

SUPPLEMENT TO “BANKS, LIQUIDITY MANAGEMENT, AND
 MONETARY POLICY”
 (*Econometrica*, Vol. 90, No. 1, January 2022, 391–454)

JAVIER BIANCHI
 Federal Reserve Bank of Minneapolis

SAKI BIGIO
 Department of Economics, UCLA and NBER

APPENDIX E: COMPENDIUM: EQUILIBRIUM CONDITIONS

E.1. *Transitional Dynamics*

HERE, WE PRESENT the set of equilibrium conditions. Given a sequence of government policies $\{i_t^m, i_t^w, i_t^g, B_t^{\text{Fed}}, M_t^{\text{Fed}}, G_t^{\text{FA}}, G_t^{\text{Fed}}, T_t^h, T_t\}$ that satisfy the Fed’s and fiscal authorities’ budget constraint, the optimal individual bank variables, $\{\bar{b}_t, \bar{a}_t, \bar{d}_t, \bar{c}_t, \Omega_t, v_t\}$, aggregate variables, $\{B_t, M_t, D_t, G_t, E_t\}$, and a system of prices and real returns $\{P_t, R_t^b, R_t^m, R_t^g, R_t^d, \bar{\chi}_t^+, \bar{\chi}_t^-\}$, the system features 18 unknowns to be determined for all t . There is only one endogenous aggregate state variable, E_t , from which the entire equilibrium is solved for. Table III presents the list of all model variables and Figure 11 presents a detailed timing of the model.

Individual Bank Variables. The portfolio solution to $\{\bar{b}_t, \bar{a}_t, \bar{d}_t\}$ and the values of $\{\Omega_t, v_t\}$ are the solution and values of the following problem:

$$\begin{aligned} \Omega_t &\equiv \max_{\{\bar{b}, \bar{a}, \bar{d}\} \geq 0} \left\{ \mathbb{E}_\omega [R_t^b \bar{b} + R_t^m \bar{a} - R_t^d \bar{d} + \bar{\chi}_t(\bar{a}, \bar{d}, \omega)]^{1-\gamma} \right\}^{\frac{1}{1-\gamma}}, \\ \bar{b} + \bar{a} - \bar{d} &= 1, \\ \bar{d} &\leq \kappa_t. \end{aligned} \tag{E.1}$$

The value of the bank’s problem is

$$v_t = \frac{1}{1-\gamma} \left[1 + (\beta(1-\gamma)\Omega_t^{1-\gamma} v_{t+1})^{\frac{1}{\gamma}} \right]^\gamma. \tag{E.2}$$

Dividends depend on $\{\Omega_t, v_t\}$ via

$$\bar{c}_t = \frac{1}{1 + [\beta(1-\gamma)v_{t+1}\Omega_{t+1}^{1-\gamma}]^{1/\gamma}}. \tag{E.3}$$

This block of equations yields the equations needed to obtain $\{\bar{b}_t, \bar{a}_t, \bar{d}_t, \bar{c}_t, \Omega_t, v_t\}$ for a given path for real rates $\{R_t^b, R_t^m, R_t^d, \bar{\chi}_t\}$.

Javier Bianchi: javier.i.bianchi@gmail.com
 Saki Bigio: sbigio@econ.ucla.edu

TABLE III
MODEL VARIABLE LIST.

Interest rates	
\bar{i}^f	average interbank market rate
i^b	nominal interest rate on loans
i^d	nominal interest rate on deposits
i^g	nominal interest rate on bonds
Individual bank variables	
Portfolios	
\bar{b}	bank loans in lending stage
\bar{m}	reserves held by banks at end of lending stage
\bar{d}	bank deposits at end of lending stage
\bar{g}	government bonds at end of lending stage
b	loans at beginning of lending stage
m	reserves held at beginning of lending stage
d	deposits owed at beginning of lending stage
g	government bonds at beginning of lending stage
\bar{b}	portfolio share in loans
\bar{m}	portfolio share in reserves
\bar{a}	portfolio share in liquid assets
\bar{d}	portfolio share in deposits
\bar{g}	portfolio share in bonds
Others	
c	bank consumption
Ω	risk-adjusted value of bank equity
R^e	return on equity
e	real equity
μ	Lagrange multiplier on capital requirement constraint
V^l, V^b	value of the bank at lending/balancing stage
V	value of the bank as a function of bank equity
Interbank market	
ω	withdrawal shock
s	surplus at beginning of balancing stage after shock ω
θ	market tightness in interbank market
f	interbank market loans
w	discount window loans
Ψ^+	probability that a bank with surplus finds a match
Ψ^-	probability that a bank with deficit finds a match
Aggregates	
E	aggregate real bank equity
Θ_t^x	intercept demand $x \in \{g, b, d, m\}$
ϵ^x	elasticity of demand $x \in \{g, b, d, m\}$
B, B^f	bank loan supply / firm demand
D, D^h	bank / household deposits
M, M^h	reserve / currency holdings
G, G^h	bank / household gov. bond holdings
P	price level
π	inflation
Government and fed policies	
i^m	nominal interest rate on reserves
i^w	nominal interest rate on discount window loans
G^{FA}	Total supply of government bonds
G^{Fed}	Fed holding of government bonds
M^{Fed}	supply of reserves
B^{Fed}	Fed holdings of private loans
W^{Fed}	discount window loans

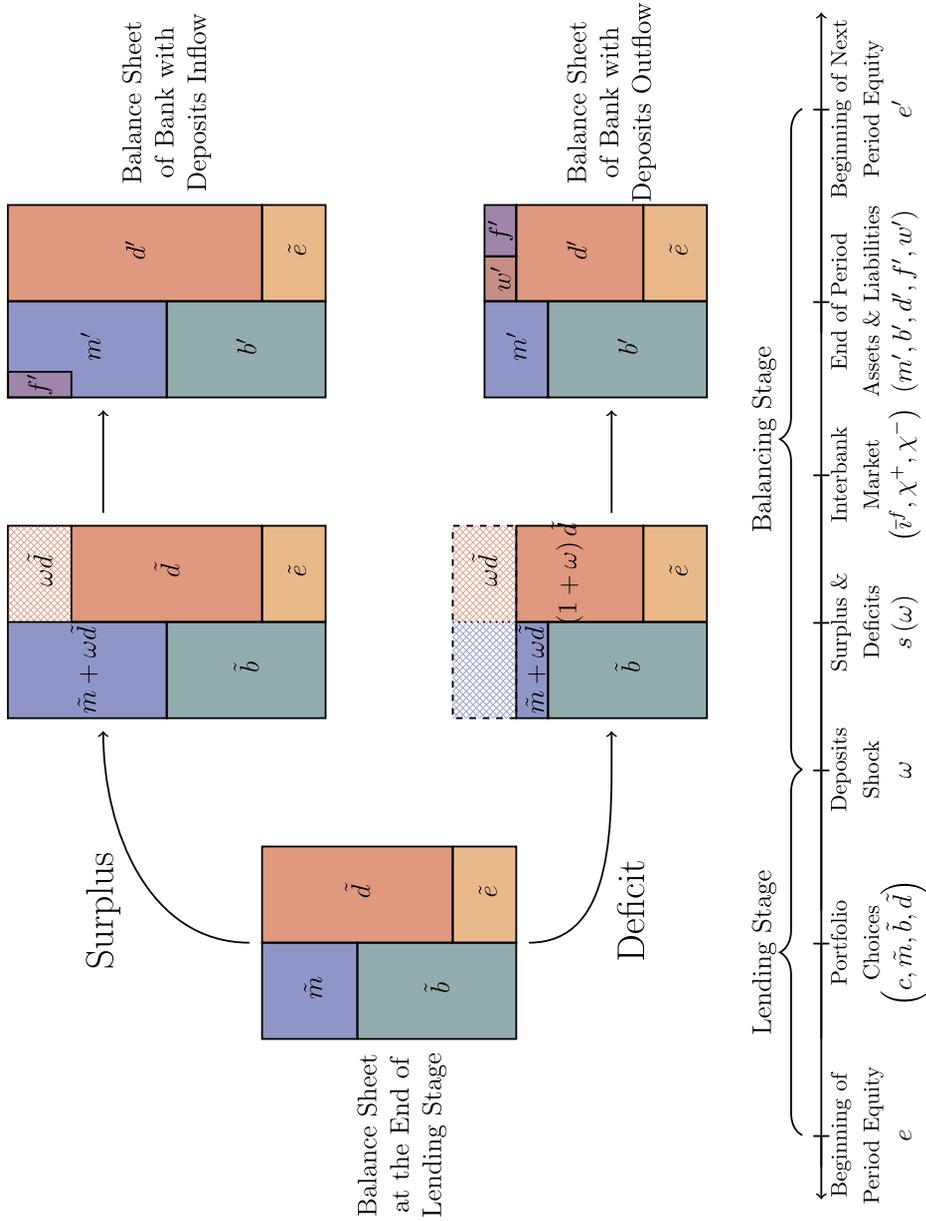


FIGURE 11.—Timeline diagram and banks' balance sheet. For illustration purposes, it is assumed that banks do not accumulate government bonds $g' = 0$ and that $(\tilde{m} = \rho \tilde{d})$.

Aggregate Banking Variables. Next, homogeneity in policy functions gives us the aggregate bank portfolio:

$$B_{t+1} = P_t \bar{b}_t (1 - \bar{c}_t) E_t, \quad (\text{E.4})$$

$$D_{t+1} = P_t \bar{d}_t (1 - \bar{c}_t) E_t, \quad (\text{E.5})$$

$$A_{t+1} = P_t \bar{a}_t (1 - \bar{c}_t) E_t. \quad (\text{E.6})$$

Real aggregate equity evolves according to

$$E_{t+1} = \left[\left((1 + i_{t+1}^b) \bar{b}_t + (1 + i_{t+1}^m) \bar{m}_t + (1 + i_{t+1}^g) \bar{g}_t - (1 + i_{t+1}^d) \bar{d}_t \right) P_t (1 - \bar{c}_t) E_t - (1 + i_{t+1}^w) W_{t+1} - P_t T_t \right] / P_{t+1}. \quad (\text{E.7})$$

This block of equations determines $\{B_t, G_t, D_t, E_t\}$ given a path for inflation and nominal rates—which together determine real rates—and transfers.

Market Clearing Conditions. The real rates and the path for prices follow from the market clearing conditions in all the asset markets:

$$\frac{B_{t+1} + B_{t+1}^{\text{FED}}}{P_t} = \Theta_t^b (R_t^b)^{\epsilon_b}, \quad (\text{E.8})$$

$$\frac{D_{t+1}}{P_t} = \Theta_t^d (R_t^d)^{\epsilon_d}, \quad (\text{E.9})$$

$$M_{t+1}^{\text{Fed}} = M_{t+1} + P_t \Theta_t^m (R_t^m)^{\epsilon_m}, \quad (\text{E.10})$$

$$G_{t+1} = G_{t+1}^{\text{FA}} - (G_{t+1}^{\text{Fed}} + P_t \Theta_t^g (R_t^g)^{\epsilon_g}), \quad (\text{E.11})$$

$$R_t^m = \frac{1 + i_t^m}{P_{t+1}/P_t}. \quad (\text{E.12})$$

Using these market clearing conditions, we determine $\{\bar{m}, \bar{a}\}$:

$$P_t \bar{a}_t \cdot (1 - \bar{c}_t) \cdot E_t = \tilde{M}_{t+1}^{\text{Fed}} - M_{t+1}^h + G_{t+1} - G_{t+1}^{\text{Fed}} - G_{t+1}^h. \quad (\text{E.13})$$

$$M_{t+1}^h + P_t \cdot \bar{m}_t (1 - \bar{c}_t) E_t = \tilde{M}_{t+1}^{\text{Fed}}. \quad (\text{E.14})$$

The last term is the definition of R_t^m . This block determines $\{P_t, R_t^b, R_t^m, R_t^d\}$ given aggregate bank variables. The return for the government bond comes from the clearing of government bonds at the balancing stage. This condition is

$$R_t^g = \begin{cases} R_t^m + \chi_t^+ & \text{if } P_t \Theta_t^g (R_t^m + \chi_t^+)^{\epsilon_g} \leq G_t^s - G_t^{\text{Fed}}, \\ \left[\frac{G_t^s - G_t^{\text{Fed}}}{P_t \Theta_t^g} \right]^{1/\epsilon_g} & \text{otherwise.} \end{cases}$$

To close the system, we need the equations that determine χ_t .

Interbank Market Block. We need to determine $\bar{\chi}_t$. This follows from the conditions obtained from Proposition 1:

$$\tilde{S}_t^- = (1 - \bar{c}_t) E_t \cdot \int_1^{\omega_t^*} \tilde{s}(\omega) d\Phi \quad \text{and} \quad \tilde{S}_t^+ = (1 - \bar{c}_t) E_t \cdot \int_{\omega_t^*}^{\infty} \tilde{s}(\omega) d\Phi,$$

where we employ the definition of reserve balances before the exchange of government bonds:

$$\tilde{s}(\omega) = \bar{a}_t + \left(\frac{1 + i_{t+1}^d}{1 + i_{t+1}^m} \right) \omega \bar{d}_t - \rho \bar{d}_t (1 + \omega).$$

The market tightness is defined as

$$\theta_t = \frac{\tilde{S}_t^-}{\tilde{S}_t^+ - \bar{g}_t},$$

and the deposit threshold as

$$\omega_t^* \equiv - \frac{\frac{\bar{a}_t}{\bar{d}_t} - \rho}{\frac{R_{t+1}^d}{R_{t+1}^m} - \rho}.$$

From here, discount-window loans are

$$W_t = (1 - \Psi^-(\theta_t))(\tilde{S}_t^- - \bar{g}_t^-), \quad (\text{E.15})$$

and the average interbank market rate, \bar{i}_t^f , is

$$\bar{i}_t^f = \phi(\theta_t) i_t^m + (1 - \phi(\theta_t)) i_t^w.$$

This system of equations gives us

$$\chi_t^- = \Psi_t^-(\bar{i}_t^f - i_t^m) + (1 - \Psi_t^-)(i_t^w - i_t^m) \quad \text{and} \quad \chi_t^+ = \Psi_t^+(\bar{i}_t^f - i_t^m). \quad (\text{E.16})$$

Note that here we take the probabilities Ψ_t^- and Ψ_t^+ as given functions of market tightness, as we do in the main text. This block determines $\bar{\chi}_t$ and the amount of discount window loans, W_t . Note that so far, we have provided enough equations to solve for $\{\bar{b}_t, \bar{a}_t, \bar{d}_t, \bar{c}_t, \Omega_t, v_t\}$, $\{B_t, M_t, D_t, G_t, E_t\}$, and $\{P_t, R_t^b, R_t^m, R_t^d, R_t^s, \bar{\chi}_t\}$.

Law of Motion for Aggregate Equity. The Fed's budget constraint is

$$E_{t+1} = (1 + (R_{t+1}^b - 1))(\bar{b}_t + \bar{b}_t^{\text{Fed}}) - (R_{t+1}^d - 1)\bar{d}_t(1 - \bar{c}_t)E_t, \quad (\text{E.17})$$

where $\bar{b}_t^{\text{Fed}} \equiv B_{t+1}^{\text{Fed}} / (P_t \cdot (1 - \bar{c}_t)E_t)$. Equation (E.17) shows that portfolio choices, market returns, and next-period Fed policies and price level determine next-period aggregate real equity.

Consolidated Government Budget Constraint. The government's budget policy sequence $\{i_t^m, i_t^w, W_t, B_t^{\text{Fed}}, M_t^{\text{Fed}}, T, T^h\}$ satisfies the following constraint:

$$\begin{aligned} & (1 + i_t^m)M_t + M_t^h + B_{t+1}^{\text{Fed}} - (G_{t+1}^{\text{FA}} - G_{t+1}^{\text{Fed}}) + W_{t+1}^{\text{Fed}} \\ & = M_{t+1}^{\text{Fed}} + (1 + i_t^b)B_t^{\text{Fed}} - (1 + i_t^s)(G_t^{\text{FA}} - G_t^{\text{Fed}}) + (1 + i_t^w)W_t^{\text{Fed}} \\ & \quad + P_t(T_t + T_t^h). \end{aligned} \quad (\text{E.18})$$

And the tax on banks satisfies

$$T_t = (i^m - \pi)\beta \frac{M_t}{P_t} + (i_t^g - \pi_t) \frac{G_t}{P_t} - (i_t^b - \pi_t) \frac{B_t^{\text{Fed}}}{P_t} - (i_t^w - i_t^m) \frac{W_t}{P_t}. \quad (\text{E.19})$$

E.2. Stationary Equilibrium

In a stationary equilibrium, inflation is constant. The stationary equilibrium conditions are summarized by replacing time subscripts with steady state subscripts ss .

Individual Bank Variables. For the individual bank variables, we have

$$c_{ss} = 1 - \beta^{\frac{1}{\gamma}} \Omega_{ss}^{1/\gamma-1}, \quad (\text{E.20})$$

$$v^{ss} = \frac{1}{1-\gamma} \left(\frac{1}{1 - (\beta \Omega_{ss}^{1-\gamma})^{\frac{1}{\gamma}}} \right)^{\gamma}, \quad (\text{E.21})$$

$$\Omega_{ss} \equiv (1 - \tau^{ss}) \max_{\{\bar{b}, \bar{a}, \bar{d}\} \geq 0} \left\{ \mathbb{E}_{\omega} [R_{ss}^b \bar{b} + R_{ss}^m \bar{a} - R_{ss}^d \bar{d} + \bar{\chi}(\bar{a}, \bar{d})]^{1-\gamma} \right\}^{\frac{1}{1-\gamma}}, \quad (\text{E.22})$$

$$\bar{b} + \bar{a} - \bar{d} = 1, \quad (\text{E.23})$$

$$\bar{d} \leq \kappa, \quad (\text{E.24})$$

where $\{\bar{b}_{ss}, \bar{a}_{ss}, \bar{d}_{ss}\}$ are the optimal choices of $\{\bar{b}, \bar{a}, \bar{d}\}$ in the problem above.

Market Clearing Conditions. The real rates and the path for prices follow from the market clearing conditions in all the asset markets:

$$\frac{B_{t+1} + B_{t+1}^{\text{Fed}}}{P_t} = \Theta_t^b (R_t^b)^{\epsilon_b}, \quad (\text{E.25})$$

$$\frac{D_{t+1}}{P_t} = \Theta_t^d (R_t^d)^{\epsilon_d}, \quad (\text{E.26})$$

$$M_{t+1}^{\text{Fed}} = M_{t+1} + P_t \Theta_t^m (R_t^m)^{\epsilon_m}, \quad (\text{E.27})$$

$$G_{t+1} = G_{t+1}^{\text{FA}} - (G_{t+1}^{\text{Fed}} + P_t \Theta_t^g (R_t^g)^{\epsilon_g}), \quad (\text{E.28})$$

$$R_t^m = \frac{1 + i_t^m}{P_{t+1}/P_t}. \quad (\text{E.29})$$

The last term is the definition of R_{ss}^m . This block determines $\{P_t, R_t^b, R_t^m, R_t^d\}$ given aggregate bank variables. The return for the government bond comes from the clearing of government bonds at the balancing stage. This condition is

$$R_{ss}^g = \begin{cases} R_{ss}^m + \chi_{ss}^+ & \text{if } \Theta_{ss}^g (R_{ss}^m + \chi_{ss}^+)^{\epsilon_g} \leq P_t (G_{ss}^{\text{FA}} - G_{ss}^{\text{Fed}}), \\ \left[\frac{G_{ss}^{\text{FA}} - G_{ss}^{\text{Fed}}}{P_t \Theta_{ss}^g} \right]^{1/\epsilon_g} & \text{otherwise.} \end{cases}$$

Notice that in a stationary equilibrium, the price level is pinned down by M_{t+1}^{Fed} , using the demand for reserves and currency. This is because reserves are obtained as a residual, given the indifference. To close the system, we need the equations that determine χ_t .

