Claims for `divergence`

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Abstract

This document contains proofs of claims that are stated but not proved in the draft.

Accounting for Richer Dependence of $\hat{c}$ on the Data

Claim 1. Suppose we begin with moments $g(D_i, \theta)$, and are interested in inference on $c = E[h(D_i, \theta)]$. Then the influence function for $\hat{c}$ is given by

$$h(D_i, \theta) - H(G'WG)^{-1} G'Wg(D_i, \theta)$$

for $H = E\left[\frac{\partial}{\partial\theta} h(D_i, \theta)\right]$.

Proof. We can define the new parameter $\tilde{\theta} = (c, \theta)$ and the new moments

$$\tilde{g}\left(D_i, \tilde{\theta}\right) = \begin{pmatrix} h(D_i, \theta) - c \\ g(D_i, \theta) \end{pmatrix}$$

with associated weight matrix

$$\tilde{W} = \begin{pmatrix} 1 & 0 \\ 0 & W \end{pmatrix}.$$

Estimation with these moments and weight matrix will yield the same estimates as a two-step approach where we first estimate $\hat{\theta}$ using $g()$ and W and then estimate $\hat{c}$ by plugging into $h()$.

This representation allows us to derive the correct influence function for $\hat{c}$ accounting for its direct dependence on the data D_i. Begin by noting that

$$\phi_c(D_i) = e_1' \Lambda_{\theta\tilde{g}} \tilde{g}\left(D_i, \tilde{\theta}\right)$$

for e_1 the first standard basis vector. Recall that

$$\Lambda_{\theta\tilde{g}} = -\left(\tilde{G}'\tilde{W}\tilde{G}\right)^{-1} \tilde{G}'\tilde{W}$$

and note that

$$\tilde{G} = \begin{pmatrix} -1 & H \\ 0 & G \end{pmatrix}.$$

Hence,

$$\tilde{W}\tilde{G} = \begin{pmatrix} -1 & H \\ 0 & WG \end{pmatrix}$$

and

$$\tilde{G}'W\tilde{G} = \begin{pmatrix} -1 & 0 \\ H' & G' \end{pmatrix} \begin{pmatrix} -1 & H \\ 0 & WG \end{pmatrix} = \begin{pmatrix} 1 & -H \\ -H' & H'H + G'WG \end{pmatrix},$$

and the sensitivity is

$$\Lambda_{\theta\tilde{g}} = -\begin{pmatrix} 1 & -H \\ -H' & H'H + G'WG \end{pmatrix}^{-1} \begin{pmatrix} -1 & 0 \\ H' & G'W \end{pmatrix}.$$

From the block matrix inverse formula,

$$\begin{pmatrix} 1 & -H \\ -H' & H'H + G'WG \end{pmatrix}^{-1} = \begin{pmatrix} 1 + H\left(G'WG\right)^{-1} H' & H\left(G'WG\right)^{-1} \\ \left(G'WG\right)^{-1} H' & \left(G'WG\right)^{-1} \end{pmatrix}.$$

Hence,

$$e_1'\Lambda_{\theta\tilde{g}} = -e_1' \begin{pmatrix} 1 + H\left(G'WG\right)^{-1} H' & H\left(G'WG\right)^{-1} \\ \left(G'WG\right)^{-1} H' & \left(G'WG\right)^{-1} \end{pmatrix} \begin{pmatrix} -1 & 0 \\ H' & G'W \end{pmatrix}$$

$$= -\begin{pmatrix} -1 & H\left(G'WG\right)^{-1} G'W \end{pmatrix} = \begin{pmatrix} 1 & -H\left(G'WG\right)^{-1} G'W \end{pmatrix}.$$

This yields the desired form for the influence function. □