1}{\psi}\right)^2}{1 - \kappa_1\nu_c}, \end{aligned} \tag{A.19}$$

and

$$A_0 = \frac{A_0^1 + A_0^2}{1 - \kappa_1},$$

where

$$A_0^1 = \log\delta + \kappa_0 + \mu\left(1 - \frac{1}{\psi}\right) + \kappa_1 A_{2,x}(1-\nu_x)(\varphi_x\bar{\sigma})^2 + \kappa_1 A_{2,c}(1-\nu_c)(\varphi_c\bar{\sigma})^2,$$

$$A_0^2 = \frac{\theta}{2}\left\{(\kappa_1 A_{1,\lambda}+1)^2\sigma_\lambda^2 + (\kappa_1 A_{2,x}\sigma_{w_x})^2 + (\kappa_1 A_{2,c}\sigma_{w_c})^2\right\}.$$

For convenience, (A.18) can be rewritten as

$$\begin{aligned} &m_{t+1} - \mathbb{E}_t[m_{t+1}] \\ &\quad = \lambda_c\sigma_{c,t}\eta_{c,t+1} + \lambda_x\sigma_{x,t}\eta_{x,t+1} + \lambda_\lambda\sigma_\lambda\eta_{\lambda,t+1} + \lambda_{w_x}\sigma_{w_x}w_{x,t+1} + \lambda_{w_c}\sigma_{w_c}w_{c,t+1}. \end{aligned}$$

Note that λ's represent the market price of risk for each source of risk. To be specific,

$$\begin{aligned} \lambda_c &= \gamma, \\ \lambda_x &= \left(\gamma - \frac{1}{\psi}\right)\frac{\kappa_1}{1-\kappa_1\rho}, \\ \lambda_\lambda &= -\frac{\theta - \kappa_1\rho_\lambda}{1-\kappa_1\rho_\lambda}, \\ \lambda_{w_x} &= \frac{\theta\left(\gamma - \frac{1}{\psi}\right)\left(1 - \frac{1}{\psi}\right)\kappa_1}{2(1-\kappa_1\nu_x)}\left(\frac{\kappa_1}{1-\kappa_1\rho}\right)^2, \\ \lambda_{w_c} &= \frac{\theta\left(\gamma - \frac{1}{\psi}\right)\left(1 - \frac{1}{\psi}\right)\kappa_1}{2(1-\kappa_1\nu_c)}. \end{aligned} \tag{A.20}$$

Similarly, rewrite (A.17) as

$$\begin{aligned} r_{c,t+1} - \mathbb{E}_t[r_{c,t+1}] = {} & \beta_{c,c}\sigma_{c,t}\eta_{c,t+1} + \beta_{c,x}\sigma_{x,t}\eta_{x,t+1} \\ & + \beta_{c,\lambda}\sigma_\lambda\eta_{\lambda,t+1} + \beta_{c,w_x}\sigma_{w_x}w_{x,t+1} + \beta_{c,w_c}\sigma_{w_c}w_{c,t+1}, \end{aligned}$$

where

$$\begin{aligned} \beta_{c,c} &= 1, \\ \beta_{c,x} &= \kappa_1 A_1, \qquad \beta_{c,\lambda} = \kappa_1 A_{1,\lambda}, \\ \beta_{c,w_x} &= \kappa_1 A_{2,x}, \qquad \beta_{c,w_c} = \kappa_1 A_{2,c}. \end{aligned} \tag{A.21}$$

The risk premium for the consumption claim is

$$\begin{aligned} &\mathbb{E}_t(r_{c,t+1} - r_{f,t}) + \frac{1}{2}\operatorname{Var}_t(r_{c,t+1}) \\ &\quad = -\operatorname{Cov}_t(m_{t+1}, r_{c,t+1}) \\ &\quad = \beta_{c,x}\lambda_x\sigma_{x,t}^2 + \beta_{c,c}\lambda_c\sigma_{c,t}^2 + \beta_{c,\lambda}\lambda_\lambda\sigma_\lambda^2 + \beta_{c,w_x}\lambda_{w_x}\sigma_{w_x}^2 + \beta_{c,w_c}\lambda_{w_c}\sigma_{w_c}^2. \end{aligned} \tag{A.22}$$

C.2. *Market Return*

Similarly, using the conjectured solution to the price-dividend ratio

$$pd_t = A_{0,m} + A_{1,m}x_t + A_{1,\lambda,m}x_{\lambda,t} + A_{2,x,m}\sigma^2_{x,t} + A_{2,c,m}\sigma^2_{c,t} + A_{2,d,m}\sigma^2_{d,t}, \tag{A.23}$$

the market return can be expressed as

$$\begin{aligned} r_{m,t+1} = {} & \kappa_{0,m} + A_{0,m}(\kappa_{1,m} - 1) + \mu_d + \kappa_{1,m}A_{2,x,m}(1-\nu_x)(\varphi_x\bar{\sigma})^2 \\ & + \kappa_{1,m}A_{2,c,m}(1-\nu_c)(\varphi_c\bar{\sigma})^2 + \kappa_{1,m}A_{2,d,m}(1-\nu_d)(\varphi_d\bar{\sigma})^2 \\ & + \left\{\phi + A_{1,m}(\kappa_{1,m}\rho - 1)\right\}x_t + (\kappa_{1,m}\rho_\lambda - 1)A_{1,\lambda,m}x_{\lambda,t} \\ & + A_{2,x,m}(\kappa_{1,m}\nu_x - 1)\sigma^2_{x,t} + A_{2,c,m}(\kappa_{1,m}\nu_c - 1)\sigma^2_{c,t} \\ & + A_{2,d,m}(\kappa_{1,m}\nu_d - 1)\sigma^2_{d,t} + \pi\sigma_{c,t}\eta_{c,t+1} + \sigma_{d,t}\eta_{d,t+1} \\ & + \kappa_{1,m}A_{1,m}\sigma_{x,t}\eta_{x,t+1} + \kappa_{1,m}A_{1,\lambda,m}\sigma_\lambda\eta_{\lambda,t+1} \\ & + \kappa_{1,m}A_{2,x,m}\sigma_{w_x}w_{x,t+1} + \kappa_{1,m}A_{2,c,m}\sigma_{w_c}w_{c,t+1} + \kappa_{1,m}A_{2,d,m}\sigma_{w_d}w_{d,t+1}. \end{aligned} \tag{A.24}$$