Interbank Market Block. We need to determine $\bar{\chi}_{ss}$. This follows from the conditions obtained from Proposition 1:

$$\tilde{S}_{ss}^- = (1 - c_{ss})E_{ss} \cdot \int_1^{\omega_{ss}^*} \tilde{s}(\omega) d\Phi \quad \text{and} \quad S_{ss}^+ = (1 - c_{ss})E_{ss} \cdot \int_{\omega_{ss}^*}^{\infty} \tilde{s}(\omega) d\Phi,$$

where we employ the definition

$$s(\omega) = \bar{a}_{ss} + \left(\frac{1 + i_{ss}^d}{1 + i_{ss}^m} \right) \omega \bar{d}_{ss} - \rho \bar{d}_{ss} (1 + \omega). \quad (\text{E.30})$$

The market tightness is defined as

$$\theta_{ss} = \frac{\tilde{S}_{ss}^-}{\tilde{S}_{ss}^+ - \bar{g}_{ss}} \quad (\text{E.31})$$

and the deposit threshold

$$\omega_{ss}^* \equiv - \frac{\frac{\bar{a}_{ss}}{\bar{d}_{ss}} - \rho}{\left(\frac{R_{ss}^d}{R_{ss}^m} - \rho \right)}. \quad (\text{E.32})$$

From here, discount window loans are

$$W_{ss} = (1 - \Psi^-(\theta_{ss})) \tilde{S}_{ss}^-, \quad (\text{E.33})$$

and the average interbank market rate is

$$\bar{i}_{ss}^f = \phi(\theta_{ss}) i_{ss}^m + (1 - \phi) \text{yields}(\theta_{ss}) i_{ss}^w.$$

This system of equations gives us

$$\chi_{ss}^- = \Psi_{ss}^-(\bar{i}_{ss}^f - i_{ss}^m) + (1 - \Psi_{ss}^-)(i_{ss}^w - i_{ss}^m) \quad \text{and} \quad \chi_{ss}^+ = \Psi_{ss}^+(\bar{i}_{ss}^f - i_{ss}^m). \quad (\text{E.34})$$

Note that here we take the probabilities Ψ_{ss}^- and Ψ_{ss}^+ as given functions of market tightness, as in the main text. This block determines $\bar{\chi}_{ss}$ and the amount of discount window loans, W_{ss} .

Law of Motion for Aggregate Equity. The steady state condition for the law of motion of bank equity is

$$1/\beta = (1 + (R_{ss}^b - 1)(\bar{b}_{ss} + \bar{b}_{ss}^{\text{Fed}}) - (R_{ss}^d - 1)\bar{d}_{ss}), \quad (\text{E.35})$$

where $\bar{b}_{ss}^{\text{Fed}} \equiv B_{t+1}^{\text{Fed}} / (P_t (1 - \bar{c}_{ss}) E_{ss})$.

Consolidated Government Budget Constraint. The policy sequence satisfies the following consolidated budget constraint:

$$T_{ss} + T_{ss}^h = \left[(i_{ss}^m - \pi_{ss}) \frac{M_{t+1}}{P_t} + (i_{ss}^g - \pi_{ss}) \frac{G_{t+1}^{\text{FA}} - G_{t+1}^{\text{Fed}}}{P_t} - (i_{ss}^b - \pi_{ss}) \frac{B^{\text{Fed}}}{P_t} \right. \\ \left. - (i_{ss}^w - i_{ss}^m) \frac{W_{t+1}}{P_t} \right]. \quad (\text{E.36})$$

The tax on banks satisfies

$$T_{ss} = E_{ss}(1 - \bar{c}_{ss})[(i_{ss}^m - \pi_{ss})\bar{m}_{ss} + (i_{ss}^g - \pi_{ss})\bar{g}_{ss} - (i_{ss}^b - \pi_{ss})b_{ss}^{\text{Fed}} - (i_{ss}^w - i_{ss}^m)\bar{w}_{ss}]. \quad (\text{E.37})$$

APPENDIX F: NONFINANCIAL SECTOR (PROOF OF PROPOSITION 3)

This Appendix describes the nonfinancial sector of the model, which closes the general equilibrium. The nonfinancial sector is composed of a representative household that supplies labor; stores wealth in deposits, government bonds, and currency; and owns shares of a representative firm. The firm uses labor for production and is subject to a working capital constraint. This block delivers an endogenous demand schedule for loans, a supply for deposits, and a demand for government bonds. Preference and technology assumptions are such that the equilibrium has no feedback from future state variables to the asset demands at period t . The assumptions make all the schedules static and autonomous. This formulation has two virtues. First, we can solve the equilibrium allocations by solving the equilibrium in the deposit market and loan markets, by solving the bank's problem that takes these schedules as given. From then, since quantities are consistent with an equilibrium demand equation from the nonfinancial sector, we know it is satisfying market clearing conditions in the labor market. If all asset markets clear, the goods market also clears. The formulation is convenient because it allows us to focus on the banking system, as we can effectively treat these schedules as exogenous functions with exogenous shocks to their intercepts. We exploit this feature in the application.

The nonfinancial sector is populated by a representative household that saves in deposits, currency, and government bonds and own shares of a productive firm. Assets are special, because different goods are bought with different assets. Similar assumptions are common in new-monetarist models (Lagos, Rocheteau, and Wright (2017)). We see this formulation as a convenient way to obtain asset demands. The firm is subject to a working capital constraint that delivers a demand for loans. The household's Bellman equation is

$$V_t^h(G, M, D, Y) = \max_{\{c^x, X', Y', h\}} \sum_{x \in \{d, g, m\}} U^x(c^x) + c^h - \frac{h^{1+\nu}}{1+\nu} + \beta^h V_{t+1}^h(G', M', D', Y'),$$

subject to the budget constraint

$$\begin{aligned} P_t \left(\sum_{x \in \{d, g, m\}} c^x + c^h \right) + \sum_{X \in \{G, M, D\}} X' + q_t Y' \\ = \sum_{X \in \{G, M, D\}} (1 + i_t^X) X + (q_t + P_t r_t^h) Y + z_t h - P_t T_t^h \end{aligned} \quad (\text{F.1})$$

and the following payment constraints:

$$P_t c^d \leq (1 + i_t^d) D^h, \quad P_t c^g \leq (1 + i_t^g) G^h, \quad P_t c^m \leq M. \quad (\text{F.2})$$

In the problem, the household supplies h hours and consumes four types of goods: c^d are goods subject to a deposits in advance constraint, (F.2); c^g are goods subject to a bond-in-advance constraint; c^m are goods subject to a currency-in-advance constraint; and c^h are goods that are not subject to any constraint and yield linear utility. The quasi-linearity in c^h is key to produce the static nature of demand schedules, because it allows us to fix

marginal utility to one in any Euler equation. Labor supply is h and has an inverse Frisch elasticity of ν , the key parameter for the effect of the loans rate on output. Also, note that β^h is the household's discount factor, which can differ from the banker's discount factor. Equation (F.1) is the household's nominal budget constraint. The right-hand side includes the value of the household's portfolio of assets, G , M , and D . These assets earn nominal interest rates paid by banks and the government; currency has no interest. The term Y is firm shares, which can be normalized to 1. The nominal price of the firm is q_t , firm profits (in real terms) are r_t^h . The wage is z_t is earned on hours worked. Finally, households pay a lump-sum tax T_t^h .

The portfolio of assets G , M , and D matters because each asset is a store of wealth in the budget constraint (F.1), but also because each asset is a special medium of exchange in the corresponding good markets. The preference specification (quasi-linear preferences) is identical to the one in Lagos and Wright (2005). Furthermore, the fact that some goods must be bought with specific assets is akin to the transaction technology in new monetarist models (Lagos, Rocheteau, and Wright (2017)), but the trading protocol stemming from random search is replaced by a Walrasian market. We employ the following utility specification for each good:

$$U^d \equiv (\bar{D}_t)^{\gamma^d} \frac{(c^d)^{1-\gamma^d}}{1-\gamma^d}, \quad U^m \equiv (\bar{M}_t)^{\gamma^m} \frac{(c^m)^{1-\gamma^m}}{1-\gamma^m}, \quad \text{and} \quad U^g \equiv (\bar{G}_t)^{\gamma^g} \frac{(c^g)^{1-\gamma^g}}{1-\gamma^g},$$

where $\{\gamma^d, \gamma^g, \gamma^m\} > 0$. This specification delivers an iso-elastic asset demand with $\{\bar{D}_t, \bar{M}_t, \bar{G}_t\}$ as demand shifts. Notice that if $c^d = \bar{D}_t$, we have $\partial U^d / \partial c^d = 1$. The presence of the linear term c^h in the utility function implies that at the household optimum, we must have $c^d \leq \bar{D}_t$. This bound will be achieved, in effect, when the household is satiated in deposits. The same holds for c^m and c^g .

Next, we present the firm's problem. The firm has access to a production technology that uses h_t^d units of labor that are transformed into $t+1$ output via a production function $y_{t+1} = A_{t+1} h_t^\alpha$. Production is scaled by A_{t+1} , a productivity shock that works as a loan demand shifter. The term A_{t+1} is known at t . The firm uses bank loans to pay workers in the first period to maximize shareholder value.

PROBLEM 15—Firm's Problem:

$$P_{t+1} r_t^h = \max_{\{B_{t+1}^d, h_t\} \geq 0} P_{t+1} y_{t+1} - (1 + i_{t+1}^b) B_{t+1}^d + (1 + i_{t+1}^d) (B_{t+1}^d - z_t h_t),$$

subject to the working capital constraint, $z_t h_t \leq B_{t+1}^d$.

In the firm's problem, the firm maximizes profits, the sum of sales minus financial expenses. The firm borrows B_{t+1}^d from banks and uses these funds to finance payroll, $z_t h_t$. What the firm does not spend is saved as deposits. Notice that in equilibrium, the firm does not save. The next proposition is a generalized version of Proposition 3.¹ It is more general because it describes the equilibrium solution to the asset demands when asset markets for the household are not necessarily satiated.

¹We use the superscript h to indicate the aggregate household holdings of a specific asset.

TABLE IV
STRUCTURAL TO REDUCED FORM PARAMETERS.

Reduced	Θ^x	ϵ^x	Θ^b	ϵ^b
Structural	$\bar{X}_t(\beta^h)^{1/\gamma^x}$	$\frac{1}{\gamma^x} - 1$	$(\alpha A_{t+1})^{-\left(\frac{\nu+1}{\alpha-(\nu+1)}\right)}$	$\frac{\nu+1}{\alpha-(\nu+1)}$

PROPOSITION F.1: *The household demand for loans, deposits, and government bonds are given by*

$$\frac{X_{t+1}^h}{P_t} \begin{cases} = \Theta_t^x (R_{t+1}^x)^{\epsilon^x}, & R_{t+1}^x \leq 1/\beta^h, \\ \geq \bar{X}_t, & R_{t+1}^x = 1/\beta^h, \\ = \infty, & \text{otherwise.} \end{cases} \quad \text{for } x \in \{m, d, g\}.$$

The firm's loan demand is

$$\frac{B_{t+1}^d}{P_t} = \Theta_t^b (R_{t+1}^b)^{\epsilon^b}.$$

Output and hours are given by

$$y_{t+1} = \left(\frac{1}{\alpha}\right)^{\frac{\alpha}{\alpha-(\nu+1)}} A_{t+1}^{\frac{(\nu+1)}{\nu+1-\alpha}} (R_{t+1}^b)^{\frac{\alpha}{\alpha-(\nu+1)}} \quad \text{and} \quad h_t = \left(\frac{1}{\alpha A_{t+1}}\right)^{\frac{1}{\alpha-(\nu+1)}} (R_{t+1}^b)^{\frac{1}{\alpha-(\nu+1)}},$$

and profits and the value of the firm are given by

$$r_{t+1}^h = A_{t+1}^{\frac{(\nu+1)}{\nu+1-\alpha}} \left(\alpha^{-\frac{\alpha}{\alpha-(\nu+1)}} - \alpha^{-\frac{\nu+1}{\alpha-(\nu+1)}} \right) \cdot (R_{t+1}^b)^{\frac{\alpha}{\alpha-(\nu+1)}} \quad \text{and} \quad q_t = \sum_{s \geq 0} (\beta^h)^s r_s^h.$$

One important thing to note is that R_{t+1}^x for $x \in \{m, d, g\}$ refers correspondingly to the real return of each asset. In the context of the household, R_{t+1}^m is the inverse inflation, not the real rate on reserves. Table IV is the conversion table from structural parameters to the reduced form parameters of the nonfinancial sector demand functions.

The rest of the Appendix proceeds with the proof.

PROOF OF PROPOSITION 3: *Derivation of Household Deposit, Bond, and Currency Demands.* To ease the notation, we remove the h superscripts from Problem F. Define the household's net worth, $e^h = (1 + i_t^d)D + (1 + i_t^m)M + (1 + i_t^g)G + (q_t + r_t)Y - T_t^h$, as the right-hand side of its budget constraint, excluding labor income. Then substitute c^h from the budget constraint and employ the definition e^h . We obtain the following value function:

$$\begin{aligned} V_t^h(G, M, D, Y) = & \max_{\{c^d, c^g, h, G', D', Y'\}} U^d(c^d) + U^g(c^g) + U^m(c^m) - \frac{h^{1+\nu}}{1+\nu} + e^h \\ & + \frac{z_t h - (P_t c^g + P_t c^d + D' + G' + M' + q_t Y')}{P_t} \\ & + \beta^h V_{t+1}^h(G', M', D', Y'), \end{aligned} \quad (\text{F.3})$$

subject to the payment in advance constraints in (F.2).

Step 1—Derivation of the Deposit, Currency, and Bond-Goods Demand. The first step is to take the first-order conditions for $\{c^d, c^g, c^m\}$. Since $\{G, D, M\}$ enter symmetrically into the problem, we express the formulas in terms of $x \in \{d, g, m\}$, an index that corresponds to each asset. From the first-order conditions in the objective of (F.3) with respect to $\frac{D}{P_t}$, $\frac{G}{P_t}$, and $\frac{M}{P_t}$, we obtain that

$$(U_{c^x}^x) = 1 + \mu_t^x,$$

where $\mu_t^x \geq 0$ are associated multipliers payment-in-advance constraints in (F.2). The multiplier is activated when $U_{c^x}^x \leq 1$, and thus $c^x \leq R_t^x \cdot \frac{X}{P_{t-1}}$. Solving for the multiplier, we obtain $\mu_t^x = \max\{U_{c^x}^x(R_t^x \cdot \frac{X}{P_{t-1}}) - 1, 0\}$. Combining this multiplier yields

$$c^x(X, t) = \min\left\{(U_{c^x}^x)^{-1}(1), R_t^x \cdot \frac{X}{P_{t-1}}\right\} \quad \text{for } x \in \{d, g, m\}. \quad (\text{F.4})$$

The expression shows that the deposit- and bond-in-advance constraints bind if the marginal utility associated with their consumption is less than one. Note that

$$U_{c^x}^x(\bar{X}) = (\bar{X})^{\gamma^x} x^{-\gamma^x} \quad \text{for } x \in \{d, g, m\}, \quad (\text{F.5})$$

and marginal utility is above 1, for $X/P_t < \bar{X}$. Then the marginal consumption as a function of real balances is

$$\frac{\partial c^x}{\partial(X'/P_t)} = \begin{cases} R_t^x, & X/P_t < \bar{X}, \\ 0, & \text{otherwise,} \end{cases} \quad \text{for } x \in \{d, g, m\}.$$

We return to this condition below to derive the demand for deposits and bonds by the non-financial sector.

Step 2—Labor Supply. The first-order condition with respect to labor supply yields a labor supply that depends only on the real wage:

$$h_t^v = z_t/P_t. \quad (\text{F.6})$$

Step 3—Deposit and Bond Demand. Next, we derive the household demand for deposits, government bonds, and currency. By taking first-order conditions with respect to D'/P_t , G'/P_t , and M'/P_t , we obtain the real balances of deposits, bonds, and currency:

$$1 = \beta^h \frac{\partial V_{t+1}^h}{\partial(X'/P_t)} = \beta^h \left[\frac{\partial U^x}{\partial c^x} \cdot \frac{\partial c^x}{\partial(X'/P_t)} + \frac{\partial U^h}{\partial c^h} \cdot \frac{\partial c^h}{\partial(X'/P_t)} \right] \quad \text{for } x \in \{d, g, m\}.$$

The first equality follows directly from the first-order condition, and the second uses the envelope theorem and the solution for the optimal consumption rule. If we shift the period in (F.4) by one, the first-order condition then becomes

$$\frac{1}{\beta^h} = \begin{cases} \frac{\partial U^x}{\partial c^x} R_t^x, & X/P_t < \bar{X}, \\ R_t^x, & \text{otherwise,} \end{cases} \quad \text{for } x \in \{d, g, m\}.$$

Finally, once we employ the definition of marginal utility, we obtain

$$\frac{1}{\beta^h} = \begin{cases} (\bar{X})^{\gamma^x} (R_t^x X/P_t)^{-\gamma^x} R_t^x, & X/P_t < \bar{X}, \\ R_t^x, & \text{otherwise,} \end{cases} \quad \text{for } x \in \{d, g, m\}.$$

Inverting the condition yields

$$X/P_t = \begin{cases} \bar{X}(\beta^h)^{1/\gamma^x} (R_t^x)^{\frac{1}{\gamma^x}-1}, & R_t^x < 1/\beta^h, \\ [\bar{X}, \infty), & R_t^x = 1/\beta^h, \\ \infty, & R_t^x > 1/\beta^h, \end{cases} \quad \text{for } x \in \{d, g, m\}.$$

Thus, we have that

$$\Theta_t^x = \bar{X}_t(\beta^h)^{1/\gamma^x} \quad \text{and} \quad \epsilon^x = \frac{1}{\gamma^x} - 1 \quad \text{for } x \in \{d, g\}.$$

This verifies the functional form for the household demand schedules. Next, we move to the firm's problem to obtain the demand for loans.

Firm Problem. In this Appendix, we allow the firm to save in deposits whatever it does not spend in wages. From the firm's problem, if we substitute the production function into the objective, we obtain

$$P_{t+1}r_{t+1}^h = \max_{B_{t+1}^d \geq 0, x_{t+1}, h_t \geq 0} P_{t+1}A_{t+1}h_t^\alpha - (1 + i_{t+1}^b)B_{t+1}^d + (1 + i_{t+1}^d)(B_{t+1}^d - z_t h_t),$$

subject to $z_t h_t \leq B_{t+1}^d$. Observe that

$$\begin{aligned} & P_{t+1}A_{t+1}h_t^\alpha - (1 + i_{t+1}^b)B_{t+1}^d + (1 + i_{t+1}^d)(B_{t+1}^d - z_t h_t) \\ &= P_{t+1}A_{t+1}h_t^\alpha - z_t h_t - (i_{t+1}^b - i_{t+1}^d)(B_{t+1}^d + z_t h_t). \end{aligned}$$

Step 4—Loans Demand. Since $i_{t+1}^b \geq i_{t+1}^d$, it is without loss of generality that the working capital constraint is binding, $z_t h_t = B_{t+1}^d$. Thus, the objective is

$$P_{t+1}A_{t+1}h_t^\alpha - (1 + i_{t+1}^b)z_t h_t.$$

The first-order condition in h_t yields

$$P_{t+1}\alpha A_{t+1}h_t^{\alpha-1} = (1 + i_{t+1}^b)z_t.$$

Dividing both sides by P_t , we obtain

$$\frac{P_{t+1}}{P_t}\alpha A_{t+1}h_t^{\alpha-1} = (1 + i_{t+1}^b)\frac{z_t}{P_t}h_t.$$

Next, we use the labor supply function, (F.6), to obtain the labor demand as a function of the loans rate:

$$\frac{P_{t+1}}{P_t}\alpha A_{t+1}h_t^{\alpha-1} = (1 + i_{t+1}^b)h_t^{\nu+1} \rightarrow R_t^b = \frac{\alpha A_{t+1}h_t^\alpha}{h_t^{\nu+1}}. \quad (\text{F.7})$$

Once we obtain the wage, if we use that the fact that the working capital constraint is binding, we have

$$\frac{B_{t+1}^d}{P_t} = h_t \frac{z_t h_t}{P_t} = h_t^{\nu+1} \rightarrow h_t = \left(\frac{B_{t+1}^d}{P_t} \right)^{\frac{1}{\nu+1}}. \quad (\text{F.8})$$

We combine (F.7) and (F.8) to obtain the demand for loans:

$$R_t^b = \alpha A_{t+1} \left(\frac{B_{t+1}^d}{P_t} \right)^{-1} \left(\frac{B_{t+1}^d}{P_t} \right)^{\frac{\alpha}{\nu+1}} \rightarrow \frac{B_{t+1}^d}{P_t} = \Theta_t (R_{t+1}^b)^{\epsilon^b}. \quad (\text{F.9})$$

Thus, the coefficients of the loans demand are

$$\Theta_t^b = (\alpha A_{t+1})^{-\epsilon^b} \quad \text{and} \quad \epsilon^b = \left(\frac{\nu+1}{\alpha - (\nu+1)} \right).$$

This concludes the elements of the proposition. Next, we present the formulas for hours, output and the market price of shares.

Step 5—Equilibrium Output and Hours. We substitute the loans demand, (F.9), into (F.8) to obtain the labor market equilibrium:

$$h_t = \left(\frac{1}{\alpha A_{t+1}} \right)^{\frac{1}{\alpha - (\nu+1)}} (R_{t+1}^b)^{\frac{1}{\alpha - (\nu+1)}}.$$

We substitute (F.8) into the production function to obtain

$$y_{t+1} = A_{t+1} \left(\frac{1}{\alpha A_{t+1}} \right)^{\frac{\alpha}{\alpha - (\nu+1)}} (R_{t+1}^b)^{\frac{\alpha}{\alpha - (\nu+1)}} \rightarrow y_{t+1} = \left(\frac{1}{\alpha} \right)^{\frac{\alpha}{\alpha - (\nu+1)}} A_{t+1}^{\frac{(\nu+1)}{\nu+1 - \alpha}} (R_{t+1}^b)^{\frac{\alpha}{\alpha - (\nu+1)}}.$$

The profit of the firm is given by

$$r_{t+1}^h = y_{t+1} - R_{t+1}^b B_{t+1} \rightarrow r_{t+1}^h = A_{t+1}^{\frac{(\nu+1)}{\nu+1 - \alpha}} \left(\alpha^{-\frac{\alpha}{\alpha - (\nu+1)}} - \alpha^{-\frac{\nu+1}{\alpha - (\nu+1)}} \right) \cdot (R_{t+1}^b)^{\frac{\alpha}{\alpha - (\nu+1)}}.$$

Step 6—Market Price. The asset price q_t then is determined as

$$q_t = \sum_{s \geq 1} (\beta^h)^s r_s^h.$$

With this, we conclude that output, hours, and the firm price are decreasing in the current (and future) loans rate. Throughout the proof, we use the labor market clearing condition, so this market clears independently of other markets. Thus, once we compute equilibria taking the schedules as exogenous in the bank's problem, it is possible to obtain output, hours, and household consumption from the equilibrium rates. By Walras' law, if asset markets clear, so does the goods market. *Q.E.D.*

APPENDIX G: PROOFS OF POLICY ANALYSIS (SECTION 3)

To present formal proofs, we define two important concepts: reserve satiation and neutrality.

DEFINITION 16—Satiation: Banks are *satiated* with reserves at period t if the liquidity premium is zero, $R_t^b = R_t^m$.

The following lemma states that banks are satiated with reserves under two conditions.

LEMMA G.1: *Banks are satiated with reserves if and only if either (Case 1) $i_t^w = i_t^m$ or (Case 2) a bank is in surplus for $\omega = \omega_{\min}$.*

To discuss policy effects, we compare an original policy sequence—with subindex o —with an alternative (shock) policy—subindex s in all of the exercises. We say that a policy is neutral relative to the other if it satisfies the following definition.

DEFINITION 17—Neutrality: Consider original and alternative policy sequences:

$$\begin{aligned} & \{ \rho_{o,t}, B_{o,t}^{\text{Fed}}, G_{o,t}^{\text{Fed}}, G_{o,t}^{\text{FA}}, M_{o,t}, W_{o,t}, T_{o,t}, \kappa_{o,t}, i_{o,t}^{\text{ior}}, i_{o,t}^w \} \quad \text{and} \\ & \{ \rho_{s,t}, B_{s,t}^{\text{Fed}}, G_{s,t}^{\text{Fed}}, G_{s,t}^{\text{FA}}, M_{s,t}, W_{s,t}, T_{s,t}, \kappa_{s,t}, i_{s,t}^{\text{ior}}, i_{s,t}^w \}. \end{aligned}$$

Policy s is *neutral*—relative to o —if the induced equilibria satisfies

$$\{ E_{o,t}, c_{o,t}, \bar{b}_{o,t}, \bar{d}_{o,t}, \bar{m}_{o,t}, \bar{g}_{o,t} \} = \{ E_{s,t}, c_{s,t}, \bar{b}_{s,t}, \bar{d}_{s,t}, \bar{m}_{s,t}, \bar{g}_{s,t} \} \quad \text{for all } t \geq 0.$$

When the condition holds, real aggregate loans and deposits are also determined and identical to those of the original allocation—and also for currency and holdings of government bonds and currency. The rest of this Appendix shows the proofs for the classic exercises in monetary policy analysis that we studied in the main text. We begin by establishing the classic results.

PROPOSITION G.1: *Consider an equilibrium sequence induced by policy $\{M_{t+1}, W_{t+1}, B_{t+1}^{\text{Fed}}, G_{t+1}^{\text{Fed}}\}$ and $\{i_t^w, i_t^m\}$. Then:*

(i) *if the sequence induces a stationary equilibrium, then an alternative policy sequence in which the Fed balance sheet is scaled by a constant $K > 0$ induces another stationary equilibrium in which the price level is scaled by K , but all real variables are the same as in the original stationary equilibrium;*

(ii) *if the components of the balance sheet of the Fed grow at rate k_t for some t , and an alternative policy that differs only in that growth rate is neutral if and only if the demand for currency is inelastic (or zero) and the Fed alters its nominal policy rates to keep $\{\frac{1+i_t^m}{1+k_t}, \frac{1+i_t^w}{1+k_t}\}$ constant across both policies.*

Part (i) establishes long-run neutrality. This result applies only to the stationary equilibrium because assets are nominal. Thus, changes at any point in time, by changing the price level, have redistributive consequences. Even if the policy is anticipated, the policy change induces a different equilibrium if policy rates are not adjusted. In the long run, however, a change in the scale of the Fed's balance sheet leads to a scaled stationary equilibrium. Part (ii) is a condition for superneutrality: the condition that changes in the inflation rate of the economy are neutral. The result says that if the Fed increases the growth rate of its nominal balance sheet by a scalar and adjusts its nominal policy rates to keep real rates constant, variations in the growth rate of its nominal balance sheet translate only into changes in the unit of account, and inflation generates no changes. It is important to note that a qualification for this result is that the demand for real balances of currency is inelastic. Otherwise, changes in inflation produce a change in the money demand by households, and inflation adjusts differently in that context. Part (ii) also can be interpreted as an approximation for mild inflation rates: as long as the currency demand is close to inelastic, changes in inflation will be close to neutral.

G.1. Proof of Lemma G.1 (Conditions for Satiation)

By definition of satiation, the right-hand side of (Loan LP-Deposit LP) must equal zero under satiation, and thus

$$0 = \bar{\chi}^+ + (\bar{\chi}^- - \bar{\chi}^+) \cdot F(\omega^*) \cdot \frac{\mathbb{E}_\omega[(R^e)^{-\gamma} \omega < \omega^*]}{\mathbb{E}_\omega[(R^e)^{-\gamma}]},$$

and

$$0 = \bar{\chi}^+.$$

This expression equals zero in two cases.

Case 1. If $i_t^w = i_t^m$, then the condition holds immediately, since $\chi^- = \chi^+ = 0$. This case is condition (i) in the proposition.

Case 2. If $i_t^w > i_t^m$, then since $\chi^- > \chi^+$ for any θ , we must have that $F(\omega^*) = 0$ and $\bar{\chi}^+ = 0$. This occurs only if $\omega^* \leq \omega_{\min}$.

Under condition (ii) of the proposition, no bank is in deficit, even for the worst shock.

G.2. Proof of Proposition G.1 Item (i)

Consider a policy sequence $\{o\}$ and an alternative policy $\{s\}$ such that:

1. $X_{s,t} = kX_{o,t}$ for some $k > 0$ for the balance sheet variables $X \in \{B^{\text{Fed}}, G^{\text{Fed}}, G^{\text{FA}}, M, W\}$;
2. policies are identical for nonbalance-sheet variables $\{\rho_{o,t}, \kappa_{o,t}, i_{o,t}^m, i_{o,t}^w\} = \{\rho_{s,t}, \kappa_{s,t}, i_{s,t}^m, i_{s,t}^w\}$.

The proposition states that the stationary equilibrium induced by either policy features identical real asset positions and price levels that satisfy $P_{s,t} = kP_{o,t}$.

The proof is by construction, and it is immediate to verify that the equilibrium conditions that determine $\{\bar{b}_{ss}, \bar{a}_{ss}, \bar{d}_{ss}, \bar{c}_{ss}, E_{ss}\}$ in Section E.2 are satisfied by the pair of policy sequences

$$\{B_{o,t}^{\text{Fed}}, G_{o,t}^{\text{Fed}}, G_{o,t}^{\text{FA}}, M_{o,t}, W_{o,t}\}_{t \geq 0}$$

and

$$\{B_{s,t}^{\text{Fed}}, G_{s,t}^{\text{Fed}}, G_{s,t}^{\text{FA}}, M_{s,t}, W_{s,t}\}_{t \geq 0}.$$

We proceed to check that $\{\bar{b}_{ss}, \bar{a}_{ss}, \bar{d}_{ss}, \bar{c}_{ss}, E_{ss}\}$ solves the set of equilibrium equations in Section E.2 in both cases.

Consider the original and alternative policies. We are considering stationary equilibria, so by hypothesis these satisfy

$$X_{a,t} = X_{a,t-1}(1 + \pi_{ss}), \quad \text{for some } \pi_{ss} \text{ and } a \in \{o, s\} \text{ and } \{B^{\text{Fed}}, G^{\text{Fed}}, G^{\text{FA}}, M, W\}.$$

By hypothesis also, inflation and nominal rates are equal under both policies. Thus, the real interest rate on reserves is equal under both policies. We check the equilibrium conditions in the order in which they appear in Section E.

First, we guess and verify that the real returns on loans and deposits are also equal under both policies. If both policies yield the same real rates, the solution for bank portfolios (the solution for Ω_t) must also be equal in both equilibria:

$$\{\bar{b}_{o,ss}, \bar{a}_{o,ss}, \bar{d}_{o,ss}, \bar{c}_{o,ss}\} = \{\bar{b}_{s,ss}, \bar{a}_{s,ss}, \bar{d}_{s,ss}, \bar{c}_{s,ss}\}.$$

Consider now the aggregate supply of loans and reserves under either policy:

$$(1 - c_{ss})\bar{b}_{ss}E_{ss} = \Theta^b (R_{ss}^b)^\epsilon - B_{t+1}^{\text{Fed}}/P_t.$$

That equation can be satisfied under both policies because $B_{o,t+1}^{\text{Fed}}/P_{o,t} = (1+g)B_{o,t+1}^{\text{Fed}}/(1+g)P_{o,t} = B_{s,t+1}^{\text{Fed}}/P_{s,t}$. This verifies that the real rate on loans is equal under both policies.

The same steps verify that R_{ss}^d is the same under both policies. Similarly, the demand for reserves and currency can be satisfied in both equations because

$$(1 - c_{ss})\bar{m}_{ss}E_{ss} = M_{o,t}/P_{o,t} = M_{a,t}/P_{a,t}.$$

A similar argument holds for the holdings of government bonds. This verifies market clearing for reserves.

Now, the ratio of surpluses to deficits is also equal under both policies:

$$\theta_{ss} \equiv S_{a,t}^-/S_{a,t}^+ \quad \text{for } a \in \{o, s\}.$$

Because θ and policy rates are equal, the liquidity cost function χ is also equal under both policies. Observe that χ is a function of θ only. With equal inflation under both policies, the liquidity return R^χ must also be equal. This verifies that all the real rates in both equilibria are the same under both policies. Since rates are the same, both policies satisfy the same law of motion for equity (18). It is immediate to verify that the consolidated government budget constraint is satisfied under both policies, once all portfolios from the private sector are identical in real terms.

G.3. Proof of Proposition G.1 Item (ii)

The proof closely follows the proof of item (i). The difference is that we prove neutrality along an equilibrium sequence, not only in a stationary equilibrium. The proof is again by construction and requires only that we verify that the equilibrium conditions that determine $\{\bar{b}_t, \bar{a}_t, \bar{d}_t, \bar{c}_t, E_t\}$ in Section E lead to the same values under both policies. Let

$$\{M_{o,t}, G_{a,t}^{\text{Fed}}, G_{a,t}^{\text{FA}}, B_{o,t}^{\text{Fed}}, W_{o,t}\}_{t \geq 0}$$

and

$$\{M_{a,t}, G_{a,t}^{\text{Fed}}, G_{a,t}^{\text{FA}}, B_{a,t}^{\text{Fed}}, W_{a,t}\}_{t \geq 0}$$

be two policy sequences. Again, to ease notation, we follow the order of the equations in Section E.

Consider the original and alternative policies. By the hypothesis of stationary equilibrium, both equilibria satisfy

$$X_{a,t} = X_{a,t-1}^{\text{Fed}}(1 + k_a),$$

$$B_{a,t}^{\text{Fed}} = B_{a,t-1}^{\text{Fed}}(1 + k_a) \quad \text{for } k_a \text{ and for } a \in \{o, s\} \text{ and } X \in \{M, G^{\text{Fed}}, G^{\text{FA}}, B^{\text{Fed}}, W\}.$$

Also, let the initial conditions be the same: $X_{s,0} = X_{o,0}$.

Then the condition for the consolidated government implies that

$$\begin{aligned} X_{s,t+1} &= (1 + k_s)^t X_{s,0} = (1 + k_s)^t X_{o,0}, \quad \text{and} \\ X_{o,t+1} &= (1 + k_o)^t X_{o,0} \end{aligned}$$

for $X \in \{M, G^{\text{Fed}}, G^{\text{FA}}, B^{\text{Fed}}, W\}$. Thus, we can relate both government policy paths via

$$X_{s,t+1} = \left(1 + \frac{k_s - k_o}{1 + k_o}\right)^t X_{o,t+1}.$$

Through the proof, we guess and verify the following.

$$\text{A.1 } \{R_{o,t}^b, R_{o,t}^d, R_{o,t}^m, R_{o,t}^g, R_{o,t}^{\bar{x}}\} = \{R_{s,t}^b, R_{s,t}^d, R_{s,t}^m, R_{s,t}^g, R_{s,t}^{\bar{x}}\}.$$

$$\text{A.2 } P_{o,0} = P_{s,0} = P_0.$$

$$\text{A.3 } (1 + \pi_{s,t}) = (1 + \pi_{o,t}) \left(1 + \frac{k_s - k_o}{1 + k_o}\right).$$

First, we verify (A.1). Under the conjecture that real returns are the same along a sequence, we have that

$$\{\bar{b}_{o,t}, \bar{a}_{o,t}, \bar{d}_{o,t}, \bar{c}_{o,t}\} = \{\bar{b}_{s,t}, \bar{a}_{s,t}, \bar{d}_{s,t}, \bar{c}_{s,t}\},$$

so the optimality conditions are satisfied in both cases.

Next, consider the aggregate supply of loans and reserve demand. Equilibrium in the loans market requires

$$(1 - c_t) \bar{b}_t E_t = \Theta^b (R_t^b)^\epsilon - B_{t+1}^{\text{Fed}} / P_t.$$

If the equation is satisfied under both policies, then we must verify that $B_{o,t+1}^{\text{Fed}} / P_{o,t} = B_{s,t+1}^{\text{Fed}} / P_{s,t}$. To see that this condition holds, recall that

$$B_{s,t+1}^{\text{Fed}} = \left(1 + \frac{k_s - k_o}{1 + k_o}\right)^t B_{s,0}^{\text{Fed}}.$$

Now, if $\pi_{s,t} - \pi_{o,t} = (k_s - k_o)/(1 + k_o)$, by (A.2), we have that

$$P_{a,t} = \prod_{\tau=1}^t (1 + \pi_{a,\tau}) P_0 \quad \text{for } a \in \{o, s\}.$$

Combined with the guess (A.3) above, we obtain

$$P_{s,t} = \prod_{\tau=1}^t (1 + \pi_{o,\tau}) \left(1 + \frac{k_s - k_o}{1 + k_o}\right) P_0 = P_{o,t} \left(1 + \frac{k_s - k_o}{1 + k_o}\right)^t.$$

Therefore,

$$B_{s,t+1}^{\text{Fed}} / P_{s,t} = \left(1 + \frac{k_s - k_o}{1 + k_o}\right)^t B_{s,0}^{\text{Fed}} / P_{s,t} = B_{o,t+1}^{\text{Fed}} / P_{o,t},$$

which shows that the real holdings of loans under both policies are equal. The arguments are identical for the equilibrium in the deposit and government bond market, but the market for Fed assets works differently.

We needed to verify that under our guess, $\{R_t^b, R_t^g, R_t^d\}$ is the same under both policies. Note that R_t^m is the same under both policies:

$$R_{o,t}^m = (1 + i_{o,t+1}^{\text{ior}}) / (1 + \pi_{o,t+1}) = (1 + i_{s,t+1}^{\text{ior}}) \left(1 + \frac{k_s - k_o}{1 + g_o}\right) / (1 + \pi_{o,t+1}),$$

and by assumption (A.3), the condition is also equal:

$$(1 + i_{s,t+1}^{\text{ior}}) \frac{(1 + \pi_{o,t+1})}{(1 + \pi_{s,t+1})} / (1 + \pi_{o,t+1}) = R_{s,t}^m.$$

Next, consider the condition for an equilibrium for Fed liabilities:

$$(1 - c_t) \bar{m}_t E_t = M_{o,t} / P_{o,t} - M_{o,t}^h / P_{o,t} = M_{s,t} / P_{s,t} - M_{s,t}^h / P_{s,t}.$$

It is important that the demand for real balances of currency is inelastic—possibly zero. Otherwise, since currency earns no interest rate, its rate of return does change with the rate of inflation. For that reason, consider that if the demand the demand for real balances is indeed inelastic, then bank reserve demand must be the same: the condition is used to verify our guess (A.3). The condition above requires

$$\frac{P_{s,t+1}}{P_{o,t+1}} = \frac{M_{s,t+1}}{M_{o,t+1}} = \left(1 + \frac{k_s - k_o}{1 + k_o}\right)^t \frac{M_{o,t+1}}{M_{o,t+1}} = \left(1 + \frac{k_s - k_o}{1 + k_o}\right)^t.$$

Then since by Assumption (A.2), initial prices are the same we have that

$$\frac{P_{s,t+1}}{P_{o,t+1}} = \frac{\prod_{\tau=1}^t (1 + \pi_{s,\tau}) P_0}{\prod_{\tau=1}^t (1 + \pi_{o,\tau}) P_0} = \left(1 + \frac{k_s - k_o}{1 + k_o}\right)^t \Rightarrow \prod_{\tau=1}^t (1 + \pi_{s,\tau}) = \prod_{\tau=1}^t (1 + \pi_{o,\tau}) \left(\frac{1 + k_s}{1 + k_o}\right).$$

Since the condition holds for all t , then A.3 is deduced from the quantity equation of reserves.

The next step is to verify that $R_t^{\bar{x}}$ is constant under both policies. For that, observe that the interbank market tightness is the same under both economies. To see that, simply note that the ratio of reserves to deposits is the same under both policies and that this is enough to guarantee that θ_t is equal under both policies. By Lemma C.3 and the condition for policy rates in the proposition— $(1 + i_{o,t}^x) = (1 + i_{s,t}^x) \left(\frac{1 + k_s}{1 + k_o}\right)$ for $x \in \{w, m\}$ —in states away from satiation,

$$\chi(\cdot; i_{s,t}^w, i_{s,t}^{\text{ior}}) = \left(\frac{1 + k_s}{1 + k_o}\right) \chi(\cdot; i_{o,t}^w, i_{o,t}^{\text{ior}}).$$

Therefore, we have that

$$R_{o,t}^x = \frac{\chi(\cdot; i_{o,t}^w, i_{o,t}^{\text{ior}})}{1 + \pi_{o,t}} = \frac{\left(\frac{1 + k_s}{1 + k_o}\right) \chi(\cdot; i_{o,t}^w, i_{o,t}^{\text{ior}})}{\left(\frac{1 + k_s}{1 + k_o}\right) (1 + \pi_{o,t})} = \frac{\chi(\cdot; i_{s,t}^w, i_{s,t}^{\text{ior}})}{(1 + \pi_{s,t})} = R_{s,t}^x.$$

This step verifies that $R_{o,t}^{\bar{x}} = R_{s,t}^{\bar{x}}$. So far, we have checked the consistency of assumptions (A.1) and (A.3) and that the policy rules for $\{\bar{b}_t, \bar{a}_t, \bar{d}_t, \bar{c}_t\}$ and the real rates are the same under both equilibria. We still need to show that the sequences for E_t are the same under both policies, that the initial price level is the same, and that the Fed's budget constraint

is satisfied under both policies. This follows immediately from the law of motion of bank equity:

$$E_{t+1} = (1 + (R_{t+1}^b - 1)\bar{b}_t - (R_{t+1}^d - 1)\bar{d}_t)(1 - \bar{c}_t)E_t,$$

which, as noted, must be the same. We have already verified that $B_{s,t+1}^{\text{Fed}}/P_{s,t} = B_{s,0}^{\text{Fed}}/P_{o,t}$. Following the same steps, we can show that real reserves M_t^{Fed}/P_t , government bonds G_t^{Fed}/P_t and G_t^{FA}/P_t and discount loans W_t^{Fed}/P_t are identical under both policies. Away from satiation, $R_{o,t}^x = R_{s,t}^x$, so that means that real income from the discount window, $\frac{W_t^{\text{Fed}}}{P_t} \left(\frac{1+i_t^w}{1+\pi_t} \right)$, is constant under both policies— τ_t is identical under both policies. Consider now $(B_0, D_0, M_0, G_0, W_0)$, the initial condition under both policies. If P_0 is same initial price under both policies, $E_{o,0} = E_{s,0}$. This is precisely the initial conditions that we need to confirm our guess that $E_{o,0} = E_{s,0}$ and $P_{o,0} = P_{s,0}$.