Given the solution for A's, A_m's can be derived as follows:

$$\begin{aligned} A_{0,m} &= \frac{A^{1st}_{0,m} + A^{2nd}_{0,m}}{1-\kappa_{1,m}}, \\ A_{1,m} &= \frac{\phi - \dfrac{1}{\psi}}{1-\kappa_{1,m}\rho}, \\ A_{1,\lambda,m} &= \frac{\rho_\lambda}{1-\kappa_{1,m}\rho_\lambda}, \\ A_{2,x,m} &= \frac{\dfrac{1}{2}\left\{(\theta-1)\kappa_1 A_1 + \kappa_{1,m}A_{1,m}\right\}^2 + (\theta-1)(\kappa_1\nu_x - 1)A_{2,x}}{1-\kappa_{1,m}\nu_x}, \\ A_{2,c,m} &= \frac{\dfrac{1}{2}(\pi-\gamma)^2 + (\theta-1)(\kappa_1\nu_c - 1)A_{2,c}}{1-\kappa_{1,m}\nu_c}, \\ A_{2,d,m} &= \frac{\dfrac{1}{2}}{1-\kappa_{1,m}\nu_d}, \end{aligned} \tag{A.25}$$

where

$$\begin{aligned} A^{1st}_{0,m} = {} & \theta\log\delta \\ & + (\theta-1)\left\{\kappa_0 + A_0(\kappa_1 - 1) + \kappa_1 A_{2,x}(1-\nu_x)(\varphi_x\bar{\sigma})^2 + \kappa_1 A_{2,c}(1-\nu_c)(\varphi_c\bar{\sigma})^2\right\} \\ & - \gamma\mu + \kappa_{0,m} + \mu_d + \kappa_{1,m}A_{2,x,m}(1-\nu_x)(\varphi_x\bar{\sigma})^2 + \kappa_{1,m}A_{2,c,m}(1-\nu_c)(\varphi_c\bar{\sigma})^2 \\ & + \kappa_{1,m}A_{2,d,m}(1-\nu_d)(\varphi_d\bar{\sigma})^2, \end{aligned}$$

$$A_{0,m}^{2\text{nd}} = \frac{1}{2}\big(\kappa_{1,m} A_{2,x,m}\sigma_{w_x} + (\theta-1)\kappa_1 A_{2,x}\sigma_{w_x}\big)^2 + \frac{1}{2}\big(\kappa_{1,m} A_{2,c,m}\sigma_{w_c} + (\theta-1)\kappa_1 A_{2,c}\sigma_{w_c}\big)^2 + \frac{1}{2}(\kappa_{1,m} A_{2,d,m}\sigma_{w_d})^2 + \frac{1}{2}\big(\kappa_{1,m} A_{1,\lambda,m}\sigma_\lambda + (\theta-1)\kappa_1 A_{1,\lambda}\sigma_\lambda + \theta\sigma_\lambda\big)^2.$$

Rewrite the market-return equation (A.24) as

$$\begin{aligned} r_{m,t+1} - \mathbb{E}_t[r_{m,t+1}] &= \beta_{m,c}\sigma_{c,t}\eta_{c,t+1} + \beta_{m,x}\sigma_{x,t}\eta_{x,t+1} + \beta_{m,d}\sigma_{d,t}\eta_{d,t+1} + \beta_{m,\lambda}\sigma_\lambda\eta_{\lambda,t+1} \\ &\quad + \beta_{m,w_x}\sigma_{w_x}w_{x,t+1} + \beta_{m,w_c}\sigma_{w_c}w_{c,t+1} + \beta_{m,w_d}\sigma_{w_d}w_{d,t+1}, \end{aligned}$$

where

$$\begin{aligned} &\beta_{m,c} = \pi, \qquad \beta_{m,x} = \kappa_{1,m}A_{1,m}, \qquad \beta_{m,d} = 1, \qquad \beta_{m,\lambda} = \kappa_{1,m}A_{1,\lambda,m}, \\ &\beta_{m,w_x} = \kappa_{1,m}A_{2,x,m}, \qquad \beta_{m,w_c} = \kappa_{1,m}A_{2,c,m}, \qquad \beta_{m,w_d} = \kappa_{1,m}A_{2,d,m}. \end{aligned} \tag{A.26}$$

The risk premium for the dividend claim is

$$\begin{aligned} &E_t(r_{m,t+1} - r_{f,t}) + \frac{1}{2}\operatorname{Var}_t(r_{m,t+1}) \\ &\quad = -\operatorname{Cov}_t(m_{t+1}, r_{m,t+1}) \\ &\quad = \beta_{m,x}\lambda_x\sigma_{x,t}^2 + \beta_{m,c}\lambda_c\sigma_{c,t}^2 + \beta_{m,\lambda}\lambda_\lambda\sigma_\lambda^2 \\ &\qquad + \beta_{m,w_x}\lambda_{w_x}\sigma_{w_x}^2 + \beta_{m,w_c}\lambda_{w_c}\sigma_{w_c}^2. \end{aligned} \tag{A.27}$$

C.3. *Risk-Free Rate*

The model-driven equation for the risk-free rate is

$$\begin{aligned} r_{f,t} &= -\mathbb{E}_t[m_{t+1}] - \frac{1}{2}\operatorname{Var}_t[m_{t+1}] \\ &= -\theta\log\delta - \mathbb{E}_t[x_{\lambda,t+1}] + \frac{\theta}{\psi}\mathbb{E}_t[g_{c,t+1}] \\ &\quad + (1-\theta)\mathbb{E}_t[r_{c,t+1}] - \frac{1}{2}\operatorname{Var}_t[m_{t+1}]. \end{aligned} \tag{A.28}$$

Subtract $(1-\theta)r_{f,t}$ from both sides and divide by θ:

$$\begin{aligned} r_{f,t} &= -\log\delta - \frac{1}{\theta}\mathbb{E}_t[x_{\lambda,t+1}] + \frac{1}{\psi}\mathbb{E}_t[g_{c,t+1}] \\ &\quad + \frac{(1-\theta)}{\theta}\mathbb{E}_t[r_{c,t+1} - r_{f,t}] - \frac{1}{2\theta}\operatorname{Var}_t[m_{t+1}]. \end{aligned} \tag{A.29}$$

From (A.11) and (A.18),

$$r_{f,t} = B_0 + B_1 x_t + B_{1,\lambda}x_{\lambda,t} + B_{2,x}\sigma_{x,t}^2 + B_{2,c}\sigma_{c,t}^2,$$

where

$$
\begin{aligned}
B_1 &= \frac{1}{\psi}, \qquad B_{1,\lambda} = -\rho_\lambda, \\
B_{2,x} &= -\frac{\left(1-\frac{1}{\psi}\right)\left(\gamma-\frac{1}{\psi}\right)\kappa_1^2}{2(1-\kappa_1\rho)^2}, \\
B_{2,c} &= -\frac{1}{2}\left(\frac{\gamma-1}{\psi}+\gamma\right),
\end{aligned}
\tag{A.30}
$$

and

$$
\begin{aligned}
B_0 = {} & -\theta\log\delta-(\theta-1)\big\{\kappa_0+(\kappa_1-1)A_0 \\
& + \kappa_1 A_{2,x}(1-\nu_x)(\varphi_x\bar{\sigma})^2+\kappa_1 A_{2,c}(1-\nu_c)(\varphi_c\bar{\sigma})^2\big\} \\
& + \gamma\mu-\frac{1}{2}\big\{(\theta-1)\kappa_1 A_{2,x}\sigma_{w_x}\big\}^2-\frac{1}{2}\big\{(\theta-1)\kappa_1 A_{2,c}\sigma_{w_c}\big\}^2 \\
& - \frac{1}{2}\big\{\big((\theta-1)\kappa_1 A_{1,\lambda}+\theta\big)^2\sigma_\lambda^2\big\}.
\end{aligned}
$$