G.4. Proof of Proposition 9

Consider two policies, o and s , and let the alternative policy feature a mix of conventional and unconventional open-market operations performed at $t = 0$ and reverted at $t = 1$ in the sense that:

1. $B_{s,1}^{\text{Fed}} = B_{o,1}^{\text{Fed}} + \Delta B^{\text{Fed}}$, $G_{s,1}^{\text{Fed}} = G_{o,0}^{\text{Fed}} + \Delta G^{\text{Fed}}$, and $M_{s,1}^{\text{Fed}} = M_{o,1}^{\text{Fed}} + \Delta M^{\text{Fed}}$, such that, $\Delta M^{\text{Fed}} = \Delta G^{\text{Fed}} + \Delta B^{\text{Fed}}$ and $\Delta M^{\text{Fed}}, \Delta G^{\text{Fed}}, \Delta B^{\text{Fed}} \geq 0$ satisfying $B_{s,1}^{\text{Fed}} \geq 0$ and $G_{s,1}^{\text{Fed}} \geq 0$;
2. for all $t \geq 0$, we have

$$\{i_{o,t}^m, i_{o,t}^w, G_{o,t}^{\text{FA}}, W_{o,t}\} = \{i_{s,t}^m, i_{s,t}^w, G_{s,t}^{\text{FA}}, W_{s,t}\};$$

3. for all $t \neq 1$, we have

$$\{i_{o,t}^m, i_{o,t}^w, M_{o,t}^{\text{Fed}}, G_{o,t}^{\text{Fed}}, G_{o,t}^{\text{FA}}, B_{o,t}^{\text{Fed}}, W_{o,t}\} = \{i_{s,t}^m, i_{s,t}^w, M_{s,t}^{\text{Fed}}, G_{s,t}^{\text{Fed}}, G_{s,t}^{\text{FA}}, B_{s,t}^{\text{Fed}}, W_{s,t}\}.$$

The statement of the proposition is that for $\lambda > 0$, the operation is neutral if and only if banks are satiated with reserves at time zero under both policies. If $\lambda \rightarrow 0$ and the economy is away from satiation, then a conventional policy, $\Delta B = 0$, is neutral, but an unconventional policy, $\Delta B > 0$, is not. We refer to neutrality as a situation in which, as we compare across both policy sequences, the total outstanding amount of loans, deposits, and bonds remains unchanged in real terms.

The proof requires an intermediate step: First, we show that if two policies induce identical real aggregate loans deposits and bond holdings, the equilibrium prices $P_{o,0} = P_{s,0}$ must be equal. Then we show for positive λ that if the price is constant, the open-market operation must have real effects away from satiation. Then we show that if banks are satiated, the policy has no effects. Finally, we show that if $\lambda = 0$, the stated results holds.

Auxiliary Lemma. First, we prove the following auxiliary lemma corresponding to the first step of the proof.

LEMMA G.2: *Consider two arbitrary policy sequences o and s , as described above. If total real loans, deposits, dividends, reserves, government bonds, and bank equity are equal across equilibria for all $t \geq 0$, then $P_{o,0} = P_{s,0}$.*

PROOF: Without loss of generality, normalize the price in the original equilibrium to $P_{o,0} = 1$, but not the price of the alternative sequence—we can always rescale the original sequence to obtain a price of one. The idea of the proof is to start from the quantity equation in one equilibrium and use real market clearing conditions to express obtain a relationship using quantities of the second equilibrium. Using the quantity equation of the second equilibrium, the result must follow.

Consider now a given bank. By hypothesis, real equity is equal in both equilibria, $E_{s,0} = E_{o,0}$ and $\bar{c}_{o,0} = \bar{c}_{s,0}$. Also, recall that

$$\{B_{o,1}^{\text{Fed}}, B_{o,1}, G_{o,1}^{\text{Fed}}, G_{o,1}, M_{o,1}^{\text{Fed}}, M_{o,1}\}$$

and

$$\{B_{s,1}^{\text{Fed}}, B_{s,1}, G_{s,1}^{\text{Fed}}, G_{s,1}, M_{s,1}^{\text{Fed}}, M_{s,1}\}$$

are the nominal loans, government bonds, and reserves of the Fed and the representative bank, respectively, under the original and alternative policies.

We use the following relationships. Since equity, dividends, and real deposits are constant, from the bank's budget constraints, we obtain

$$B_{o,1} - B_{s,1}/P_{s,1} = A_s/P_{s,1} - A_o. \quad (\text{G.1})$$

Also, we know that since market clearing must hold in the loans market under both equilibrium sequences,

$$\begin{aligned} \Theta^b(R_{o,1}^b)^{\epsilon^b} &\equiv B_{o,1}^{\text{Fed}} + B_{o,1} \\ &= (B_{s,1}^{\text{Fed}} + B_{s,1})/P_{s,1}, \end{aligned} \quad (\text{G.2})$$

—recall that $P_{o,0} = 1$.

We now exploit the quantity equations of both equilibria through the following relationship:

$$\begin{aligned} M_{o,t}^{\text{Fed}} + G_{o,t}^{\text{FA}} - G_{o,t}^{\text{Fed}} - G_{o,t}^h + \Delta M - \Delta G &= M_{1,t}^{\text{Fed}} + G_{1,t}^{\text{FA}} - G_{1,t}^{\text{Fed}} - G_{1,t}^h \\ &= P_{s,0} \bar{a}_{s,0} (1 - \bar{c}_{s,0}) E_{s,0} \\ &= P_{s,0} (\bar{b}_{o,0} + \bar{a}_{o,0} - \bar{b}_{s,0}) (1 - \bar{c}_{s,0}) E_{s,0} \\ &= P_{s,0} (\bar{b}_{o,0} + \bar{a}_{o,0}) (1 - \bar{c}_{s,0}) E_{s,0} \cdots \\ &\quad - (\Theta^b(R_{o,1}^b)^{\epsilon^b} - B_{s,1}^{\text{Fed}}). \end{aligned} \quad (\text{G.3})$$

The first equality uses just the relationship between both policies; the second equality follows from the quantity equation (22), which holds under the alternative equilibrium; the third equality uses (G.1) expressed in terms of the bank's portfolio; and fourth follows from (G.2). Substituting out $B_{s,1}^{\text{Fed}}$ from the last term into (G.3), we obtain

$$\begin{aligned} M_{o,t}^{\text{Fed}} + G_{o,t}^{\text{FA}} - G_{o,t}^{\text{Fed}} - G_{o,t}^h + \Delta M - \Delta G \\ = \cdots P_{s,0} (\bar{b}_{o,0} + \bar{a}_{o,0}) (1 - \bar{c}_{s,0}) E_{s,0} - (\Theta^b(R_{o,1}^b)^{\epsilon^b} - (B_{o,1}^{\text{Fed}} + \Delta B)). \end{aligned}$$

Then, using that $\Delta M = \Delta B + \Delta G$, the equation simplifies to

$$M_{o,t}^{\text{Fed}} + G_{o,t}^{\text{FA}} - G_{o,t}^{\text{Fed}} - G_{o,t}^h = P_{s,0}(\bar{b}_{o,0} + \bar{a}_{o,0})(1 - \bar{c}_{s,0})E_{s,0} - (\Theta^b(R_{o,1}^b)^{\epsilon^b} - B_{o,1}^{\text{Fed}}).$$

Using the loans clearing condition $\Theta^b(R_{o,1}^b)^{\epsilon^b} - B_{o,1}^{\text{Fed}} = \bar{b}_{o,0}(1 - \bar{c}_{s,0})E_{s,0}$, we obtain

$$\begin{aligned} M_{o,t}^{\text{Fed}} + G_{o,t}^{\text{FA}} - G_{o,t}^{\text{Fed}} - G_{o,t}^h &= P_{s,0}(\bar{b}_{o,0} + \bar{a}_{o,0})(1 - \bar{c}_{s,0})E_{s,0} - \bar{b}_{o,0}(1 - \bar{c}_{s,0})E_{s,0} \\ &= (P_{s,0} - 1)\bar{b}_{o,0}(1 - \bar{c}_{s,0})E_{s,0} + P_{s,0}\bar{a}_{o,0}(1 - \bar{c}_{s,0})E_{s,0} \end{aligned}$$

Finally, using the quantity equation (22) applied to the first equilibrium, $M_{o,t}^{\text{Fed}} + G_{o,t}^{\text{FA}} - G_{o,t}^{\text{Fed}} - G_{o,t}^h = \bar{a}_{o,0}(1 - \bar{c}_{s,0})E_{s,0}$, we obtain

$$0 = (P_{s,0} - 1)\bar{b}_{o,0}(1 - \bar{c}_{s,0})E_{o,0} + (P_{o,0} - 1)\bar{a}_{o,0}(1 - \bar{c}_{o,0})E_{o,0}.$$

Since this equation is independent of ΔM ; $\bar{b}_{o,0}$, $\bar{c}_{o,0}$, $E_{o,0}$, $\bar{a}_{o,0}$ are all positive numbers; and any price is positive, it must be that $P_o = P_s = 1$. *Q.E.D.*

Next, we establish the main results.

Item 1: Nonneutrality Away From Satiation and $\lambda > 0$. First, we argue that if the policy change is neutral away from satiation, we reach a contradiction. Assume that the policy is indeed neutral. If policy s is neutral with respect to policy o , real assets, real asset returns, dividends, and bank equity must be equal across both equilibria. Consider Loan LP. Since real loans are the same, R^b must be the same in both equilibria. Also, R^m must be the same. This is the case because by $t = 1$, the policy is reversed, and thus the equilibrium and the price must be the same. Since by assumption, the policy is neutral, the $t = 0$ price must also be equal, as shown in Lemma G.2. Hence, since i^m is constant across both policies, then R^m must be the same. However, since under one equilibrium liquid assets are lower, but deposits are the same by assumption, the liquidity premium cannot be the same—a contradiction.

Item 2: Neutrality Under Satiation. Next, we verify that under satiation, the policy change has no effects. The key to verifying the result is showing that if the economy is under satiation under both policies, and if the bank's portfolio changes in the exact opposite direction as the Fed's portfolio, the policy is neutral—that is, we guess that there are no crowding in or crowding out effects. Thus, we guess that

$$B_{o,1} - \Delta B = B_{s,1}, \quad G_{o,1} - \Delta G = G_{s,1}, \quad \text{and} \quad M_{o,1} + \Delta M = M_{s,1}.$$

If the allocation is the same in real terms, then by Lemma G.2, $t = 0$ and $t = 1$ prices are the same, and $P_{o,1} = P_{s,1}$. Under satiation, we also know that $R^b = R^m = R^g$. Hence, the aggregate quantity of loans and bonds is equal under both policies. Hence, clearing in the loans market implies

$$\begin{aligned} \Theta^b(R_{o,1}^b)^{\epsilon^b} &= \Theta^b(R_{o,1}^m)^{\epsilon^b} = \frac{B_{o,1} + B_{o,1}^{\text{Fed}}}{P_{o,1}} = \frac{B_{o,1} - \Delta B + B_{o,1}^{\text{Fed}} + \Delta B}{P_{o,1}} \dots \\ &= \frac{B_{s,1} + B_{s,1}^{\text{Fed}}}{P_{s,1}} = \Theta^b(R_{s,1}^b)^{\epsilon^b}. \end{aligned}$$

In the bond market,

$$\begin{aligned}\Theta^g(R_{o,1}^g)^{\epsilon^g} &= \Theta^g(R_{o,1}^m)^{\epsilon^g} = \frac{G_1^{\text{FA}} - G_{o,1} + G_{o,1}^{\text{Fed}}}{P_{o,1}} = \frac{G_1^{\text{FA}} - G_{o,1} - \Delta G + G_{o,1}^{\text{Fed}} + \Delta G}{P_{o,1}} \dots \\ &= \frac{G_1^{\text{FA}} - G_{s,1} + G_{s,1}^{\text{Fed}}}{P_{s,1}} = \Theta^g(R_{s,1}^g)^{\epsilon^g},\end{aligned}$$

and in the deposit market,

$$\begin{aligned}\frac{D_{o,1}}{P_{o,1}} &= \frac{B_{o,1} + G_{o,1} + M_{o,1}}{P_{o,1}} + E_{o,1} = \frac{B_{o,1} + G_{o,1} + M_{o,1} - \Delta B - \Delta G + \Delta M}{P_{o,1}} + E_{o,1} \dots \\ &= \frac{B_{s,1} + G_{s,1} + M_{s,1}}{P_{o,1}} + E_{s,1} = \frac{D_{s,1}}{P_{s,1}}.\end{aligned}$$

Thus, all market clearing conditions are satisfied. If banks start under satiation, and the increase in total liquid assets is positive, then banks are satiated under both policies. Under both policies, banks are indifferent between loan, bond, and reserve holdings; thus, the guess is consistent with bank equilibrium choices. Since the law of motion of equity is unchanged across both equilibria, the path of dividends is also the same, which verifies the guess that the price level is the same in both cases.

Finally, let us explain why the qualifications $B_{s,1}^{\text{Fed}} \geq 0$ and $G_{s,1}^{\text{Fed}} \geq 0$ are necessary. Note that if $B_{o,1} < -\Delta B$, then banks will no longer hold loans and $R^b < R^m$. Thus, their nonnegativity constraint will be binding. Hence, the argument in the proposition does not follow through. Indeed, if the operation exceeds the bank's holdings, the policy may have real effects because the Fed will induce greater amounts of lending and generate a fiscal cost. Similarly, if $G_{o,1} < -\Delta G$ while holding fixed G^{FA} , the nonfinancial sector must reduce its holdings of reserves and will also have real effects.

Item 3: Limit Case as $\lambda \rightarrow 0$. Finally, we verify that if $\lambda \rightarrow 0$, conventional policies are neutral, but unconventional policies are not. Recall a conventional policy is one in which $\Delta B = 0$, $\Delta G > 0$, and an unconventional policy is one in which $\Delta B > 0$. Also, recall that if $\lambda \rightarrow 0$, then $\bar{\chi}_{o,1}^+ = 0$ and $(\bar{\chi}_{o,1}^- - \bar{\chi}_{o,1}^+) = R_{o,1}^w$ for any interbank market tightness. Thus, we have that the Loan LP becomes

$$R^b - R^m = R_{o,1}^w \cdot F(\omega^*) \cdot \frac{\mathbb{E}_\omega[(R^e)^{-\gamma} | \omega < \omega^*]}{\mathbb{E}_\omega[(R^e)^{-\gamma}]},$$

where $\omega^* \equiv -(\bar{a}/\bar{d} - \rho)/(R_{t+1}^d/R_{t+1}^m - \rho)$. In turn, the bond return satisfies

$$R^g = R^m.$$

First, we verify that the conventional policy is neutral. Suppose it is. By Lemma G.2, $t = 0$ and $t = 1$ prices are the same: $P_{o,1} = P_{s,1}$. Thus, $R_{o,1}^m = R_{s,1}^m$. Since the price level is the same and the policy is exclusively a conventional policy, $\Delta M = \Delta G$. Under this guess, if the policy does not crowd out household bonds, $G_{o,1} + M_{o,1} = G_{o,1} + M_{o,1} + \Delta M - \Delta G = G_{s,1} + M_{s,1}$. This, in turn, implies that $\bar{a}_{o,0} = \bar{a}_{s,0}$, and thus $\omega_{o,0}^* = \omega_{s,0}^*$. Since the threshold remains unchanged, and tightness does not affect the loans nor the bond premium, the guess is therefore verified.

To close the proposition, we verify that the unconventional policy has an effect. Suppose it does not. Then, prices do not change, again by Lemma G.2. We know that

$$G_{o,1} + M_{o,1} = G_{o,1} + M_{o,1} + \Delta M - \Delta G - \Delta B = G_{s,1} + M_{s,1} - \Delta B.$$

Thus, since $\Delta B \neq 0$, the real value of liquid assets under the original and alternative policies differ. Hence, we have a contradiction: either the threshold ω^* differs across both policies, or the deposits adjust, or both. In either case, the liquidity premium must be different across both policies. The result follows.

G.5. Proof of Proposition 11 and Corollary 12

We first demonstrate Proposition 11 and then prove the bound in Corollary 12. For the rest of this proof, we avoid time subscripts under the understanding that the condition applies to stationary equilibria. A stationary equilibrium satisfies four equilibrium conditions under any Friedman rule. We have:

1. A stationarity condition:

$$1/\beta = (\bar{a} + R^b(1 + \bar{d} - \bar{a}) - R^d \bar{d}); \quad (\text{G.4})$$

2. two stationary clearing conditions:

$$B = \Theta^b(\bar{R})^{\epsilon^b} \quad \text{and} \quad D = \Theta^d(\bar{R})^{\epsilon^d};$$

3. an aggregate budget balance:

$$B = D + (1 - \bar{a})\tilde{E}. \quad (\text{G.5})$$

Here, \tilde{E} is the steady-state equity after dividends. These conditions hold regardless of the stationary dividend.

PROOF OF PROPOSITION 11: There are four possible outcomes: either capital requirements bind or not, and either $\bar{a} > 0$ or $a = 0$. We develop observations for each case. We first observe that under the Friedman rule, $R^b \geq R^m$, with equality if $a > 0$. We first investigate the cases where $a = 0$.

Case I: $\bar{a} = 0$ and capital requirements do not bind.

If the capital requirement does not bind and $\bar{a} = 0$, then we know that $R^b \geq R^m$ and $R^b = R^d$. Because the Friedman rule eliminates the liquidity premium, the stationary condition (G.4) requires

$$\frac{1}{\beta} = (\bar{a} + R^b(1 - \bar{a} + \bar{d}) - R^d \bar{d}) = R^b,$$

where the first equality is the definition of equity returns and the second equality uses $R^b = R^d$ and $a = 0$. Thus, we have that $R^b = R^d = 1/\beta$ and $R^m \leq 1/\beta$.

In this case, the stationary equilibrium loans and deposits are given by

$$B = \Theta^b(1/\beta)^{\epsilon^b} \quad \text{and} \quad D = \Theta^d(1/\beta)^{\epsilon^d}.$$

Thus, (G.5) becomes

$$\tilde{E} = \Theta^b(1/\beta)^{\epsilon^b} - \Theta^d(1/\beta)^{\epsilon^d}. \quad (\text{G.6})$$

If capital requirements are indeed satisfied, it must be that

$$\tilde{E} \geq \frac{1}{\kappa} \Theta^d (1/\beta)^{\epsilon^d}. \quad (\text{G.7})$$

Combining, (G.6)–(G.7) yields

$$\Theta^b (1/\beta)^{\epsilon^b} \geq \frac{\kappa + 1}{\kappa} \Theta^d (1/\beta)^{\epsilon^d}. \quad (\text{G.8})$$

If the condition is not satisfied, then it is not possible to have a stationary equilibrium under the Friedman rule with $a = 0$ and where capital requirements do not bind. We summarize this case with the following observation.

REMARK 18: If $\bar{a} = 0$ and capital requirements do not bind, then $R^b = R^d = 1/\beta$ and $R^m \leq 1/\beta$, and condition (24) must hold.

Case II: capital requirements binds and $\bar{a} = 0$.