C.4. *Linearization Parameters*

For any asset, the linearization parameters are determined endogenously by the following system of equations:

$$
\begin{aligned}
\bar{pd}_i &= A_{0,i}(\bar{pd}_i)+\sum_{j\in\{c,x,d\}} A_{2,i,j}(\bar{pd}_i)\times(\varphi_j\bar{\sigma})^2, \\
\kappa_{1,i} &= \frac{\exp(\bar{pd}_i)}{1+\exp(\bar{pd}_i)}, \\
\kappa_{0,i} &= \log\big(1+\exp(\bar{pd}_i)\big)-\kappa_{1,i}\bar{pd}_i.
\end{aligned}
$$

The solution is determined numerically by iteration until reaching a fixed point of $\bar{pd}_i$.

C.5. *Deriving the Intertemporal Marginal Rate of Substitution (MRS)*

We consider a representative-agent endowment economy modified to allow for time-preference shocks. The representative agent has Epstein and Zin (1989) recursive preferences and maximizes her lifetime utility

$$
V_t = \max_{C_t}\Big[(1-\delta)\lambda_t C_t^{\frac{1-\gamma}{\theta}}+\delta\big(\mathbb{E}_t\big[V_{t+1}^{1-\gamma}\big]\big)^{\frac{1}{\theta}}\Big]^{\frac{\theta}{1-\gamma}}
$$

subject to budget constraint

$$
W_{t+1}=(W_t-C_t)R_{c,t+1},
$$

where W_t is the wealth of the agent, $R_{c,t+1}$ is the return on all invested wealth, γ is risk aversion, $\theta=\frac{1-\gamma}{1-1/\psi}$, and ψ is intertemporal elasticity of substitution. The ratio $\frac{\lambda_{t+1}}{\lambda_t}$ determines how agents trade off current versus future utility and is referred to as the time-preference shock (see Albuquerque, Eichenbaum, Luo, and Rebelo (2016)).

First conjecture a solution for $V_t = \phi_t W_t$. The value function is homogeneous of degree 1 in wealth; it can now be written as

$$\phi_t W_t = \max_{C_t}\Big[(1-\delta)\lambda_t C_t^{\frac{1-\gamma}{\theta}} + \delta\big(\mathbb{E}_t\big[(\phi_{t+1}W_{t+1})^{1-\gamma}\big]\big)^{\frac{1}{\theta}}\Big]^{\frac{\theta}{1-\gamma}} \tag{A.31}$$

subject to

$$W_{t+1} = (W_t - C_t)R_{c,t+1}.$$

Epstein and Zin (1989) showed that the above dynamic program has a maximum.

Using the dynamics of the wealth equation, we substitute W_{t+1} into (A.31) to derive

$$\phi_t W_t = \max_{C_t}\Big[(1-\delta)\lambda_t C_t^{\frac{1-\gamma}{\theta}} + \delta(W_t - C_t)^{\frac{1-\gamma}{\theta}}\big(\mathbb{E}_t\big[(\phi_{t+1}R_{c,t+1})^{1-\gamma}\big]\big)^{\frac{1}{\theta}}\Big]^{\frac{\theta}{1-\gamma}}. \tag{A.32}$$

At the optimum, $C_t = b_t W_t$, where b_t is the consumption-wealth ratio. Using (A.32) and shifting the exponent on the braces to the left-hand side, and dividing by W_t, yields

$$\phi_t^{\frac{1-\gamma}{\theta}} = (1-\delta)\lambda_t\left(\frac{C_t}{W_t}\right)^{\frac{1-\gamma}{\theta}} + \delta\left(1-\frac{C_t}{W_t}\right)^{\frac{1-\gamma}{\theta}}\big(\mathbb{E}_t\big[(\phi_{t+1}R_{c,t+1})^{1-\gamma}\big]\big)^{\frac{1}{\theta}} \tag{A.33}$$

or simply

$$\phi_t^{\frac{1-\gamma}{\theta}} = (1-\delta)\lambda_t b_t^{\frac{1-\gamma}{\theta}} + \delta(1-b_t)^{\frac{1-\gamma}{\theta}}\big(\mathbb{E}_t\big[(\phi_{t+1}R_{c,t+1})^{1-\gamma}\big]\big)^{\frac{1}{\theta}}. \tag{A.34}$$

The first-order condition with respect to the consumption choice yields

$$(1-\delta)\lambda_t b_t^{\frac{1-\gamma}{\theta}-1} = \delta(1-b_t)^{\frac{1-\gamma}{\theta}-1}\big(\mathbb{E}_t\big[(\phi_{t+1}R_{c,t+1})^{1-\gamma}\big]\big)^{\frac{1}{\theta}}. \tag{A.35}$$

Plugging (A.35) into (A.34) yields

$$\begin{aligned}\phi_t &= (1-\delta)^{\frac{\theta}{1-\gamma}}\lambda_t^{\frac{\theta}{1-\gamma}}\left(\frac{C_t}{W_t}\right)^{\frac{1-\gamma-\theta}{1-\gamma}} \\ &= (1-\delta)^{\frac{\psi}{\psi-1}}\lambda_t^{\frac{\psi}{\psi-1}}\left(\frac{C_t}{W_t}\right)^{\frac{1}{1-\psi}}.\end{aligned} \tag{A.36}$$

The lifetime value function is $\phi_t W_t$, with the solution to ϕ_t stated above. This expression for ϕ_t is important: It states that the maximized lifetime utility is determined by the consumption-wealth ratio.