If the capital requirement binds and $\bar{a} = 0$, we know that $R^b > R^d$ and $R^b > R^m$. Since capital requirements bind, after dividend equity must equal $\tilde{E} = \frac{1}{\kappa} \Theta^d (R^d)^{\epsilon^d}$. Again, because the Friedman rule eliminates the liquidity premium, the stationary condition (G.4) to bank equity is $1/\beta$:

$$1/\beta = R^b + (R^b - R^d)\kappa.$$

Rewriting this expression yields a relationship between R^d and R^b :

$$R^b = \frac{\frac{1}{\beta} + \kappa R^d}{(1 + \kappa)}. \quad (\text{G.9})$$

We have the following observation, which we proof consequently.

REMARK 19: If the capital requirement binds and $\bar{a} = 0$, we have that $R^d < 1/\beta$.

Suppose the contrary. Then

$$R^b = \frac{\frac{1}{\beta} + \kappa R^d}{(1 + \kappa)} > \frac{\frac{1}{\beta} + \kappa \frac{1}{\beta}}{(1 + \kappa)} = \frac{1}{\beta}.$$

But if $R^b > 1/\beta$ and $R^b > R^d$, this implies that the return on equity is above $1/\beta$. Clearly, a contradiction. Hence, Remark 2 must hold.

Substituting the equilibrium conditions into the aggregate budget constraint yields

$$\Theta^b (R^b)^{\epsilon^b} = \Theta^d (R^d)^{\epsilon^d} + \frac{1}{\kappa} \Theta^d (R^d)^{\epsilon^d} = \Theta^d (R^d)^{\epsilon^d} \left(\frac{\kappa + 1}{\kappa} \right).$$

Substituting (G.9) and rearranging produces

$$(1 + \kappa) \left(\frac{\Theta^d}{\Theta^b} \frac{1 + \kappa}{\kappa} \right)^{1/\epsilon^b} (R^d)^{\epsilon^d/\epsilon^b} = \frac{1}{\beta} + \kappa R^d. \quad (\text{G.10})$$

This is the same expression for \bar{R}^d in Proposition 11. We have the following property.

REMARK 20: Equation (G.10) has a unique solution.

To see this, note that the left-hand side is decreasing in R^d and the right is increasing in R^d . For $R^d = 0$, the left-hand side is above $1/\beta$, so the solution must be unique. With this, and using (G.9), we obtain \bar{R}^b as in the proposition.

Next, we must verify that the unique solution indeed holds for $R^d < 1/\beta$ as needed; see Remark 2. Since the right-hand side of (G.10) is increasing and the left is decreasing, $\bar{R}^d < 1/\beta$ if and only if

$$(1 + \kappa) \left(\frac{\Theta^d}{\Theta^b} \frac{1 + \kappa}{\kappa} \right)^{1/\epsilon^b} (1/\beta)^{\epsilon^d/\epsilon^b} < \frac{1}{\beta} + \kappa 1/\beta.$$

Rearranging the terms leads to

$$\Theta^b (1/\beta)^{\epsilon^b} < \frac{\kappa + 1}{\kappa} \Theta^d (1/\beta)^{\epsilon^d}.$$

This is enough to reach the following conclusion.

REMARK 21: If $\bar{a} = 0$ and capital requirements bind, then $R^m \leq \bar{R}^b \leq 1/\beta$ and condition (24) must be violated.

We can combine this result and Remark 1, to obtain the following.

REMARK 22: Whether condition (24) holds or not, if $\bar{a} = 0$, then $R^m \leq \min\{1/\beta, \frac{1/\beta + \kappa \bar{R}^d}{(1 + \kappa)}\}$ for $(1 + \kappa) \left(\frac{\Theta^d}{\Theta^b} \frac{1 + \kappa}{\kappa} \right)^{1/\epsilon^b} (\bar{R}^d)^{\epsilon^d/\epsilon^b} = \frac{1}{\beta} + \kappa \bar{R}^d$.

Next, we move to the cases where $\bar{a} > 0$.

Case III: capital requirements do not bind and $\bar{a} > 0$.

In this case, we know that $R^b = R^m = R^d$. The (G.4) becomes

$$1/\beta = (\bar{a} + R^m(1 + \bar{d} - \bar{a}) - R^m \bar{d}) = (R^m - \bar{a}(R^m - 1)).$$

Hence, we have that

$$\bar{a} = \frac{R^m - 1/\beta}{R^m - 1}.$$

This implies that $R^m > 1/\beta$. We have the following remark.

REMARK 23: If capital requirements do not bind and $\bar{a} > 0$, then $R^m > 1/\beta$.

Case IV: capital requirements bind and $\bar{a} > 0$.

So far, we have shown conditions for stationary equilibria in which $a = 0$, which hold only if $R^m \leq 1/\beta$ —with an exact threshold given in Remark 5. Remark 6 shows that if capital requirements do not bind and if $R^m > 1/\beta$, then $\bar{a} > 0$. To complete the statement of

the proposition, we need to show if condition (24) is not satisfied and $R^m \leq \bar{R}^b = \frac{1}{\beta} \frac{1+\kappa\beta\bar{R}^d}{1+\kappa}$, where \bar{R}^d solves

$$(1 + \kappa) \left(\frac{\Theta^d}{\Theta^b} (1 + \kappa^{-1}) \right)^{1/\epsilon^b} (\bar{R}^d)^{\epsilon^d/\epsilon^b} = \frac{1}{\beta} + \kappa \bar{R}^d,$$

then $\bar{a} = 0$ and the capital requirement binds. Hence, any $R^m > \bar{R}^b$ must feature $\bar{a} > 0$ establishing that $R^b = R^m$.

To prove this, we assume by contradiction that $\bar{a} > 0$. By assumption,

$$R^b = R^m \leq \bar{R}^b = \frac{1}{\beta} \frac{1 + \kappa\beta\bar{R}^d}{1 + \kappa},$$

and $a > 0$. Under the stated assumptions, (G.4) becomes

$$1/\beta = (R^m - (R^m - 1)\bar{a} + (R^m - R^d)\kappa). \quad (\text{G.11})$$

However, we also know that

$$1/\beta = (\bar{R}^b + (\bar{R}^b - \bar{R}^d)\kappa). \quad (\text{G.12})$$

Hence, we have the following condition.

REMARK 24: If condition (24) is not satisfied, capital requirements bind, $\bar{a} > 0$, and $R^m \leq \bar{R}^b$, then it would be the case that $R^d < \bar{R}^d$.

The remark can be shown to hold simply by noticing that if $R^m < \bar{R}^b$ and $\bar{a} > 0$, then it must be that $R^d < \bar{R}^d$, by comparing (G.11) and (G.12).

Next, solving for \bar{a} from (G.11) yields

$$\bar{a} = \frac{R^m - 1/\beta + (R^m - R^d)\kappa}{R^m - 1}.$$

Observe that by monotonicity

$$\Theta^b(R^m)^{\epsilon^b} > \Theta^b(\bar{R}^b)^{\epsilon^b},$$

and also

$$\Theta^d(R^d)^{\epsilon^d} (1 + \kappa^{-1}(1 - \bar{a})) < \Theta^d(\bar{R}^d)^{\epsilon^d} (1 + \kappa^{-1}(1 - \bar{a})) < \Theta^d(\bar{R}^d)^{\epsilon^d}.$$

Then, if we substitute real rates into (G.5), we obtain

$$\Theta^b(R^m)^{\epsilon^b} = \Theta^d(R^d)^{\epsilon^d} + \frac{1 - \bar{a}}{\kappa} \cdot \Theta^d(R^d)^{\epsilon^d} = \Theta^d(R^d)^{\epsilon^d} \left(\frac{\kappa + (1 - \bar{a})}{\kappa} \right).$$

Substituting the inequalities above,

$$\Theta^b(\bar{R}^b)^{\epsilon^b} < \Theta^b(R^m)^{\epsilon^b} = \Theta^d(R^d)^{\epsilon^d} \left(\frac{\kappa + (1 - \bar{a})}{\kappa} \right) < \Theta^d(\bar{R}^d)^{\epsilon^d} \left(\frac{\kappa + 1}{\kappa} \right).$$

However, this contradicts the definition of $\{\bar{R}^b, \bar{R}^d\}$. Hence, we reach the following conclusion.

REMARK 25: If condition (24) is not satisfied and $R^m \leq \bar{R}^b$, then we have that $\bar{a} = 0$ and capital requirements bind.

Collecting the Results.

Consider the cases in which condition (24) holds. Combining remarks (1) and (6), we have the following remark.

REMARK 26: If (24) holds, then the loans rate associated with the Friedman rule is

$$R^{b,FR} = \begin{cases} 1/\beta & \text{if } R^m < 1/\beta, \\ R^m & \text{if } R^m \geq 1/\beta. \end{cases} \quad (\text{G.13})$$

Moreover, $\bar{a} = 0$, and the capital requirement does not bind if and only if $R^m \leq 1/\beta$. If $R^m \leq 1/\beta$, the stationary deposit rate is also $1/\beta$.

Now consider the cases in which condition (24) does not hold. Combining remarks (5) and (8), we have the following result.

REMARK 27: If (24) does not hold,

$$R^{b,FR} = \begin{cases} \bar{R}^b & \text{if } R^m < \bar{R}^b, \\ R^m & \text{if } R^m \geq \bar{R}^b, \end{cases} \quad (\text{G.14})$$

where

$$\bar{R}^b = \frac{1}{\beta} \frac{1 + \kappa\beta\bar{R}^d}{1 + \kappa} \leq \frac{1}{\beta},$$

where \bar{R}^d is the stationary deposit rate that is the unique solution to

$$(1 + \kappa) \left(\frac{\Theta^d}{\Theta^b} (1 + \kappa^{-1}) \right)^{1/\epsilon^b} (\bar{R}^d)^{\epsilon^d/\epsilon^b} = \frac{1}{\beta} + \kappa\bar{R}^d.$$

Moreover, $\bar{a} = 0$ if and only if $R^m \leq \bar{R}^b$. In this case, the capital requirement binds.

Combining remarks (9) and (10), we obtain the statement of Proposition 11. *Q.E.D.*

PROOF OF COROLLARY 12: The proof is immediate. First, observe that if (24) holds, then we had showed that the \bar{R}^d that solves

$$(1 + \kappa) \left(\frac{\Theta^d}{\Theta^b} (1 + \kappa^{-1}) \right)^{1/\epsilon^b} (\bar{R}^d)^{\epsilon^d/\epsilon^b} = \frac{1}{\beta} + \kappa\bar{R}^d,$$

is above $\frac{1}{\beta}$. Thus, $\bar{R}^b > 1/\beta$. Otherwise, if (24) does not hold, the solution is less than $1/\beta$ and $\bar{R}^b < 1/\beta$. This implies that a compact way to write $R^{b,FR}$ is

$$R^{b,FR} = \begin{cases} \min\{\bar{R}^b, 1/\beta\} & \text{if } R_{ss}^m < \min\{\bar{R}^b, 1/\beta\}, \\ R^m & \text{if } R_{ss}^m \geq \min\{\bar{R}^b, 1/\beta\}. \end{cases}$$

Then, since by assumption of the corollary $R^m \geq \min\{\bar{R}^b, 1/\beta\} = R^{b,FR}$ and $R^b \geq R^m$ in any equilibrium, the bound follows. *Q.E.D.*

APPENDIX H: EFFICIENT ALLOCATIONS AND PROOF OF PROPOSITION 13

In this Appendix, we derive the efficient allocations under the assumption that $\beta^h = \beta$. We also show that a version of the Friedman rule, with the appropriate choice of R^m , can implement the first-best allocation, provided that capital requirements are sufficiently ample. We let a planner maximize a weighted average of households' and bankers' utility subject to the resource constraint for goods and labor. The planner's problem is as follows.

PROBLEM 28—Planner's Problem: The unconstrained planner's problem is given by

$$\max_{\{c_t, c_t^d, c_t^g, c_t^m, h_t\}_{t=0,1,\dots}} \sum \beta^t \left[(1 - \varpi) \left(\sum_{x \in \{d, g, m\}} U^x(c_t^x) + c_t^h - \frac{h_t^{1+\nu}}{1+\nu} \right) + \varpi \cdot u(c_t) \right],$$

subject to the resource constraint

$$\sum_{x \in \{d, g, m\}} c_t^x + c_t^h + c_t = y_t \quad (\text{H.1})$$

and the technological constraint

$$y_t = A_t h_{t-1}^\alpha. \quad (\text{H.2})$$

The initial labor input h_{-1} is given.

Here, we use ϖ for the Pareto weight on the banker's consumption. The next proposition characterizes an optimal allocation.

PROPOSITION H.1: *The optimality conditions of the unconstrained planner problem are*

$$U_{c_t^x}^x(\bar{X}) = 1 = \frac{\varpi}{(1 - \varpi)} \phi u'(c_t) \quad \text{for } x \in \{d, g, m\} \quad (\text{H.3})$$

and

$$\beta \alpha A_{t+1} h_t^{\alpha-1} = h_t^\nu. \quad (\text{H.4})$$

The proposition states that in the first best allocation, the planner equalizes the labor wedge to zero and equalizes all the marginal utility across goods to one. The latter is optimal because the marginal rate of transformation is one across all goods. Notice also that the planner's solution is characterized by a sequence of static problems, so there are no dynamic trade-offs in the allocation. We say an allocation is efficient if it coincides with the planner's solution for some ϖ .

Next, we state a detailed version of Proposition 13.

PROPOSITION H.2: *Consider a competitive equilibrium and a version of the Friedman rule in which the Fed sets $R_t^m = R_t^w = 1/\beta$ (and $\pi_t = \beta - 1$) and adjusts M_t^{Fed} and G_t^{Fed} such that $R^g = 1/\beta$. A necessary condition for this Friedman rule to induce an efficient allocation in its stationary equilibrium is that*

$$\Theta^b (1/\beta)^{\epsilon^b} \geq \left(\frac{1 + \kappa}{\kappa} \right) \Theta^d (1/\beta)^{1/\epsilon^d}.$$

If the condition is violated, the first-best is not attainable.

We proceed with a proof.

H.1. Decentralization and the Friedman Rule (Proof of Propositions H.1 and H.2)

We begin with the proof of Proposition H.1.

PROOF: If we substitute out c^h from the resource constraint into the objective, replacing y_t from the technological constraint yields a modified objective function:

$$\max_{\{c_t, c_t^d, c_t^g, c_t^m, h_t\}} \sum_{t=0,1,\dots} \beta^t \left[(1 - \varpi) \left(\sum_{x \in \{d,g,m\}} U^x(c_t^x) + A_t h_{t-1}^\alpha \right) - \sum_{x \in \{d,g,m\}} c^x + c_t - \frac{h_t^{1+\nu}}{1+\nu} \right] + \varpi \cdot u(c_t).$$

The conditions are verified by taking first-order conditions with respect to $\{c_t, c_t^d, c_t^g, c_t^m, h_t\}$. Q.E.D.

We now move to proof Proposition H.2.

PROOF: Note that a necessary condition for efficiency is that $R^d = R^g = 1/\beta$ and $\frac{1}{1+\pi} = 1/\beta$. This follows directly from the household's optimality condition in (F.4)—the case in which each asset-in-advance constraint is slack. In that case, the household's allocation across goods coincides with the planner problem's unconstrained condition, (H.3). Similarly, for the firm's problem, the optimality condition (F.7) coincides with (H.4) for $R^b = 1/\beta$. Also notice that it is possible for $R^b = R^d = R^g = R^m = 1/\beta$ only if there is no liquidity premium.

Assume that the efficiency condition holds at every period. Then loans and deposits are given by

$$B_t = \Theta^b (1/\beta)^{\epsilon^b} \quad \text{and} \quad D_t = \Theta^d (1/\beta)^{1/\epsilon^d}.$$

From the bank's budget constraint and portfolio constraints, using these quantities, we have that

$$A_t + \Theta^b (1/\beta)^{\epsilon^b} = \Theta^d (1/\beta)^{1/\epsilon^d} + E_t, \tag{H.5}$$

$$E_t \geq \frac{1}{\kappa} \Theta^d (1/\beta)^{1/\epsilon^d}, \tag{H.6}$$

$$A_t \geq 0. \tag{H.7}$$

A condition for stationarity is

$$1/\beta = (\bar{a}_{ss} + R_{ss}^b (1 + \bar{d} - \bar{a}) - R_{ss}^d \bar{d}), \tag{H.8}$$

and replacing the efficiency condition, $R^b = R^d = 1/\beta$, yields

$$1/\beta = (1/\beta - 1/\beta(1 - \bar{a})).$$

This condition implies that $A_t \geq 0$.

Combining (H.5) and (H.7), we obtain

$$E_{ss} = \Theta^b (1/\beta)^{\epsilon^b} - \Theta^d (1/\beta)^{1/\epsilon^d}.$$

Substituting this result into the capital requirement condition yields

$$\Theta^b(1/\beta)^{\epsilon^b} \geq \left(\frac{1+\kappa}{\kappa}\right)\Theta^d(1/\beta)^{1/\epsilon^d}.$$

Hence, the necessary conditions for efficiency in Proposition 13.

Q.E.D.

APPENDIX I: PROOF OF PROPOSITION 14

Here, we present the proof of Proposition 14 in Section 5.2 regarding the pass-through of monetary policy. In this Appendix, we present a more general version of the proposition in the text. Specifically, we derive the comparative statics with respect to changes in the the interest on reserves under two scenarios: (i) keeping the discount window rate constant, and (ii) keeping the spread in both policy rates constant. For ease of exposition, we restrict to the cases in which $\bar{b}^{\text{fed}} = 0$, but the result can be extended along that dimension without difficulty. Let $\mathcal{L}\mathcal{P}_y^x$ denote the derivative of the liquidity premium of asset x with respect to portfolio holdings of asset y . The general version of of Proposition 14 is as follows.

PROPOSITION I.1: *Consider stationary equilibria. Consider an increase from a stationary level of r^m that leaves the stationary level of r^w constant or leaves the corridor spread, $\Delta = r^w - r^m$, constant. If capital requirements are binding, then the increase in r^m unambiguously increases r^b . Then, if capital requirements bind, in the region where capital requirements bind,*

$$\begin{aligned} \frac{dr^b}{dr^m} &= \frac{1 + \epsilon^d \cdot \frac{(\bar{b} + \bar{b}^{\text{Fed}}) r^b}{\kappa R^d}}{1 + \epsilon^d \cdot \frac{(\bar{b} + \bar{b}^{\text{Fed}}) r^b}{\kappa R^d} - \mathcal{L}\mathcal{P}_a^b \frac{(\bar{b} + \bar{b}^{\text{Fed}})}{R^b} \left(\epsilon^d \cdot \frac{\bar{b}}{\kappa} - \epsilon^b\right)} \\ &\times \left(1 - \frac{\mathbb{E}_\omega[\chi(\theta)]}{\Delta} \cdot \mathbb{I}[dr^w = 0]\right) \in [0, 1], \end{aligned}$$

and $\frac{dr^b}{dr^m} = 1$, when banks are satiated with reserves. If capital requirements do not bind and the deposit supply is perfectly elastic at r^d , the pass-through is ambiguous and given by

$$\begin{aligned} \frac{dr^b}{dr^m} &= \frac{((\mathcal{L}\mathcal{P}_d^b + \mathcal{L}\mathcal{P}_d^d))r^b + (\mathcal{L}\mathcal{P}_b^b + \mathcal{L}\mathcal{P}_b^d)r^d}{\mathcal{L}\mathcal{P}_b^b r^d + \mathcal{L}\mathcal{P}_d^d r^b + (\mathcal{L}\mathcal{P}_b^b(\mathcal{L}\mathcal{P}_b^d + \mathcal{L}\mathcal{P}_d^d) - \mathcal{L}\mathcal{P}_b^d(\mathcal{L}\mathcal{P}_b^b + \mathcal{L}\mathcal{P}_d^d))\bar{b}} \\ &\times \left(1 - \frac{\mathbb{E}_\omega[\chi(\theta)]}{\Delta} \cdot \mathbb{I}[dr^w = 0]\right). \end{aligned}$$

PROOF: We first prove the result for the case with a perfectly elastic deposit supply schedule and binding capital requirements. We then relax one assumption at a time for the general result. First, recall that the slopes of the liquidity yield function are given by

$$\chi^+ = (i^w - i^m) \left(\frac{\bar{\theta}}{\theta}\right)^\eta \left(\frac{\theta^\eta \bar{\theta}^{1-\eta} - \theta}{\bar{\theta} - 1}\right) \quad \text{and} \quad \chi^- = (i^w - i^m) \left(\frac{\bar{\theta}}{\theta}\right)^\eta \left(\frac{\theta^\eta \bar{\theta}^{1-\eta} - 1}{\bar{\theta} - 1}\right). \quad (\text{I.1})$$

Thus, we can write them as

$$\chi^+ = \Delta q^+(\theta) \quad \text{and} \quad \chi^- = \Delta q^-(\theta).$$

Clearly, $\{q^+, q^-\} \in [0, 1]^2$. We proof the results for the case in which $dr^w > 0$, but the steps are the same to obtain the general result above. *Q.E.D.*

Case #1: Infinitely Elastic Deposit Supply and Binding Capital Requirements. The gist of the proof is to perform a comparative statics analysis with respect to r^m on the following subsystem of equilibrium equations:

$$1 = \beta(1 + r^b(\bar{b} + \bar{b}^{\text{Fed}}) - r^d \bar{d}) \tag{I.2}$$

and

$$r^b = r^m + \mathbb{E}_\omega[\bar{\chi}], \tag{I.3}$$

where

$$\mathbb{E}_\omega[\bar{\chi}] = \int_{-1}^{\omega^*} \bar{\chi}^- f(\omega) d\omega + \int_{\omega^*}^{\infty} \bar{\chi}^+ f(\omega) d\omega.$$

This subsystem is the loans premium and the stationarity condition for equity.

Then, taking total differentials with respect to r^m on (I.2) and (I.3), respectively, we obtain

$$(\bar{b} + \bar{b}^{\text{Fed}}) \frac{dr^b}{dr^m} + r^b \frac{d\bar{b}}{dr^m} = 0 \tag{I.4}$$

and

$$\frac{dr^b}{dr^m} = 1 + \frac{d[\mathbb{E}[\bar{\chi}]]}{dr^m}. \tag{I.5}$$

Then we have that

$$\frac{d[\mathbb{E}[\bar{\chi}]]}{dr^m} = -\mathbb{E}_\omega[q(\theta)] + \Delta \frac{d\bar{b}}{dr^m}, \tag{I.6}$$

where

$$\mathcal{L}\mathcal{P}_b^b \equiv \left[\mathbb{E} \left[(\bar{\chi}^- - \bar{\chi}^+) f(\omega^*) \frac{d\omega^*}{d\bar{b}} + \mathbb{E}_\omega[\bar{\chi}_\theta f(\omega) d\omega] \frac{d\theta^*}{d\bar{b}} \right] \right] > 0.$$

We employ Leibnitz's rule. Thus, substituting the expressions we obtain

$$\frac{dr^b}{dr^m} - \mathcal{L}\mathcal{P}_b^b \frac{d\bar{b}}{dr^m} = 1 - \mathbb{E}_\omega[q(\theta)] > 0.$$

The system (I.4) and (I.5) in matrix form is represented as

$$\begin{bmatrix} \bar{b} + \bar{b}^{\text{Fed}} & r^b \\ 1 & -\mathcal{L}\mathcal{P}_b^b \end{bmatrix} \cdot \begin{bmatrix} \frac{dr^b}{dr^m} \\ \frac{d\bar{b}}{dr^m} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 - \mathbb{E}_\omega[q(\theta)] \end{bmatrix}. \tag{I.7}$$

Inverting the matrix on the left yields the solution to the local comparative statics of (I.2) and (I.3):

$$\begin{bmatrix} \frac{dr^b}{dr^m} \\ \frac{d\bar{b}}{dr^m} \end{bmatrix} = \begin{bmatrix} \bar{b} + \bar{b}^{\text{Fed}} & r^b \\ 1 & -\mathcal{L}\mathcal{P}_b^b \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 1 - \mathbb{E}_\omega[q(\theta)] \end{bmatrix}.$$

To compute the solution, we need only the upper right element of the inverse matrix. By construction, that term is

$$\frac{dr^b}{dr^m} = \frac{r^b}{\mathcal{L}\mathcal{P}_b^b(\bar{b} + \bar{b}^{\text{Fed}}) + r^b} (1 - \mathbb{E}_\omega[q(\theta)]) > 0.$$

Similarly, we can also sign the portfolio share:

$$\frac{d\bar{b}}{dr^m} = -\frac{(\bar{b} + \bar{b}^{\text{Fed}})}{\mathcal{L}\mathcal{P}_b^b(\bar{b} + \bar{b}^{\text{Fed}}) + r^b} ((1 - \mathbb{E}_\omega[q(\theta)])) < 0.$$

Observation 1. Notice that under satiation, $q(\theta) = \mathcal{L}\mathcal{P}_b^b = 0$. Thus, the pass-through is one for one. Away from satiation, the pass-through is less than one, because $\mathcal{L}\mathcal{P}_b^b \bar{b} > 0$.

Observation 2. Notice that for fixed Δ , the result goes through, since $(1 - \mathbb{E}_\omega[q(\theta)])$ is replaced by 1.

Case #2: Finitely Elastic Deposit Supply and Binding Capital Requirements. We now move to a more general result, with an elastic deposit supply schedule. Equilibrium in the loan supply and deposit requires

$$\begin{aligned} (\bar{b} + \bar{b}^{\text{fed}}) \cdot \beta \cdot E_{ss} &= (\Theta^b)^{-1} \cdot (r^b + 1)^{\epsilon^b}, \\ \kappa \cdot \beta \cdot E_{ss} &= (\Theta^d)^{-1} (r^d + 1)^{\epsilon^d}. \end{aligned}$$

Combining both conditions yields a single equilibrium condition that we append to the equilibrium system (I.2) and (I.3):

$$\frac{(\bar{b} + \bar{b}^{\text{fed}})}{\kappa} = \frac{\Theta^d}{\Theta^b} \cdot \frac{(r^b + 1)^{\epsilon^b}}{(r^d + 1)^{\epsilon^d}}. \quad (\text{I.8})$$

We write (I.8) in differential form:

$$\frac{1}{\kappa} \frac{d\bar{b}}{dr^m} - \epsilon^b \frac{\Theta^b}{\Theta^d} \cdot \frac{(r^b + 1)^{\epsilon^b - 1}}{(r^d + 1)^{\epsilon^d}} \frac{dr^b}{dr^m} + \epsilon^d \frac{\Theta^d}{\Theta^d} \cdot \frac{(r^b + 1)^{\epsilon^b}}{(r^d + 1)^{\epsilon^d + 1}} \frac{dr^d}{dr^m} = 0.$$

Substituting (I.8), this expression is written as

$$\frac{1}{\kappa} \frac{d\bar{b}}{dr^m} - \epsilon^b \frac{(\bar{b} + \bar{b}^{\text{fed}})}{\kappa} \frac{1}{R^b} \frac{dr^b}{dr^m} + \epsilon^d \cdot \frac{(\bar{b} + \bar{b}^{\text{fed}})}{\kappa} \frac{1}{R^d} \frac{dr^d}{dr^m} = 0.$$

In addition, the differential form of (I.2) is now

$$(\bar{b} + \bar{b}^{\text{Fed}}) \frac{dr^b}{dr^m} + r^b \frac{d\bar{b}}{dr^m} - \kappa \frac{dr^d}{dr^m} = 0, \quad (\text{I.9})$$

which replaces (I.4).

Hence, in matrix form, the local comparative statics is given by

$$\begin{bmatrix} \frac{dr^b}{dr^m} \\ \frac{d\bar{b}}{dr^m} \\ \frac{dr^d}{dr^m} \end{bmatrix} = \begin{bmatrix} (\bar{b} + \bar{b}^{\text{Fed}}) & r^b & -\kappa \\ 1 & -\mathcal{L}\mathcal{P}_b^b & 0 \\ -\epsilon^b \frac{(\bar{b} + \bar{b}^{\text{Fed}})}{\kappa} \frac{1}{R^b} & \frac{1}{\kappa} & \epsilon^d \cdot \frac{(\bar{b} + \bar{b}^{\text{Fed}})}{\kappa} \frac{1}{R^d} \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ (1 - \mathbb{E}_\omega[q(\theta)]) \\ 0 \end{bmatrix}.$$

We can use standard linear algebra tools to obtain the solution to the pass-through to the credit rate. In this case,

$$\begin{aligned} \frac{dr^b}{dr^m} &= - \frac{\left\| \begin{bmatrix} r^b & -\kappa \\ \frac{1}{\kappa} & \epsilon^d \cdot \frac{(\bar{b} + \bar{b}^{\text{Fed}})}{\kappa} \frac{1}{R^d} \end{bmatrix} \right\|}{\left\| \begin{bmatrix} (\bar{b} + \bar{b}^{\text{Fed}}) & r^b & -\kappa \\ 1 & -\mathcal{L}\mathcal{P}_b^b & 0 \\ -\epsilon^b \frac{(\bar{b} + \bar{b}^{\text{Fed}})}{\kappa} \frac{1}{R^b} & \frac{1}{\kappa} & \epsilon^d \cdot \frac{(\bar{b} + \bar{b}^{\text{Fed}})}{\kappa} \frac{1}{R^d} \end{bmatrix} \right\|} (1 - \mathbb{E}_\omega[q(\theta)]) \\ &= - \frac{\epsilon^d \cdot \frac{(\bar{b} + \bar{b}^{\text{Fed}})}{\kappa} \frac{r^b}{R^d} + 1}{-\mathcal{L}\mathcal{P}_b^b \epsilon^d \cdot \frac{(\bar{b} + \bar{b}^{\text{Fed}})}{\kappa} \frac{\bar{b}}{R^d} - 1 + \epsilon^b \frac{(\bar{b} + \bar{b}^{\text{Fed}})}{\kappa} \frac{1}{R^b} \kappa \mathcal{L}\mathcal{P}_b^b - \epsilon^d \cdot \frac{(\bar{b} + \bar{b}^{\text{Fed}})}{\kappa} \frac{r^b}{R^d}} \\ &\quad \times (1 - \mathbb{E}_\omega[q(\theta)]) \\ &= \frac{1 + \epsilon^d \cdot \frac{(\bar{b} + \bar{b}^{\text{Fed}})}{\kappa} \frac{1}{R^d} r^b}{1 + \epsilon^d \cdot \frac{(\bar{b} + \bar{b}^{\text{Fed}})}{\kappa} \frac{r^b}{R^d} + \mathcal{L}\mathcal{P}_b^b \epsilon^d \cdot \frac{(\bar{b} + \bar{b}^{\text{Fed}})}{\kappa} \frac{(\bar{b} + \bar{b}^{\text{Fed}})}{R^d} - \epsilon^b \frac{(\bar{b} + \bar{b}^{\text{Fed}})}{\kappa} \frac{1}{R^b} \kappa \mathcal{L}\mathcal{P}_b^b} \\ &\quad \times (1 - \mathbb{E}_\omega[q(\theta)]). \end{aligned}$$

Since all terms are positive, the solution holds ($\epsilon^b < 0$); this step proves the first statement of the Proposition. Note simply that $\mathcal{L}\mathcal{P}_a^b = -\mathcal{L}\mathcal{P}_b^a$.

Observation 3. Notice that under satiation, $q(\theta) = \vartheta = \mathcal{L}\mathcal{P}_b^b$. Thus, the pass-through is one for one. Away from satiation, the pass-through is less than one, because

$$\mathcal{L}\mathcal{P}_b^b \epsilon^d \cdot \frac{(\bar{b} + \bar{b}^{\text{Fed}})}{\kappa} \frac{(\bar{b} + \bar{b}^{\text{Fed}})}{R^d} > \epsilon^b \frac{(\bar{b} + \bar{b}^{\text{Fed}})}{\kappa} \frac{1}{R^b} \kappa \mathcal{L}\mathcal{P}_b^b.$$

Case #3: Infinitely Elastic Deposit Supply and Nonbinding Capital Requirements. In this case, the equilibrium system is given by (I.2) and (I.3), but now we also include the deposit liquidity premium. In this case,

$$r^d = r^m + \mathbb{E}_\omega[\bar{\chi}] - \mathbb{E}_\omega[\bar{\chi} \cdot \omega]. \quad (\text{I.10})$$

Once the deposit share is free to move, the differential form of (I.2) is

$$(\bar{b} + \bar{b}^{\text{Fed}}) \frac{dr^b}{dr^m} + r^b \frac{d\bar{b}}{dr^m} - r^d \frac{d\bar{d}}{dr^m} = 0, \quad (\text{I.11})$$

which replaces (I.4).

From the loan LP,

$$\frac{dr^b}{dr^m} = 1 + \frac{d[\mathbb{E}[\bar{\chi}]]}{dr^m},$$

where

$$\frac{d[\mathbb{E}[\bar{\chi}]]}{dr^m} = \mathcal{L}\mathcal{P}_b^b \frac{d\bar{b}}{dr^m} + \mathcal{L}\mathcal{P}_d^b \frac{d\bar{d}}{dr^m}.$$

Following the same notation as before,

$$\mathcal{L}\mathcal{P}_b^b \equiv \left[\left[(\bar{\chi}^- - \bar{\chi}^+) f(\omega^*) \frac{d\omega^*}{d\bar{b}} + \mathbb{E}[\bar{\chi}_\theta f(\omega) d\omega] \frac{d\theta^*}{d\bar{b}} \right] \right] > 0,$$

and

$$\mathcal{L}\mathcal{P}_d^b \equiv \left[\left[(\bar{\chi}^- - \bar{\chi}^+) f(\omega^*) \frac{d\omega^*}{d\bar{d}} + \mathbb{E}[\bar{\chi}_\theta f(\omega) d\omega] \frac{d\theta^*}{d\bar{d}} \right] \right] < 0.$$

The differential form of (I.10) is

$$0 = 1 + \frac{d[\mathbb{E}_\omega[\bar{\chi}]]}{dr^m} + \frac{d[\mathbb{E}_\omega[\bar{\chi} \cdot \omega]]}{dr^m},$$

where

$$\frac{d[\mathbb{E}_\omega[\bar{\chi} \cdot \omega]]}{dr^m} = \Delta_b^d \frac{d\bar{b}}{dr^m} + \Delta_d^d \frac{d\bar{d}}{dr^m},$$

$$\mathcal{L}\mathcal{P}_b^d \equiv - \left[\left[(\bar{\chi}^- - \bar{\chi}^+) \omega^* f(\omega^*) \frac{d\omega^*}{d\bar{b}} + \mathbb{E}[\omega \bar{\chi}_\theta f(\omega) d\omega] \frac{d\theta^*}{d\bar{b}} \right] \right],$$

and

$$\mathcal{L}\mathcal{P}_d^d \equiv - \left[\left[(\bar{\chi}^- - \bar{\chi}^+) \omega^* f(\omega^*) \frac{d\omega^*}{d\bar{d}} + \mathbb{E}[\omega \bar{\chi}_\theta f(\omega) d\omega] \frac{d\theta^*}{d\bar{d}} \right] \right].$$

We also have that $\mathcal{L}\mathcal{P}_b^d > 0$, $\mathcal{L}\mathcal{P}_d^d > 0$.

As in the previous two examples, we construct the matrix representation of the comparative statics:

$$\begin{bmatrix} dr^b \\ \frac{dr^m}{d\bar{b}} \\ \frac{d\bar{b}}{dr^m} \\ \frac{d\bar{d}}{dr^m} \end{bmatrix} = \begin{bmatrix} (\bar{b} + \bar{b}^{\text{Fed}}) & r^b & -r^d \\ 1 & -\mathcal{L}\mathcal{P}_b^b & -\mathcal{L}\mathcal{P}_b^d \\ 1 & -(\mathcal{L}\mathcal{P}_b^b + \mathcal{L}\mathcal{P}_b^d) & -(\mathcal{L}\mathcal{P}_d^b + \mathcal{L}\mathcal{P}_d^d) \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ (1 - \mathbb{E}_\omega[q(\theta)]) \\ 0 \end{bmatrix}.$$

To obtain the solution to the pass-through, we do the same calculation as in the earlier step:

$$\frac{dr^b}{dr^m} = - \frac{\begin{vmatrix} r^b & -r^d \\ -(\mathcal{L}\mathcal{P}_b^b + \mathcal{L}\mathcal{P}_b^d) & -(\mathcal{L}\mathcal{P}_d^b + \mathcal{L}\mathcal{P}_d^d) \end{vmatrix}}{\begin{vmatrix} (\bar{b} + \bar{b}^{\text{Fed}}) & r^b & -r^d \\ 1 & -\mathcal{L}\mathcal{P}_b^b & -\mathcal{L}\mathcal{P}_b^d \\ 1 & -(\mathcal{L}\mathcal{P}_b^b + \mathcal{L}\mathcal{P}_b^d) & -(\mathcal{L}\mathcal{P}_d^b + \mathcal{L}\mathcal{P}_d^d) \end{vmatrix}} \quad (\text{I.12})$$

$$= \frac{((\mathcal{L}\mathcal{P}_d^b + \mathcal{L}\mathcal{P}_d^d)r^b + (\mathcal{L}\mathcal{P}_b^b + \mathcal{L}\mathcal{P}_b^d)r^d}{\mathcal{L}\mathcal{P}_b^b r^d + \mathcal{L}\mathcal{P}_d^d r^b + (\mathcal{L}\mathcal{P}_b^b(\mathcal{L}\mathcal{P}_b^d + \mathcal{L}\mathcal{P}_d^d) - \mathcal{L}\mathcal{P}_b^d(\mathcal{L}\mathcal{P}_b^b + \mathcal{L}\mathcal{P}_d^b))\bar{b}}. \quad (\text{I.13})$$

Using $\mathcal{L}\mathcal{P}_b^b = -\mathcal{L}\mathcal{P}_a^b$, $\mathcal{L}\mathcal{P}_b^d = -\mathcal{L}\mathcal{P}_a^d$, we obtain

$$\frac{((\mathcal{L}\mathcal{P}_d^b + \mathcal{L}\mathcal{P}_d^d)r^b - (\mathcal{L}\mathcal{P}_a^b + \mathcal{L}\mathcal{P}_a^d)r^d}{-\mathcal{L}\mathcal{P}_a^b r^d + \mathcal{L}\mathcal{P}_d^d r^b - (\mathcal{L}\mathcal{P}_a^b(\mathcal{L}\mathcal{P}_b^d + \mathcal{L}\mathcal{P}_d^d) + \mathcal{L}\mathcal{P}_a^d(\mathcal{L}\mathcal{P}_d^b - \mathcal{L}\mathcal{P}_a^b))\bar{b}}.$$

Observation 4. In this case, the sign is ambiguous and depends on the sign of

$$\mathcal{L}\mathcal{P}_b^b r^d + \mathcal{L}\mathcal{P}_d^d r^b + \mathcal{L}\mathcal{P}_b^b(\mathcal{L}\mathcal{P}_b^d + \mathcal{L}\mathcal{P}_d^d)\bar{b} \geq \mathcal{L}\mathcal{P}_b^d(\mathcal{L}\mathcal{P}_b^b + \mathcal{L}\mathcal{P}_d^b)\bar{b}.$$

This concludes the proof of Proposition 14.

APPENDIX J: EXISTENCE, UNIQUENESS, AND CONVERGENCE UNDER FRIEDMAN RULE

This Appendix characterizes the existence and uniqueness of a stationary equilibria when the bank has log preferences ($\gamma = 1$) and the Fed eliminates all distortions $R^m = R^w$ and sets R^m low enough that banks do not hold liquid assets. We can treat these results as holding for an approximation in which bank dividends are close to constant and interbank market distortions are not too large.

J.1. Dynamical Properties

In this section, we study the dynamical properties of the model. We fully characterize these dynamics when banks have log utility and the Fed carries out a policy of no distortions in the interbank market. Both assumptions simplify the analysis. Although the results are not general, for small deviations around that policy, the dynamic properties should be similar.

Stationary Equilibrium and Policy Effects With Satiation. We begin describing the transitional dynamics of the model when the Fed carries out a policy that satiates the market with reserves via $i_t^w = i_t^m$ by setting a sufficiently low value for i_t^m . For simplicity, we set the supply of government bonds to zero and assume the Fed does not purchase loans. By inducing satiation and maintaining an equal amount of reserves as Fed loans, the Fed eliminates the liquidity premium of loans. Thus, a spread between loans and deposits results only from capital requirements. This characterization is useful because it describes the dynamics of the model in absence of any distortions.

For this section, it is useful to define the inverse demand elasticity of loans and supply elasticity of deposits: $\bar{\epsilon}^x \equiv (\epsilon^x)^{-1}$ for $x \in \{d, b\}$, respectively. Also, intercept of the inverse demand for loans and supply of deposits are $\bar{\Theta}^x \equiv (\Theta^x)^{-1/\epsilon^x}$ for $x \in \{d, b\}$. We obtain the following characterization.