Equation (A.35) can be rewritten as

$$(1-\delta)^{\theta}\lambda_t^{\theta}\left(\frac{b_t}{1-b_t}\right)^{-\frac{\theta}{\psi}} = \delta^{\theta}\mathbb{E}_t\big[(\phi_{t+1}R_{c,t+1})^{1-\gamma}\big]. \tag{A.37}$$

Consider the term $\phi_{t+1}R_{c,t+1}$:

$$\phi_{t+1}R_{c,t+1} = (1-\delta)^{\frac{\psi}{\psi-1}}\lambda_{t+1}^{\frac{\psi}{\psi-1}}\left(\frac{C_{t+1}}{W_{t+1}}\right)^{\frac{1}{1-\psi}}R_{c,t+1}. \tag{A.38}$$

After substituting the wealth constraint, $\frac{C_{t+1}}{W_{t+1}} = \frac{C_{t+1}/C_t}{W_t/C_t - 1} \cdot \frac{1}{R_{c,t+1}} = \frac{G_{t+1}}{R_{c,t+1}} \cdot \frac{b_t}{1-b_t}$, into the above expression, it follows that

$$\phi_{t+1} R_{c,t+1} = (1-\delta)^{\frac{\psi}{\psi-1}} \lambda_{t+1}^{\frac{\psi}{\psi-1}} \left(\frac{b_t}{1-b_t}\right)^{\frac{1}{1-\psi}} \left(\frac{G_{t+1}}{R_{c,t+1}}\right)^{\frac{1}{1-\psi}} R_{c,t+1}. \tag{A.39}$$

After some intermediate tedious manipulations,

$$\delta^\theta (\phi_{t+1} R_{c,t+1})^{1-\gamma} = \delta^\theta (1-\delta)^\theta \lambda_{t+1}^\theta \left(\frac{b_t}{1-b_t}\right)^{-\frac{\theta}{\psi}} G_{t+1}^{-\frac{\theta}{\psi}} R_{c,t+1}^\theta. \tag{A.40}$$

Taking expectations and substituting the last expression into (A.37) yields

$$\delta^\theta \mathbb{E}_t \left[\left(\frac{\lambda_{t+1}}{\lambda_t}\right)^\theta G_{t+1}^{-\frac{\theta}{\psi}} R_{c,t+1}^{\theta-1} R_{c,t+1} \right] = 1. \tag{A.41}$$

From here, we see that the MRS in terms of observables is

$$M_{t+1} = \delta^\theta \left(\frac{\lambda_{t+1}}{\lambda_t}\right)^\theta G_{t+1}^{-\frac{\theta}{\psi}} R_{c,t+1}^{\theta-1}. \tag{A.42}$$

The log of MRS is

$$m_{t+1} = \theta \log \delta + \theta x_{\lambda,t+1} - \frac{\theta}{\psi} g_{t+1} + (\theta - 1) r_{c,t+1}, \tag{A.43}$$

where $x_{\lambda,t+1} = \log(\frac{\lambda_{t+1}}{\lambda_t})$.

D. STATE-SPACE REPRESENTATIONS OF THE EMPIRICAL MODELS

Below we describe the state-space representation for the LRR model. The state-space representation for the cash-flow-only specifications can be obtained by eliminating the asset returns ($r_{m,t+1}$ and $r_{f,t}$) from the set of measurement equations.

D.1. *Measurement Equations*

In order to capture the correlation structure between the measurement errors at monthly frequency, we assumed in the main text that 12 months of consumption growth data are released at the end of each year. We will now present the resulting measurement equation. To simplify the exposition, we assume that the monthly consumption data are released at the end of the quarter (rather than at the end of the year). In the main text, the measurement equation is written as

$$y_{t+1} = A_{t+1}\big(D + Z s_{t+1} + Z^v s_{t+1}^v(h_{t+1}, h_t) + \Sigma^u u_{t+1}\big), \quad u_{t+1} \sim N(0, I). \tag{A.44}$$

The selection matrix A_{t+1} accounts for the deterministic changes in the vector of observables, y_{t+1}. Recall that monthly observations are available only starting in 1959:M1. For the sake of exposition, suppose prior to 1959:M1 consumption growth was available at quarterly frequency. We further assume that dividend growth data are always available in the form of time-aggregated quarterly data. Then (we are omitting some of the o superscripts for observed series that we used in the main text):

1. Prior to 1959:M1:

(a) If $t+1$ is the last month of the quarter:

$$y_{t+1} = \begin{bmatrix} g^q_{c,t+1} \\ g^q_{d,t+1} \\ r_{m,t+1} \\ r_{f,t} \end{bmatrix}, \qquad A_{t+1} = \begin{bmatrix} \frac{1}{3} & \frac{2}{3} & 1 & \frac{2}{3} & \frac{1}{3} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

(b) If $t+1$ is not the last month of the quarter:

$$y_{t+1} = \begin{bmatrix} g^q_{d,t+1} \\ r_{m,t+1} \\ r_{f,t} \end{bmatrix}, \qquad A_{t+1} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

2. From 1959:M1 to present:

(a) If $t+1$ is the last month of the quarter:

$$y_{t+1} = \begin{bmatrix} g_{c,t+1} \\ g_{c,t} \\ g_{c,t-1} \\ g^q_{d,t+1} \\ r_{m,t+1} \\ r_{f,t} \end{bmatrix}, \qquad A_{t+1} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

(b) If $t+1$ is not the last month of the quarter:

$$y_{t+1} = \begin{bmatrix} g^q_{d,t+1} \\ r_{m,t+1} \\ r_{f,t} \end{bmatrix}, \qquad A_{t+1} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

The relationship between observations and states (ignoring the measurement errors) is given by the approximate analytical solution of the LRR model described in Section C:

$$\begin{aligned} g_{c,t+1} &= \mu_c + x_t + \sigma_{c,t}\eta_{c,t+1}, \\ g_{d,t+1} &= \mu_d + \phi x_t + \pi\sigma_{c,t}\eta_{c,t+1} + \sigma_{d,t}\eta_{d,t+1}, \\ r_{m,t+1} &= \{\kappa_{0,m} + (\kappa_{1,m} - 1)A_{0,m} + \mu_d\} \\ &\quad + (\kappa_{1,m}A_{1,m})x_{t+1} + (\phi - A_{1,m})x_t + (\kappa_{1,m}A_{1,\lambda,m})x_{\lambda,t+1} \\ &\quad - A_{1,\lambda,m}x_{\lambda,t} + \pi\sigma_{c,t}\eta_{c,t+1} + \sigma_{d,t}\eta_{d,t+1} \\ &\quad + (\kappa_{1,m}A_{2,x,m})\sigma^2_{x,t+1} - A_{2,x,m}\sigma^2_{x,t} \\ &\quad + (\kappa_{1,m}A_{2,c,m})\sigma^2_{c,t+1} - A_{2,c,m}\sigma^2_{c,t} + (\kappa_{1,m}A_{2,d,m})\sigma^2_{d,t+1} - A_{2,d,m}\sigma^2_{d,t}, \\ r_{f,t} &= B_0 + B_1 x_t + B_{1,\lambda}x_{\lambda,t} + B_{2,x}\sigma^2_{x,t} + B_{2,c}\sigma^2_{c,t}, \\ &\quad \eta_{i,t+1}, \eta_{\lambda,t+1}, w_{i,t+1} \sim N(0,1), i \in \{c, x, d\}. \end{aligned} \tag{A.45}$$