PROPOSITION J.1—Transitions Under Friedman Rule: *Consider a policy sequence such that $i_t^w = i_t^m$, $B_t^{\text{Fed}} = G_t^{\text{Fed}} = 0$, and $M_t = G_t = 0$.*

(a) *Real aggregate bank equity follows:*

$$E_{t+1} = (R_t^b + \kappa \min\{(R_t^b - R_t^d), 0\})\beta E_t, \quad \text{with } E_0 > 0 \text{ given.}$$

The dynamics are given by a critical threshold:

$$E_\kappa \equiv \frac{1}{\beta} \left[\frac{\bar{\Theta}^b / \bar{\Theta}^d}{(1 + \kappa)^{-\bar{\epsilon}^b} \kappa^{\bar{\epsilon}^d}} \right]^{\frac{1}{\bar{\epsilon}^d - \bar{\epsilon}^b}}.$$

If $E_t > E_\kappa$, then $\{R_t^b, R_t^d, \bar{d}_t\}$ solve

$$R_t^b = \bar{\Theta}^b (\beta E_t (1 + \bar{d}_t))^{\bar{\epsilon}^b} = \bar{\Theta}^d (\beta E_t \bar{d}_t)^{\bar{\epsilon}^d} = R_t^d.$$

Otherwise, $\bar{d}_t = \kappa$, and

$$R_t^b = \bar{\Theta}^b (\beta E_t (1 + \kappa))^{\bar{\epsilon}^b}, \quad \bar{\Theta}^d (\beta E_t \kappa)^{\bar{\epsilon}^d} = R_t^d.$$

(b) *There $\exists!$ steady-state level of $E_{ss} > 0$. The steady state features binding capital requirements if and only if*

$$\Theta^b (1/\beta)^{\epsilon^b} < \Theta^d (1/\beta)^{\epsilon^d} (1 + \kappa^{-1}). \quad (\text{J.1})$$

If capital requirements do not bind at steady state, then E_{ss} solves

$$E_{ss} = \frac{\Theta^b (1/\beta)^{\epsilon^b} - \Theta^d (1/\beta)^{\epsilon^d}}{\beta}.$$

Otherwise, E_{ss} solves

$$1/\beta = \bar{\Theta}^b (\beta E_{ss} (1 + \kappa))^{\bar{\epsilon}^b} (\kappa + 1) - \kappa \bar{\Theta}^d (\beta E_{ss} \kappa)^{\bar{\epsilon}^d}.$$

(c) *If $\frac{(1-1/\epsilon^b)}{(1+1/\epsilon^d)} \geq \frac{\kappa}{(1+\kappa)}$ and capital requirements bind at steady state, then E_t converges to E_{ss} monotonically.*

In the paper, the calibration satisfies these parameter restrictions.

J.2. Proof of Proposition J.1

The proof of the proposition is presented in three steps. First, we derive a threshold equity level at which capital requirements are binding. Second, we prove that there can be at most one steady state. Third, we provide conditions such that the equilibrium features binding reserve requirements. Finally, we derive the sufficient condition for monotone convergence. We then establish the result for the rate of inflation and the determination of the price level.

Part 1—Law of Motion of Bank Equity. As shown in the Proof of Proposition 7, under log utility, $\bar{c}_t = (1 - \beta)$. Then the law of motion in (18) becomes

$$E_{t+1} = (R_t^b + \kappa \min\{R_t^b - R_t^d, 0\})\beta E_t. \quad (\text{J.2})$$

This follows directly by substituting $\bar{b} = 1 + \bar{d}$ and noticing that the equity constraint binds if $(R_t^b - R_t^d)$; if not, deposits do not affect equity. This is enough to show that the law of motion of bank equity satisfies the difference equation in the proposition. Thus, we have obtained a law of motion for bank equity in real terms. We use this to establish convergence. Consider now the condition such that capital requirements are binding for a given $E_t = E$. For that, we need that $R_t^b > R_t^d$. Using the inverse of the loan demand function, we can write R_t^b in terms of the supply of loans using the market clearing condition:

$$R_t^b = \bar{\Theta}^b (\bar{b}\beta E_t)^{\bar{\epsilon}^b}.$$

If the capital requirement constraint binds,

$$R_t^b = \bar{\Theta}^b (\beta E_t (1 + \kappa))^{\bar{\epsilon}^b}.$$

Using the result that capital requirements are binding when $R_t^b > R_t^d$, we obtain

$$\bar{\Theta}^b (\beta E_t (1 + \kappa))^{\bar{\epsilon}^b} \geq \bar{\Theta}^d (\beta E_t \kappa)^{\bar{\epsilon}^d}.$$

Clearing E at equality delivers a threshold:

$$E_\kappa \equiv \frac{1}{\beta} \left[\frac{\bar{\Theta}^b / \bar{\Theta}^d}{(1 + \kappa)^{-\bar{\epsilon}^b} \kappa^{\bar{\epsilon}^d}} \right]^{\frac{1}{\bar{\epsilon}^d - \bar{\epsilon}^b}},$$

such that for any $E < E_\kappa$, capital requirements are binding in a transition. Thus, the law of motion of capital is broken into a law of motion for the binding and nonbinding capital requirements regions.

We obtain

$$E_{t+1} = \bar{\Theta}^b (\beta E_t (1 + \kappa))^{1 + \bar{\epsilon}^b} - \bar{\Theta}^d (\beta E_t \kappa)^{1 + \bar{\epsilon}^d} \quad \text{for } E_t \leq E_\kappa$$

and

$$E_{t+1} = \bar{\Theta}^b ((1 + d_t)\beta E_t)^{\bar{\epsilon}^b} \beta E_t \quad \text{for } E_t > E_\kappa.$$

Here, we substituted $\bar{d} = \kappa$ in (J.2) for the law of motion in the constrained region and $\bar{d}_t(R_t^b - R_t^d) = 0$ in the second region.

Part 2—Uniqueness of Steady State. Here, we show that there cannot be more than one steady state level of real bank equity. We prove this in a couple of steps. First, we ask whether there can be more than one steady state in each region—that is, in the binding and nonbinding regions. We show that there can be only one steady state in each region. Then, we ask if two steady states can coexist, given that they must lie in separate regions. The answer is no.

To see this, define

$$\Gamma(E) \equiv \bar{\Theta}^b (\beta(1 + \kappa))^{1+\bar{\epsilon}^b} E^{\bar{\epsilon}^b} - \bar{\Theta}^d (\beta(1 + \kappa))^{1+\bar{\epsilon}^d} E^{\bar{\epsilon}^d}.$$

If a steady state exists in the binding region, it must satisfy the following condition:

$$1 = \Gamma(E_{ss}) \quad \text{and} \quad E_{ss} \leq E_\kappa.$$

It is straightforward to verify that

$$\Gamma'(E) < 0, \quad \lim_{E \rightarrow 0} \Gamma(E) \rightarrow \infty, \quad \text{and} \quad \lim_{E \rightarrow \infty} \Gamma(E) \rightarrow -\infty.$$

Since the function is decreasing and starts at infinity and ends at minus infinity, there can be at most one steady state—with positive E —in the constrained region, $E_{ss} < E_\kappa$.

In the unconstrained region, $E_{ss} \geq E_\kappa$, a steady state occurs only when

$$1 = R_t^b \beta.$$

We need to find the level of equity that satisfies that condition. Also, we know that $R^d = R^b$ in the unconstrained region. Thus, the supply of loans in the unconstrained region is given by

$$\beta E_t + \bar{\Theta}^d (R^b)^{\bar{\epsilon}^d},$$

the sum of real bank equity plus real deposits. Thus, we can define the equilibrium rate on loans through the implicit map, $\tilde{R}^b(E)$, that solves

$$\tilde{R}^b(E) \equiv \{ \tilde{R} | \tilde{R} = \bar{\Theta}^b (\beta E_t + \bar{\Theta}^d (\tilde{R})^{\bar{\epsilon}^d})^{\bar{\epsilon}^b} \}.$$

If we can show that $\tilde{R}^b(E)$ is a function and $\tilde{R}^b(E) = \beta^{-1}$ for only one E , then we know that there can be at most one steady state in the unconstrained region. To show that $\tilde{R}^b(E)$ is a function, we must show that there is a unique value of \tilde{R}^b for any E . Note that $\tilde{R}^b(E) = \tilde{R}$ for \tilde{R} that solves

$$\bar{\Theta}^b (\tilde{R})^{\bar{\epsilon}^b} - \bar{\Theta}^d (\tilde{R})^{\bar{\epsilon}^d} = \beta E.$$

Since the first term on the left is decreasing and the second is increasing, this function is monotone, and thus its inverse is a function; that is, $\tilde{R}^b(E)$ is a function. Observe that

$$\lim_{\tilde{R} \rightarrow 0} \bar{\Theta}^b (\tilde{R})^{\bar{\epsilon}^b} - \bar{\Theta}^d (\tilde{R})^{\bar{\epsilon}^d} = \infty, \quad \text{and} \quad \lim_{\tilde{R} \rightarrow \infty} \bar{\Theta}^b (\tilde{R})^{\bar{\epsilon}^b} - \bar{\Theta}^d (\tilde{R})^{\bar{\epsilon}^d} = -\infty,$$

so $\tilde{R}^b(E)$ exists for any positive E . Since \tilde{R}^b is decreasing in E and defined everywhere, there exists at most one value for E such that $\tilde{R}^b(E) = (\beta)^{-1}$. This shows that there exists at most one steady state in the unconstrained region.

Next, we need to show that if there exists a steady state in which $E_{ss} \leq E_\kappa$, there cannot exist another steady state in which $E_{ss} \geq E_\kappa$. To see this, suppose that there \exists a steady state in the unconstrained region. Thus, there exists some value $E_u > E_\kappa$ such that

$$\tilde{R}^b(E_u) = 1/\beta.$$

Since \tilde{R}^b is decreasing and $E_u > E_\kappa$, by assumption, we obtain that

$$1/\beta < \tilde{R}^b(E_\kappa) = R^b(\beta E_\kappa(1 + \kappa)), \quad (\text{J.3})$$

where the equality follows from the definition of E_κ .

As a false hypothesis, suppose that there is another steady state in which $E_c < E_\kappa$. Then, using the law of motion for equity in the constrained region, we get the result that

$$\begin{aligned} R^b(\beta E_c(1 + \kappa)) &= 1/\beta - \kappa(R^b(\beta E_c(1 + \kappa)) - R^d(\beta E_c \kappa)), \\ R^b(\beta E_c(1 + \kappa)) &< 1/\beta, \end{aligned} \quad (\text{J.4})$$

where the second line follows from $R^b > R^d$ for any $E_c < E_\kappa$. Thus,

$$R^b(\beta E_\kappa(1 + \kappa)) < R^b(\beta E_c(1 + \kappa)) < \beta^{-1}$$

because R^b is decreasing. However, (J.4) and (J.3) cannot hold at the same time. Thus, there $\exists!$ steady state with positive real equity.

Part 3—Conditions for Capital Requirements Binding at Steady State. We have shown in Appendix G.5 that a condition for a steady state with slack capital requirements is

$$\Theta^b(1/\beta)^{\epsilon^b} \geq \Theta^d(1/\beta)^{\epsilon^d}(1 + \kappa^{-1}).$$

Then the steady state level of equity is

$$E_{ss} = \frac{\Theta^b(1/\beta)^{\epsilon^b} - \Theta^d(1/\beta)^{\epsilon^d}}{\beta}.$$

If the condition is violated, we use the stationarity condition:

$$1/\beta = R^b(\beta E(1 + \kappa)) + (R^b(\beta E(1 + \kappa)) - R^d(\beta E \kappa))\kappa.$$

This allows equity to grow at the point where the constraint begins to bind.

Part 4—Conditions for Monotone Convergence. Assume that parameters satisfy the conditions for a steady state with binding capital requirements. Observe that if $E_t > E_\kappa$, then $E_{t+1} < E_t$ since $R_t^b < (\beta)^{-1}$ for all $E > E_\kappa$. Thus, any sequence that starts from $E_0 > E_\kappa$ eventually abandons the region. Thus, without loss of generality, we need only to establish monotone convergence within the $E < E_\kappa$ region.

Now consider $E_t < E_{ss}$. We must show that E_{t+1} also satisfies $E_{t+1} < E_{ss}$ if that is the case. Employing the law of motion of equity in the constrained region, we notice that

$$E_{t+1} - E_{ss} = \bar{\Theta}^b(\beta E_t(1 + \kappa))^{1+\bar{\epsilon}^b} - \bar{\Theta}^d(\beta E_t \kappa)^{1+\bar{\epsilon}^d} - E_{ss}.$$

Define $g(E) \equiv \Gamma(E)E$. Thus,

$$\begin{aligned} E_{t+1} - E_{ss} &= \Gamma(E_t)E_t - E_{ss} \\ &= - \int_{E_t}^{E_{ss}} g'(e) de. \end{aligned}$$

It is enough to show that $g'(e) > 0$ for any e . We verify that under the parameter assumptions, this is indeed the case. Note that

$$\begin{aligned} g'(e) &= (1 + \bar{\epsilon}^b) \bar{\Theta}^b (\beta(1 + \kappa))^{1 + \bar{\epsilon}^b} e^{\bar{\epsilon}^b} - (1 + \bar{\epsilon}^d) \bar{\Theta}^d (\beta\kappa)^{1 + \bar{\epsilon}^d} e^{\bar{\epsilon}^d} \\ &= (1 + \bar{\epsilon}^b) R^b (\beta(1 + \kappa)e) \beta(1 + \kappa) - (1 + \bar{\epsilon}^d) R^d (\beta\kappa e) \beta\kappa, \end{aligned}$$

where the second line follows from the definition of R^b and R^d and the result that capital requirements are binding in $E < E_{ss}$. Furthermore, since in this region, $R^b > R^d$ for all $E < E_{\kappa}$, then a sufficient condition for $g'(E) > 0$ is to have

$$(1 + \bar{\epsilon}^b) \beta(1 + \kappa) \geq (1 + \bar{\epsilon}^d) \beta\kappa.$$

Thus, a sufficient condition for monotone convergence is

$$\frac{1 + 1/\bar{\epsilon}^b}{1 + 1/\bar{\epsilon}^d} \geq \frac{\kappa}{1 + \kappa}.$$

APPENDIX K: CALIBRATION

K.1. Data Sources

Most data series are obtained from the Federal Reserve Bank of St. Louis Economic Research Database (FRED ©) and are available at the FRED [website](#). The original data sources for each series are collected by the Board of Governors of the Federal Reserve System (US). We use the following series.

Aggregate Variables. For aggregate variables, we use the following:

- Total reserves:
 - Total Reserves of Depository Institutions, Billions of Dollars, Monthly, Not Seasonally Adjusted, <https://fred.stlouisfed.org/series/TOTRESNS>
- Bank equity:
 - Total Equity Capital for Commercial Banks in United States, Thousands of Dollars, Not Seasonally Adjusted (USTEQC), <https://fred.stlouisfed.org/series/USTEQC>
 - This data is available only at a quarterly frequency. We interpolate the series linearly from quarter to quarter to obtain the monthly series.
- The volume of interbank market loans:
 - Board of Governors of the Federal Reserve System (US), Interbank Loans, All Commercial Banks [IBLACBW027NBOG], H.8 Assets and Liabilities of Commercial Banks in the United States, <https://fred.stlouisfed.org/series/IBLACBW027NBOG>
- The volume of discount window loans:

- Discount Window Borrowings of Depository Institutions from the Federal Reserve [DISCBORR], H.3 Aggregate Reserves of Depository Institutions and the Monetary Base,

<https://fred.stlouisfed.org/series/DISCBORR>

- Bank deposits:
 - Board of Governors of the Federal Reserve System (US), Deposits, All Commercial Banks [DPSACBM027NBOG],
<https://fred.stlouisfed.org/series/DPSACBM027NBOG>
- Bank credit:
 - Board of Governors of the Federal Reserve System (US), Commercial and Industrial Loans, All Commercial Banks [BUSLOANS],
<https://fred.stlouisfed.org/series/BUSLOANS>

Series for Interest Rates. For series on interest rates, we use the following data sources:

- The interest on discount window loans:
 - Board of Governors of the Federal Reserve System (US), Primary Credit Rate [DPCREDIT],
<https://fred.stlouisfed.org/series/DPCREDIT>
- The interest on reserves:
 - Board of Governors of the Federal Reserve System (US), Interest Rate on Excess Reserves [IOER],
<https://fred.stlouisfed.org/series/IOER>
- Interest rate on deposits:
 - We use the series used in Drechsler, Savov, and Schnabl (2017),
academic.oup.com/qje/article-abstract/132/4/1819/3857743.
- The government bond rate:
 - Board of Governors of the Federal Reserve System (US), 3-Month Treasury bill: Secondary Market Rate [TB3MS],
<https://fred.stlouisfed.org/series/TB3MS>

Open-Market Operations. The series that corresponds to open-market operations is the ratio of a measure of the Fed's assets, normalized by total bank credit. During the crisis, the Fed's balance sheet grows for multiple factors, including swaps to foreign governments and direct loans to institutions such as American International Group (AIG). We consider the purchase of government bonds as the equivalent of conventional OMO. For unconventional OMO, we consider the sum of mortgage-backed securities and federal agency securities. These series are weekly and aggregated to the monthly level. The references for these series are as follows:

- Total bank credit:
 - Board of Governors of the Federal Reserve System (US), Bank Credit of All Commercial Banks [TOTBKCR],
<https://fred.stlouisfed.org/series/TOTBKCR>
- Treasury bills:
 - Board of Governors of the Federal Reserve System (US), Assets: Securities Held Outright: U.S. Treasury Securities [WSHOTS],
<https://fred.stlouisfed.org/series/WSHOTS>
- Federal agency paper:

- Board of Governors of the Federal Reserve System (US), Assets: Securities Held Outright: Federal Agency Debt Securities [WSHOFDSL],
<https://fred.stlouisfed.org/series/WSHOFDSL>
- Mortgage-Backed Securities:
 - Board of Governors of the Federal Reserve System (US), Assets: Securities Held Outright: Mortgage-Backed Securities: Wednesday Level [WSHOMCB],
<https://fred.stlouisfed.org/series/WSHOMCB>

Ratio Series. The series for ratios are derived as follows:

- Portfolio shares \bar{g} , \bar{d} :
 - The series for the data analogues of $\{\bar{g}, \bar{d}\}$ are constructed using the micro data from commercial banks from Phillip Schnabl. We take the series for Treasury securities that mature in less than 3 months and those that do so between 3 months and 1 year—typically government bonds are thought of as Treasury securities with maturity below a year. The series for the data analogue of \bar{d} is obtained as the sum of total liabilities divided by equity. Then we aggregate across banks and divide by the equity series. The raw data are available on Philip Schnabl's website,
<http://pages.stern.nyu.edu/~pschnabl/data.html>
- Portfolio shares \bar{m} :
 - We take the series for cash assets for all commercial banks. The series includes vault cash and reserves held by banks. The series is available at the website for the Board of Governors of the Federal Reserve System (US), (Cash Assets, All Commercial Banks/Total Assets, All Commercial Banks)*100,
<https://fred.stlouisfed.org/graph/?g=IW4>
 - We divide the series by the difference between all commercial bank assets minus liabilities.
- Liquidity premium:
 - The liquidity premium that is used to construct the return on loans is obtained from Del Negro, Eggertsson, Ferrero, and Kiyotaki (2017),
<https://www.aeaweb.org/articles?id=10.1257/aer.20121660>
- Liquidity premia used in the empirical analysis:
 - The data are obtained from Nagel (2016). The data extend to December 2011,
<https://www.dropbox.com/s/hroo56worw6sueb/LiqPremia.zip?dl=0>

Federal Funds Interest Rate Distribution.