In order to reproduce (A.45) and the measurement error structure described in Sections 2.1 and 3.2, we define the vectors of states s_{t+1} and s^v_{t+1} as

$$s_{t+1} = \begin{bmatrix} x_{t+1} \\ x_t \\ x_{t-1} \\ x_{t-2} \\ x_{t-3} \\ x_{t-4} \\ \sigma_{c,t}\eta_{c,t+1} \\ \sigma_{c,t-1}\eta_{c,t} \\ \sigma_{c,t-2}\eta_{c,t-1} \\ \sigma_{c,t-3}\eta_{c,t-2} \\ \sigma_{c,t-4}\eta_{c,t-3} \\ \sigma_\epsilon\epsilon_{t+1} \\ \sigma_\epsilon\epsilon_t \\ \sigma_\epsilon\epsilon_{t-1} \\ \sigma_\epsilon\epsilon_{t-2} \\ \sigma_\epsilon\epsilon_{t-3} \\ \sigma_\epsilon\epsilon_{t-4} \\ \sigma^q_\epsilon\epsilon^q_{t+1} \\ \sigma^q_\epsilon\epsilon^q_t \\ \sigma^q_\epsilon\epsilon^q_{t-1} \\ \sigma^q_\epsilon\epsilon^q_{t-2} \\ \sigma_{d,t}\eta_{d,t+1} \\ \sigma_{d,t-1}\eta_{d,t} \\ \sigma_{d,t-2}\eta_{d,t-1} \\ \sigma_{d,t-3}\eta_{d,t-2} \\ \sigma_{d,t-4}\eta_{d,t-3} \\ x_{\lambda,t+1} \\ x_{\lambda,t} \end{bmatrix}, \qquad s^v_{t+1} = \begin{bmatrix} \sigma^2_{x,t+1} \\ \sigma^2_{x,t} \\ \sigma^2_{c,t+1} \\ \sigma^2_{c,t} \\ \sigma^2_{d,t+1} \\ \sigma^2_{d,t} \end{bmatrix}. \tag{A.46}$$

It can be verified that the coefficient matrices D, Z, Z^v, and Σ^e are given by

$$Z = \left[\begin{array}{cccccccccccccccccccccccccccc}
0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -\frac{1}{3} & -\frac{1}{3} & \frac{2}{3} & -\frac{2}{3} & \frac{1}{3} & \frac{1}{3} & 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{\phi}{3} & \frac{2\phi}{3} & \phi & \frac{2\phi}{3} & \frac{\phi}{3} & \frac{\pi}{3} & \frac{2\pi}{3} & \pi & \frac{2\pi}{3} & \frac{\pi}{3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{3} & \frac{2}{3} & 1 & \frac{2}{3} & \frac{1}{3} & 0 & 0 \\
\mu_{r,1} & \mu_{r,2} & 0 & 0 & 0 & 0 & \mu_{r,3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu_{r,4} & 0 & 0 & 0 & 0 & \mu_{r,5} & \mu_{r,6} \\
0 & B_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & B_{1,\lambda} & 0
\end{array}\right],$$

$$Z^v = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
\mu_{r,7} & \mu_{r,8} & \mu_{r,9} & \mu_{r,10} & \mu_{r,11} & \mu_{r,12} \\
0 & B_{2,x} & 0 & B_{2,c} & 0 & 0
\end{bmatrix}, \qquad D = \begin{bmatrix} \mu \\ \mu \\ \mu \\ \mu \\ \mu \\ 3\mu_d \\ \mu_{r,0} \\ B_0 \end{bmatrix},$$

$$\Sigma^u = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \sigma^a_{d,\epsilon} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \sigma_{f,\epsilon}
\end{bmatrix}.$$

The coefficients $\mu_{r,0}$ to $\mu_{r,12}$ are obtained from the solution of the LRR model:

$$\begin{bmatrix} \mu_{r,0} \\ \mu_{r,1} \\ \mu_{r,2} \\ \mu_{r,3} \\ \mu_{r,4} \\ \mu_{r,5} \\ \mu_{r,6} \end{bmatrix} = \begin{bmatrix} \kappa_{0,m} + A_{0,m}(\kappa_{1,m} - 1) + \mu_d \\ \kappa_{1,m} A_{1,m} \\ \phi - A_{1,m} \\ \pi \\ 1 \\ \kappa_{1,m} A_{1,\lambda,m} \\ -A_{1,\lambda,m} \end{bmatrix}, \qquad \begin{bmatrix} \mu_{r,7} \\ \mu_{r,8} \\ \mu_{r,9} \\ \mu_{r,10} \\ \mu_{r,11} \\ \mu_{r,12} \end{bmatrix} = \begin{bmatrix} \kappa_{1,m} A_{2,x,m} \\ -A_{2,x,m} \\ \kappa_{1,m} A_{2,c,m} \\ -A_{2,c,m} \\ \kappa_{1,m} A_{2,d,m} \\ -A_{2,d,m} \end{bmatrix}.$$

D.2. *State-Transition Equations*

Using the definition of s_{t+1} in (A.46), we write the state-transition equation as

$$s_{t+1} = \Phi s_t + v_{t+1}(h_t). \tag{A.47}$$

Conditional on the volatilities h_t, this equation reproduces the law of motion of the two persistent conditional mean processes

$$\begin{aligned} x_{t+1} &= \rho x_t + \sigma_{x,t} \eta_{x,t+1}, \\ x_{\lambda,t+1} &= \rho_\lambda x_{\lambda,t} + \sigma_\lambda \eta_{\lambda,t+1}, \end{aligned} \tag{A.48}$$

and it contains some trivial relationships among the measurement-error states. The matrices Φ and $v_{t+1}(h_t)$ are defined as

$$\Phi = \left[\begin{array}{*{28}{c}}
\rho & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
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0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_\lambda & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0
\end{array}\right]$$

and

$$v_{t+1}(h_t) = \begin{bmatrix} \sigma_{x,t}\eta_{x,t+1} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \sigma_{c,t}\eta_{c,t+1} \\ 0 \\ 0 \\ 0 \\ 0 \\ \sigma_\epsilon \epsilon_{t+1} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \sigma_\epsilon^q \epsilon_{t+1}^q \\ 0 \\ 0 \\ 0 \\ \sigma_{d,t}\eta_{d,t+1} \\ 0 \\ 0 \\ 0 \\ 0 \\ \sigma_\lambda \eta_{\lambda,t+1} \\ 0 \end{bmatrix}.$$

The law of motion of the three persistent conditional log volatility processes is given by

$$h_{t+1} = \Psi h_t + \Sigma_h w_{t+1}, \tag{A.49}$$

where

$$h_{t+1} = \begin{bmatrix} h_{x,t+1} \\ h_{c,t+1} \\ h_{d,t+1} \end{bmatrix}, \qquad \Psi = \begin{bmatrix} \rho_{h_x} & 0 & 0 \\ 0 & \rho_{h_c} & 0 \\ 0 & 0 & \rho_{h_d} \end{bmatrix},$$

$$\Sigma_h = \begin{bmatrix} \sigma_{h_x}\sqrt{1-\rho_{h_x}^2} & 0 & 0 \\ 0 & \sigma_{h_c}\sqrt{1-\rho_{h_c}^2} & 0 \\ 0 & 0 & \sigma_{h_d}\sqrt{1-\rho_{h_d}^2} \end{bmatrix},$$

$$w_{t+1} = \begin{bmatrix} w_{x,t+1} \\ w_{c,t+1} \\ w_{d,t+1} \end{bmatrix}.$$

We express

$$\sigma_{x,t} = \varphi_x \sigma \exp(h_{x,t}), \qquad \sigma_{c,t} = \varphi_c \sigma \exp(h_{c,t}), \qquad \sigma_{d,t} = \varphi_d \sigma \exp(h_{d,t}),$$

which delivers the dependence on h_t in the above definition of $v_{t+1}(\cdot)$. $\varphi_c = 1$ is normalized.

E. POSTERIOR INFERENCE

E.1. *Model With Asset Prices*

To construct a posterior sampler for the LRR model (see Section 5 for estimation results), we use a particle-filter approximation of the likelihood function, constructed as follows. Our state-space representation, given the measurement equation (A.44) and the state-transition equations (A.47) and (A.49), is linear conditional on the volatility states (h_{t+1}, h_t). The particle filter uses a swarm of particles $\{z_t^j, W_t^j\}_{j=1}^M$ to approximate

$$\mathbb{E}\big[h(z_t)|Y_{1:t}\big] \approx \frac{1}{M}\sum_{j=1}^{M} W_t^j h\big(z_t^j\big). \tag{A.50}$$

Throughout this section, we omit the parameter vector Θ from the conditioning set. Here $h(\cdot)$ is an integrable function of z_t, and the approximation $\approx$, under suitable regularity conditions, can be stated formally in terms of a strong law of large numbers and a central limit theorem. In general, z_t^j would be composed of h_t^j, h_{t-1}^j, and s_t^j. However, given that the state-space model is linear conditional on (h_t, h_{t-1}), we can replace s_t^j by

$$\big[\text{vec}\big(\mathbb{E}\big[s_t|h_t^j, h_{t-1}^j, Y_{1:t}\big]\big), \text{vech}\big(\text{Var}\big[s_t|h_t^j, h_{t-1}^j, Y_{1:t}\big]\big)\big]',$$

where vech$(\cdot)$ stacks the non-redundant elements of a symmetric matrix. The use of the vector of conditional means and covariance terms for s_t in the definition of the particle z_t^j leads to a variance reduction in the particle-filter approximation of the likelihood function. The implementation of the particle filter is based on Algorithm 13 in Herbst and Schorfheide (2015). The particle-filter approximation of the likelihood function is embedded into a fairly standard random-walk Metropolis–Hastings algorithm (see Chapter 9 of Herbst and Schorfheide (2015)).

E.2. *Models Without Asset Prices*

The estimation of the cash-flow-only models in Sections 2 and 3 is considerably easier because the volatility states do not affect the conditional means of the observables. As before in the model with asset prices, the state variables are the model-implied monthly cash flows and the latent volatility processes $h_{i,t}$. Let Θ_{cf} denote the parameters that denote the cash-flow processes, Θ_h the parameters that control the evolution of the volatility processes, and H^T the sequence of latent volatilities.

The MCMC algorithm iterates over three conditional distributions: First, a Metropolis–Hastings step is used to draw from the posterior of Θ_{cf} conditional on $(Y, (H^T)^{(s)}, \Theta_h^{(s-1)})$. Second, we draw the sequence of stochastic volatilities H^T conditional on $(Y, \Theta_{\text{cf}}^{(s)}, \Theta_h^{(s-1)})$ using the algorithm developed by Kim, Shephard, and Chib (1998). It consists of transforming a nonlinear and non-Gaussian state-space form into a linear and approximately

Gaussian one, which allows the use of simulation smoothers such as those of Carter and Kohn (1994) to recover estimates of the residuals $\eta_{i,t}$. Finally, we draw from the posterior of the coefficients of the stochastic volatility processes, Θ_h, conditional on $(Y, H^{T(s)}, \Theta_{\mathrm{cf}}^{(s)})$.

F. SUPPLEMENTARY FIGURES AND TABLES

This section provides supplementary empirical results that are referenced in the main paper.

- Table A-I: provides estimates of alternative specifications of the consumption growth model considered in Section 2.2 and supplements Table I.

(i) Table A-II: provides estimates of a bivariate cash-flow model in which consumption and dividends are cointegrated. These estimates are referenced in the part *Cointegration of Dividends and Consumption* of Section 3.2.

- Table A-III: appeared in the main text of an earlier version of the paper. Comparing the estimates of ρ from Table VI based on cash-flow data only to the estimate obtained in Table VII by estimating the LRR model based on cash flow and asset return data, we observed that the posterior mean increases from 0.94 and 0.95, respectively, to 0.99 once asset returns are included. To assess the extent to which the increase in ρ leads to a decrease in fit of the consumption growth process, we re-estimate model (4) conditional on various choices of ρ between 0.90 and 0.99 and recompute the marginal data density for consumption growth. The results are summarized in the table. The key finding is that the drop in the marginal data density by changing ρ from $\hat{\rho}$ to 0.99 is small, indicating that there essentially is no tension between the parameter estimates obtained with and without asset prices.
- Figure A-1: contains further posterior predictive checks for the R^2 values associated with consumption and return predictability regressions. It supplements Figure 9 in Section 5.3 and shows how the model-implied predictive distribution of the R^2's changes as different sources of risk are switched off. These results are mentioned in the main paper in Footnote 25.
- Figure A-2: appeared in the main text of an earlier version of the paper. It examines the model's implication with respect to the long horizon correlation between consumption growth (dividend growth) and returns—that is, the Hth horizon correlation

$$\operatorname{corr}\left(\sum_{h=1}^{H} r_{m,t+h}, \sum_{h=1}^{H} \Delta c_{t+h}\right).$$

Our model performs well along this dimension. Under the "Benchmark" specifications (all shocks are active), the 10-year consumption growth and 10-year return have a correlation of 0.3, but with a very wide credible interval that encompasses −0.2 to 0.7, which importantly contains the data estimate. The analogous correlation credible interval for dividend growth ranges from 0 to 0.8, with the data at 0.4 and again very close to the model median estimate. It is noteworthy that these correlation features are primarily driven by "Growth and Volatility Risks." Albuquerque et al. (2016) highlighted that preference shocks improve the LRR model-performance for these long horizon correlations. The "Preference Risk" subplots provide the correlations when all shocks except $x_{\lambda,t}$ are shut down. These plots show that the preference shocks improve fit by generating lower credible intervals for consumption, yet deteriorate fit by generating way too large long horizon correlations for dividends.

TABLE A-I

POSTERIOR MEDIAN ESTIMATES OF CONSUMPTION GROWTH PROCESSES[a]

					Posterior Estimates								
					State-Space Model/Measurement Error Specification								
		Prior Distribution			M&A	No ME	No ME AR(2)	M	M $\rho_\epsilon \neq 0$	M $\rho_\eta \neq 0$	M NoAveOut	IID	ARMA(1, 2)
	Distr.	5%	50%	95%	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
μ_c	N	-0.007	0.0016	0.100	0.0016	0.0016	0.0016	0.0016	0.0016	0.0016	0.0016	0.0016	0.0016
ρ	U	-0.90	0	0.90	0.918	-0.287	-0.684	0.918	0.918	0.919	0.919	-	0.913
ρ_2	U	-0.90	0	0.90	-	-	-0.353	-	-	-	-	-	-
φ_x	U	0.05	0.5	0.95	0.681	-		0.669	0.704	0.644	0.681	-	-
	U	0.1	1.0	1.9	-	1.12	0.482	-	-	-	-	-	-
σ	IG	0.0008	0.0019	0.0061	0.0018	0.0022	0.0027	0.0018	0.0017	0.0019	0.0018	0.0033	0.0032
σ_ϵ	IG	0.0008	0.0019	0.0061	0.0018	-		0.0018	0.0019	0.0018	0.0018	-	-
σ_ϵ^a	IG	0.0007	0.0029	0.0386	0.0011	-	-	-	-	-	-	-	-
ρ_ϵ	U	-0.90	0	0.90	-	-	-	-	0.060	-	-	-	-
ρ_η	U	-0.90	0	0.90	-	-	-	-	-	-0.046	-	-	-
ζ_1	N	-8.2	0	8.2	-	-	-	-	-	-	-	-	-1.14
ζ_2	N	-8.2	0	8.2	-	-	-	-	-	-	-	-	0.302
$\ln p(Y)$					2887.1	2870.8	2870.3	2886.2	2883.9	2885.8	2886.5	2863.2	2884.0

[a]The estimation sample is from 1959:M2 to 2014:M12. We denote the persistence of the growth component x_t by ρ (and ρ_2 if follows an AR(2) process), the persistence of the measurement errors by ρ_ϵ, and the persistence of $\eta_{c,t}$ by ρ_η. We report posterior median estimates for the following measurement error specifications of the state-space model: (1) monthly and annual measurement errors (M&A); (2) no measurement errors (no ME); (3) no measurement errors with AR(2) process for x_t (no ME AR(2)); (4) monthly measurement errors (M); (5) serially correlated monthly measurement errors (M, $\rho_\epsilon \neq 0$); (6) serially correlated consumption shocks $\eta_{c,t}$ (M, $\rho_\eta \neq 0$, $\rho > \rho_\eta$); (7) monthly measurement errors that do not average out at annual frequency (M, NoAveOut). In addition we report results for the following models: (8) consumption growth is i.i.d.; (9) consumption growth is ARMA(1, 2).

TABLE A-II

POSTERIOR ESTIMATES: COINTEGRATION OF CONSUMPTION AND DIVIDENDS

	Distr.	Prior 5%	Prior 50%	Prior 95%	Posterior 5%	Posterior 50%	Posterior 95%
			Consumption				
ρ	U	-0.9	0	0.9	0.907	0.951	0.984
φ_x	U	0.05	0.50	0.95	0.314	0.515	0.946
σ	IG	0.0008	0.0019	0.0061	0.0022	0.0028	0.0034
ρ_{h_c}	N^T	0.27	0.80	0.999	0.976	0.992	0.999
$\sigma^2_{h_c}$	IG	0.0013	0.0043	0.0283	0.0012	0.0037	0.0117
			Dividends				
ϕ_{dc}	U	-9.0	0	9.0	-7.10	-5.66	-4.64
ρ_s	U	-0.9	0	0.9	0.997	0.998	0.999
φ_s	U	15	150	285	86.5	148.0	241.2
ρ_{h_s}	N^T	0.27	0.80	0.999	0.995	0.998	0.999
$\sigma^2_{h_s}$	IG	0.0007	0.0029	0.0392	0.0008	0.0014	0.0028
			Measurement Errors				
σ_ϵ	IG	0.0008	0.0019	0.0061	0.0010	0.0012	0.0015
σ^a_ϵ	IG	0.0008	0.0029	0.0387	0.0005	0.0044	0.0109
$\sigma^a_{d,\epsilon}$	IG	0.0008	0.0029	0.0387	-	0.10	-

Notes: We utilize the mixed-frequency approach in the estimation: For consumption we use annual data from 1930 to 1959 and monthly data from 1960:M1 to 2014:M12; we use monthly dividend annual growth data from 1930:M1 to 2014:M12. For consumption we adopt the measurement error model of Section 2.1. We allow for annual consumption measurement errors ϵ_t^a during the periods from 1930 to 1948. We impose monthly measurement errors ϵ_t when we switch from annual to monthly consumption data from 1960:M1 to 2014:M12. We fix $\mu_c = 0.0016$ and $\mu_d = 0.0010$ at their sample averages. Moreover, we also fix the measurement error variances $(\sigma^a_{d,\epsilon})^2$ and $(\sigma_{f,\epsilon})^2$ at 1% of the sample variance of dividend growth and the risk-free rate, respectively. N, N^T, G, IG, and U denote normal, truncated (outside of the interval $(-1, 1)$) normal, gamma, inverse gamma, and uniform distributions, respectively.

TABLE A-III

MARGINAL DATA DENSITIES FOR CONSUMPTION GROWTH MODEL

Estimation Sample	Fixed ρ 0.90	0.94	0.95	0.97	0.99	Estimated ρ
1959–2014	2925.9	2935.9	2935.5	2934.8	2927.5	2930.1 ($\hat{\rho} = 0.95$)
1930–2014	2912.7	2914.2	2913.3	2912.1	2909.3	2909.9 ($\hat{\rho} = 0.94$)

Notes: We estimate the consumption-only model (4) conditional on various choices of ρ ("Fixed ρ") and compute marginal data densities. We also report the marginal data densities for the estimated values of ρ ("Estimated ρ") based on the posterior mean estimates (in parentheses) from Table 3.

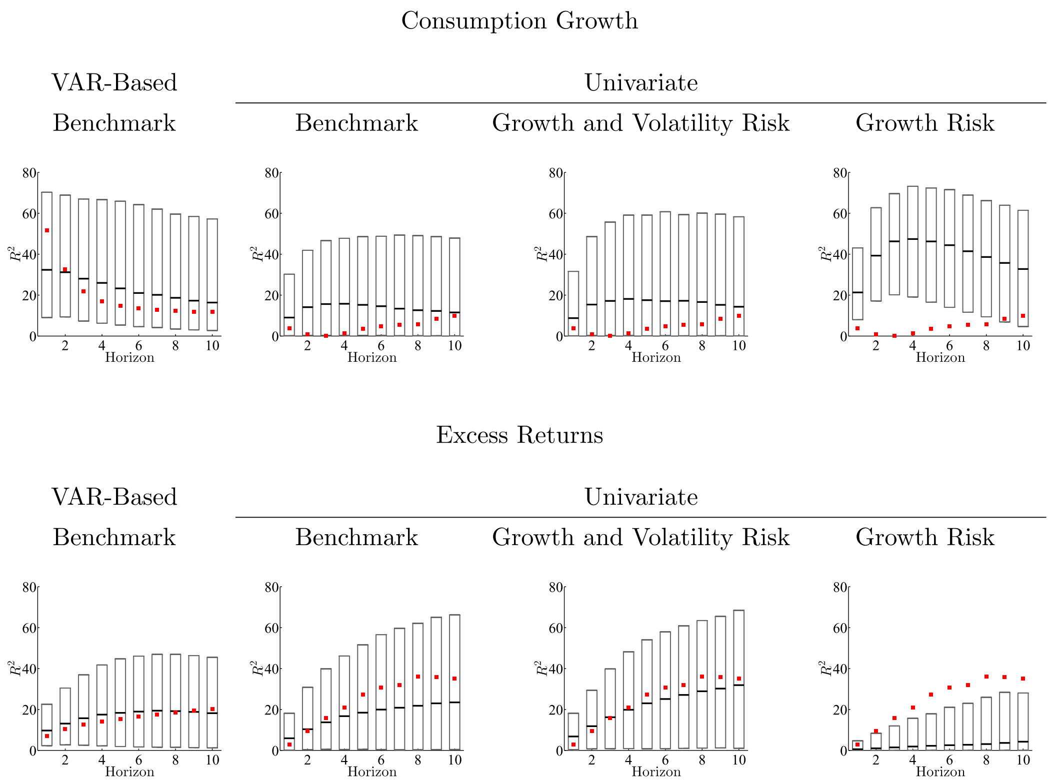

FIGURE A-1.—Predictability checks. *Notes*: We fix the parameters at their posterior median estimates. The red squares represent R^2 values obtained from the actual data. The boxes represent 90% posterior predictive intervals and the horizontal lines represent medians. The "Benchmark" case is based on simulations with all five state variables x_t, $x_{\lambda,t}$, $\sigma^2_{x,t}$, $\sigma^2_{c,t}$, and $\sigma^2_{d,t}$; "Growth and Volatility Risk" is based on x_t and $\sigma^2_{x,t}$ only; "Growth Risk" is based on x_t only. The horizon is measured in years. The VAR-based R^2's are constructed as in Hodrick (1992).

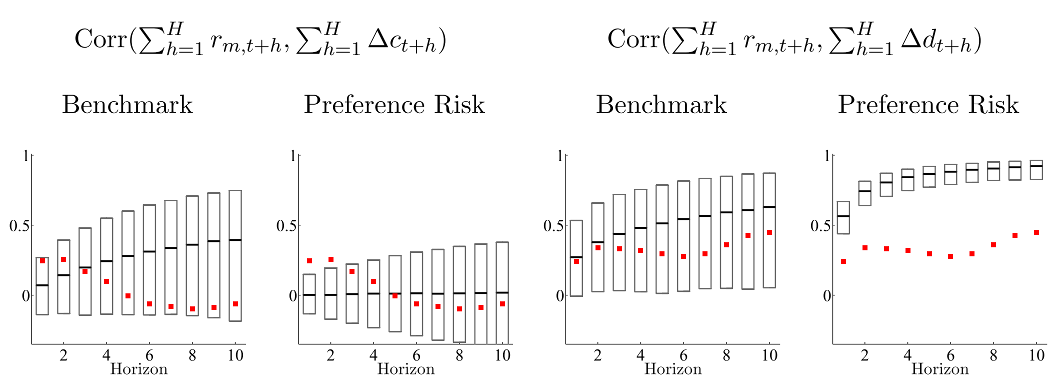

FIGURE A-2.—Correlation between market return and cash-flow growth rates. *Notes*: We fix the parameters at their posterior median estimates. The "Benchmark" case is based on simulations with all five state variables x_t, $x_{\lambda,t}$, $\sigma^2_{x,t}$, $\sigma^2_{c,t}$, and $\sigma^2_{d,t}$; "Preference Risk" is based on $x_{\lambda,t}$ only.

REFERENCES

ALBUQUERQUE, R., M. EICHENBAUM, V. LUO, AND S. REBELO (2016): "Valuation Risk and Asset Pricing," *Journal of Finance*, 71 (6), 2861–2904. [11,19]

BEELER, J., AND J. CAMPBELL (2012): "The Long-Run Risks Model and Aggregate Asset Prices: An Empirical Assessment," *Critical Finance Review*, 1, 141–182. [2]

CAMPBELL, J., AND R. SHILLER (1988): "The Dividend-Price Ratio and Expectations of Future Dividends and Discount Factors," *Review of Financial Studies*, 1, 195–227. [5]

CARTER, C. K., AND R. KOHN (1994): "On Gibbs Sampling for State Space Models," *Biometrika*, 81 (3), 541–553. [19]

EPSTEIN, L., AND S. ZIN (1989): "Substitution, Risk Aversion and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework," *Econometrica*, 57, 937–969. [10,11]

HERBST, E., AND F. SCHORFHEIDE (2015): *Bayesian Estimation of DSGE Models*. Princeton University Press. [18]

HODRICK, R. (1992): "Dividend Yields and Expected Stock Returns: Alternative Procedures for Inference and Measurement," *Review of Financial Studies*, 5, 357–386. [2,22]

KIM, S., N. SHEPHARD, AND S. CHIB (1998): "Stochastic Volatility: Likelihood Inference and Comparison With ARCH Models," *Review of Economic Studies*, 65, 361–393. [18]

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