- The series for the dispersion in the Fed funds rates are obtained from the New York Federal Reserve Bank. The NY Fed provides two data sets, one for the daily minimum and maximum and another that includes quantiles. We use both data sets. In Section 4, we use the max–min spread because the length of the data is longer.
 - To construct the series FF Range, we construct the monthly average of the daily distance between the max and the min of the Fed funds distribution and average over the month. The original series are found here:
<https://apps.newyorkfed.org/markets/autorates/fed-funds-search-page>
- When we perform the financial crisis counterfactuals in Section 5.3, we reconstruct the Fed funds rate among banks. We also use that data in the robustness checks to Section 4 in the Appendix. We use the quantiles of the Fed funds.
 - The data available from the NY Fed also include the max and min, 99, 75, 50, 25, and 1st quantiles, and the standard deviation of daily Fed funds rate. The data are available here:

<https://www.newyorkfed.org/medialibrary/media/markets/ff-volumes-oct2006-feb2016.xlsx>

K.2. Construction of Bank and Nonbank Fed Funds

Mapping the model to the data after October 2008 requires accounting for two additional features. First, in this period, the average Fed funds rate fell below the interest on reserves, the analogue of R^m in the model. Second, the 1-month T-Bill rate, the analogue of R^g in the model, also traded below the interest on reserves. One important consideration in accounting for both features in the model is that many trades in the Fed funds market are transactions between banks, which were eligible to receive interest on reserves, and other institutions, which were not. Thus, after 2008, the average Fed funds rate reflects in part transactions that occur because of that regulatory arbitrage, with an interest rate below the rate on reserves. We argue that to execute the trade with nonbanks, banks need to use government bonds as collateral. Thus, these trades generate an additional value of holding government bonds. Next, we describe how we add these features into the model to address these issues and reconstruct series of Fed funds rate corresponding to trades only among banks.

Nonbank Fed Funds Participants. We introduce a set of nonbanks (nb) that hold reserves and participate in the Fed funds market, but do not receive interest on reserves. In particular, we assume that by the end of the balancing stage, banks with a surplus of government bonds will match (in one round) with nonbanks—banks in deficit, by that stage, have already sold all of their government bonds. We assume that all banks are matched with a nonbank on a per-bond basis. Once a match occurs, banks and nonbanks trade one unit of government bonds for one unit of reserves. The position is reversed by the end of the period, but the interest is paid to the agent that holds the asset overnight. We call this transaction a “Repo.”

When a bank meets a nonbank, they solve the following Nash bargaining problem:

$$R^{f,nb} = \arg \max_R (R - 1/(1 + \pi))^{1-\eta^b} (R^m - R)^{\eta^b}.$$

The first term in parentheses is the surplus minus the outside option for the nonbank: the non-bank earns R instead of storing the reserve at no interest. The second term in parentheses is the surplus for the bank: the bank earns R^m but pays R . The bank’s bargaining power is η^b . The solution to the rate in a Repo transaction is

$$R^{f,nb} = 1 + (1 - \eta^b)(R^m - 1). \quad (\text{K.1})$$

Now, a bank that ends in surplus not only earns R^m but also the gains from the regulatory arbitrage, $R^m - R^{f,nb}$. Therefore, (Gov. Bond LP) is now modified to obtain

$$R^g + (R^m - R^{f,nb}) = R^m + \chi^+. \quad (\text{K.2})$$

This indifference condition follows the same equilibrium relationship as in the version of the model without nonbanks. However, it accounts for the fact that the return on government bonds is now R^g plus the value that banks can extract from nonbanks by pledging bonds in the repo market, $R^m - R^{f,nb}$. Equations (K.1) and (K.2) account for the fact that the government bond and the Fed funds rate trade below R^m . To see this, note that for a sufficiently large η^b and reserves are sufficiently abundant χ^+ will be low enough so that both $R^{f,nb} < R^m$ and $R^g < R^m$.

Construction of the Fed Funds Analogue. Next, we explain how we approximate $\hat{R}^{f,nb}$ and $\hat{R}^{f,ib}$, the average Fed funds rate among bank trades, using data on the distribution of Fed funds rates. Conceptually, the average Fed funds rate is the average Fed funds rates between interbank and non-interbank transactions:

$$R^f = (1 - \nu^{nb}) \cdot R^{f,ib} + \nu^{nb} \cdot R^{f,nb}, \quad (\text{K.3})$$

where ν^{nb} is the fraction of Fed fund trades that occur among banks and nonbanks. From the data, we observe R^f at a given point in time. Thus, with a data counterpart for $\hat{\nu}^{nb}$ and $\hat{R}^{f,nb}$ we could reconstruct $\hat{R}^{f,ib}$. We observe data on the 1st, 25th, 75th, and 99th percentiles of the Federal funds distribution, respectively $\{R_{t,1}^f, R_{t,25}^f, R_{t,75}^f, R_{t,99}^f\}$, at a given date t . To reconstruct the data analogue $\hat{\nu}_t^{nb}$ for each t , we find the pair of contiguous percentiles $\{R_{t,x}^f, R_{t,y}^f\}$ such that the interest rate on reserves fell within that interval; that is, $R_t^m \in [R_{t,x}^f, R_{t,y}^f]$. Naturally, we attribute all trades executed below \hat{R}_t^m to trades between banks and nonbanks. Thus, the mass of trades with nonbank trades will be $F(R_{t,x}^f)$ plus a fraction of trades that fell within the $[R_{t,x}^f, R_t^m]$ interval. We approximate the date assuming a uniform distribution among the trades within that interval. Hence, the data analogue of ν^{nb} is

$$\hat{\nu}_t^{nb} = F(R_{t,x}^f) + \frac{R_t^m - R_{t,x}^f}{R_{t,y}^f - R_{t,x}^f} \cdot [F(R_{t,y}^f) - F(R_{t,x}^f)]. \quad (\text{K.4})$$

For the analogue of $\hat{R}^{f,nb}$, we reconstruct it using the same approximation to the distribution of rates; that is,

$$\begin{aligned} \hat{R}^{f,nb} &= \frac{1}{2} \left(\sum_{\{x,y\} \in [\dots]} (R_{t,y}^f - R_{t,x}^f) \cdot [F(R_{t,y}^f) - F(R_{t,x}^f)] \right) \\ &\quad + \frac{1}{2} \left((R_t^m - R_{t,x}^f) \frac{R_t^m - R_{t,x}^f}{R_{t,y}^f - R_{t,x}^f} \cdot [F(R_{t,y}^f) - F(R_{t,x}^f)] \right). \end{aligned}$$

We use this construction to obtain $\hat{R}^{f,ib}$ and \hat{R}^g using (K.3) and (K.2), respectively. We use the monthly moving average series for $\hat{R}^{f,ib}$ and \hat{R}^g in the procedure that follows.

K.3. Calibration Procedure

In the procedure, we infer steady state parameters using data analogues for the volume of interbank loans; discount window loans, $\{W, F\}$; interest rates $\{R_{ss}^d, R_{ss}^m, R_{ss}^f, R_{ss}^w, R_{ss}^w\}$; the loan liquidity premium $\{LP_{ss}\}$ and bank portfolio shares on reserve and government bond holdings; and the capital requirement, $\{\bar{m}_{ss}, \bar{g}_{ss}, \kappa\}$.

Here, we explain how we deduce $\{\hat{\sigma}_{ss}, \hat{\lambda}_{ss}, \hat{\eta}_{ss}, \hat{\sigma}_{ss}^\delta, \hat{\Theta}_{ss}^b, \hat{\Theta}_{ss}^d\}$ in Section 5.1.

1. Obtain $\hat{\Psi}_{ss}^-$ from

$$\hat{\Psi}_{ss}^- = \frac{\hat{\Psi}_{ss}^- S_{ss}^-}{S_{ss}^-} = \frac{F_{ss}}{W_{ss} + F_{ss}}.$$

2. Obtain

$$\hat{\lambda}_{ss} = \log\left(\frac{1}{1 - \hat{\Psi}_{ss}^-}\right)$$

by inverting (30) under the assumption that $\theta_{ss} < 1$.

3. Deduce $\hat{\omega}_{ss}^*$ from the definition (E.32), substituting $\rho = 0$ and $\{\bar{a}, \bar{d}\} = \{\bar{m}_{ss}, \bar{g}_{ss}\}$, and obtain

$$\hat{\omega}_{ss}^* = -\frac{\bar{m}_{ss} + \bar{g}_{ss}}{\kappa} / \frac{R_{ss}^d}{R_{ss}^m}.$$

4. Deduce $\hat{\sigma}_{ss}$ as the solution $\hat{\sigma}$ that solves

$$\frac{W}{A} = (1 - \hat{\Psi}_{ss}^-) \Phi(\hat{\omega}_{ss}^*; \hat{\sigma}) \left(\frac{\bar{m}_{ss} + \bar{g}_{ss}}{\kappa + 1} + \frac{R_{ss}^d}{R_{ss}^m} \mathbb{E}[\omega | \omega < \hat{\omega}_{ss}^*; \hat{\sigma}] \frac{\kappa}{\kappa + 1} \right). \quad (\text{K.5})$$

This step uses (E.33), where S_{ss}^- is obtained by integrating (E.30) among all $\omega < \hat{\omega}_{ss}^*$, dividing by all assets.

5. We deduce a value for $\hat{\theta}_{ss}$ from

$$\hat{\theta}_{ss} = \frac{\Phi(\hat{\omega}_{ss}^*; \hat{\sigma}) \left(\bar{m}_{ss} + \bar{g}_{ss} + \frac{R_{ss}^d}{R_{ss}^m} \mathbb{E}[\omega | \omega < \hat{\omega}_{ss}^*; \hat{\sigma}] \kappa \right)}{(1 - \Phi(\hat{\omega}_{ss}^*; \hat{\sigma})) \left(\bar{m}_{ss} + \bar{g}_{ss} + \frac{R_{ss}^d}{R_{ss}^m} \mathbb{E}[\omega | \omega > \hat{\omega}_{ss}^*; \hat{\sigma}] \kappa \right) - \bar{g}_{ss}}. \quad (\text{K.6})$$

This step uses S_{ss}^- obtained by integrating (E.30) among all $\{\omega < \hat{\omega}_{ss}^*\}$ and S_{ss}^+ obtained by integrating (E.30) among all $\omega > \hat{\omega}_{ss}^*$ and applying the formula (E.31).

6. We deduce $\hat{\Psi}_{ss}^+$ from the clearing condition in the Fed funds market $\hat{\Psi}_{ss}^+ = \hat{\Psi}_{ss}^- \cdot \hat{\theta}_{ss}$

7. We deduce $\hat{\chi}_{ss}^+$ using (E.34); thus, $\hat{\chi}_{ss}^+ = \hat{\Psi}_{ss}^+ \cdot (R^f - R^m)$.

8. We deduce $\hat{\eta}_{ss}$, which solves

$$\hat{\chi}_{ss}^+ = (R^w - R^m) \left(\frac{\hat{\theta}_{ss}}{\hat{\theta}_{ss}} \right)^{\hat{\eta}_{ss}} \left(\frac{\hat{\theta}_{ss}^{\hat{\eta}_{ss}} \hat{\theta}_{ss}^{1-\hat{\eta}_{ss}} - \hat{\theta}_{ss}}{\hat{\theta}_{ss} - 1} \right), \quad (\text{K.7})$$

where

$$\hat{\theta}_{ss} = \begin{cases} 1 + (\hat{\theta}_{ss} - 1) \exp(\hat{\lambda}_{ss}) & \text{if } \hat{\theta}_{ss} \leq 1, \\ 1 + ((\hat{\theta}_{ss})^{-1} - 1) \exp(\hat{\lambda}_{ss})^{-1} & \text{if } \hat{\theta}_{ss} > 1, \end{cases}$$

which direct from (31).

9. We also deduce $\hat{\chi}_{ss}^-$ using

$$\hat{\chi}_{ss}^- = (R_{ss}^w - R_{ss}^m) \left(\frac{\hat{\theta}_{ss}}{\hat{\theta}_{ss}} \right)^{\hat{\eta}_{ss}} \left(\frac{\hat{\theta}_{ss}^{\hat{\eta}_{ss}} (\hat{\theta}_{ss})^{1-\hat{\eta}_{ss}} - 1}{\hat{\theta}_{ss} - 1} \right),$$

which is also obtained from (31).

10. We deduce a volatility of defaults, $\hat{\sigma}_{ss}^\delta$, and κ from the solution to

$$(\bar{b}_{ss}, 1 + \kappa - (\bar{b}_{ss}, \kappa) \equiv \underset{\bar{b} \leq \kappa, \bar{a} + \bar{b} - \kappa = 1}{\operatorname{argmax}} \left\{ \mathbb{E}[(1 - \delta) R_{ss}^b \bar{b} + R_{ss}^m \bar{a} - R_{ss}^d \bar{d} + \bar{\chi}_{ss}(\bar{a}, \bar{d}, \omega)]^{1-\gamma} \right\}^{\frac{1}{1-\gamma}},$$

where the expectation \mathbb{E} is over δ and ω .

11. We obtain $\hat{\Theta}_{ss}^b$ by inverting (E.25):

$$\hat{\Theta}_{ss}^b = (\bar{b}_{ss} + \bar{b}_{ss}^{\text{Fed}}) \cdot \hat{\beta}_{ss} \cdot E_{ss} \cdot (R_{ss})^{\epsilon^b}.$$

12. We obtain $\hat{\Theta}_{ss}^d$ by inverting (E.26):

$$\hat{\Theta}_{ss}^d = \bar{d} \cdot \hat{\beta}_{ss} \cdot E_{ss} \cdot (R_{ss}^d)^{\epsilon^d}.$$

APPENDIX L: APPENDIX TO SECTION 4: ROBUSTNESS ANALYSIS

The next sections present additional corroborating evidence to the evidence presented in Section 4.

Estimates With Other Liquidity Measures. A first robustness check runs the same regressions as in Table I, but using two other measures of liquidity premia. The first measure is a classic measure advocated by [Stock and Watson \(1989\)](#) and [Friedman and Kuttner \(1993\)](#): the 3-month spread between the AAA commercial paper and the 3-month T-Bill. The second measure is the TED spread, the difference between the 3-month Treasury bill and the 3-month US dollar LIBOR. In this case, the data availability is longer. It spans from July 2000 to February 2016, when the NY Fed stopped reporting the daily max and min values of the Fed Funds market. Table V reports the results. The pattern is consistent in significance and in magnitude with the earlier estimation in Table I.

The next robustness check compares the results with two popular measures introduced by [Gilchrist and Zakrajšek \(2012\)](#): the GZ Spread and the GZ excess-bond premium (EBP). The authors construct these measures by taking individual fixed-income securities and discounting their promised cash-flows according to zero-coupon US Treasury yields. This delivers a spread for each security. The “GZ excess bond premium” of each security is constructed as the portion of the overall credit spread that cannot be accounted for by individual predictors of default, nor bond-specific characteristics. Specifically, the authors regress their credit spreads on a firm-specific measure of expected default and

TABLE V
LIQUIDITY PREMIA AND INTERBANK SPREADS—ROBUSTNESS CHECKS.

	(1) CP Spread	(2) CP Spread	(3) CP Spread	(4) TD Spread	(5) TD Spread	(6) TD Spread
FF Range	0.352 (17.72)	0.320 (14.71)	0.293 (13.05)	0.587 (16.86)	0.609 (15.63)	0.534 (13.92)
FFR		0.0206 (3.26)	0.0267 (4.20)		-0.0137 (-1.23)	0.00430 (0.40)
VIX			0.115 (3.53)			0.315 (5.70)
Constant	0.00714 (0.44)	-0.00882 (-0.53)	-0.345 (-3.57)	0.0893 (3.13)	0.101 (3.36)	-0.816 (-5.00)
Observations	184	184	184	188	188	188
Adjusted R^2	0.631	0.649	0.670	0.602	0.603	0.661

Note: t statistics in parentheses.

TABLE VI
LIQUIDITY PREMIA AND INTERBANK SPREADS—ROBUSTNESS CHECKS.

	(1) GZ Spread	(2) GZ Spread	(3) GZ Spread	(4) GZ EBP	(5) GZ EBP	(6) GZ EBP
FF Range	0.536 (4.01)	0.769 (5.29)	0.150 (1.87)	0.463 (4.91)	0.526 (4.98)	0.112 (1.62)
FFR		-0.147 (-3.55)	0.00114 (0.05)		-0.0395 (-1.31)	0.0598 (3.07)
VIX			2.598 (22.50)			1.738 (17.39)
Constant	2.302 (21.00)	2.422 (21.70)	-5.139 (-15.07)	-0.127 (-1.64)	-0.0947 (-1.17)	-5.152 (-17.46)
Observations	188	188	188	188	188	188
Adjusted R^2	0.075	0.129	0.767	0.110	0.113	0.663

Note: t statistics in parentheses.

a vector of bond-specific characteristics; the residual of this regression is the GZ excess bond premium.

Table VI reports the results. The pattern resembles the earlier estimation in Table I, but the regressions lose power once we control for the VIX, a measure of dispersion that remains significant. This feature indicated that bonds can have liquidity premia that correlate with the cycle, but are independent of the liquidity premia among near-money assets.

Other Spreads and Placebos. Table VII reports two additional sets of robustness checks. Regressions (1–3) in Table VII are the same as regression (3) in Table I, except that the measure “FF Range” is replaced by three alternative measures of interbank mar-

TABLE VII
LIQUIDITY PREMIA AND INTERBANK SPREADS—ROBUSTNESS CHECKS.

	(1) GC Spread	(2) GC Spread	(3) GC Spread	(4) 10y AAA Spr	(5) Note Spr	(6) OF Spr
FF 99-1	0.207 (7.26)					
FFR	0.0555 (6.14)	0.0774 (9.66)	0.0429 (8.36)	-0.0912 (-8.33)	-0.00383 (-1.84)	0.00384 (4.28)
VIX	0.138 (3.05)	0.200 (4.57)	0.0584 (2.07)	0.851 (15.12)	0.0943 (8.95)	0.00462 (1.01)
FF 75-25		0.658 (6.48)				
FF std			1.051 (10.08)			
FF Range				-0.0383 (-0.98)	0.00375 (0.60)	-0.00146 (-0.54)
Constant	-0.411 (-2.84)	-0.585 (-4.11)	-0.184 (-2.08)	-0.696 (-4.19)	-0.239 (-7.17)	-0.0165 (-1.14)
Observations	63	63	138	188	138	138
Adjusted R^2	0.791	0.768	0.660	0.685	0.486	0.121

Note: t statistics in parentheses.

TABLE VIII
LIQUIDITY PREMIA AND INTERBANK SPREADS—ROBUSTNESS CHECKS.

	(1) GC Spread	(2) GC Spread	(3) GC Spread	(4) CD Spread	(5) CD Spread	(6) CD Spread
FF Range	0.238 (8.92)	0.162 (6.60)	0.157 (6.19)	0.581 (13.09)	0.529 (10.83)	0.513 (10.23)
FFR		0.0465 (6.72)	0.0482 (6.63)		0.0322 (2.34)	0.0377 (2.62)
VIX			0.0281 (0.80)			0.0886 (1.28)
Constant	0.0308 (1.35)	-0.0691 (-2.90)	-0.153 (-1.42)	-0.0110 (-0.29)	-0.0802 (-1.70)	-0.345 (-1.62)
Observations	90	90	90	90	90	90
Adjusted R^2	0.469	0.646	0.645	0.657	0.674	0.676

Note: t statistics in parentheses.

ket dispersion. The first two measures are FF 99-1, which corresponds to the monthly average of the daily spread between the 99th and 1st quantiles of the Fed Funds distribution, and FF 75-25, which corresponds to the 75th and 25th quantiles. Finally, FF std is the monthly average daily standard deviation of the Fed funds rates. The time series for quantiles are shorter than FF range, ranging only from January 2006 through December 2018. The overall fit is similar, and the magnitude of the coefficient of FF 99-1 series in particular is very similar to FF Range, not surprisingly. The coefficient for the FF 75-25 series is larger, which is unsurprising, since the standard deviation of this series is larger. The FF std series is also significantly correlated, and the coefficient is even larger.

Regressions (4–6) in Table VII employ other measures of liquidity premia in Nagel (2016), which are interpreted as placebo tests. These are the series for the 10y AAA to T-Bill corporate bond spread, the Note T-Bill spread and the spread between on the run and off the run bonds. In none of these cases, is the FF Range variable significant.

Subsamples. As final robustness check, we rerun the regressions in Table VIII, but this time, we limit the sample to the pre-crisis period from July 2000 through December 2007. Again, the pattern is the same. The FF Range variable remains a significantly correlated variable with other measures of spreads. By contrast, the VIX index is no longer significantly correlated with measures of spreads.

APPENDIX M: ALGORITHMS

This Appendix presents the numerical algorithms that we use to solve the model. We first present the algorithm to solve the stationary equilibrium. We then present the algorithm to solve for transitional dynamics.

M.1. Stationary Equilibrium

The stationary equilibrium of the model can be conveniently reduced to solving a system of two nonlinear equations in two unknowns (R_{ss}^b, θ_{ss}). In a stationary equilibrium, all nominal variables grow at a constant rate (in this case, zero) and real variables are constant. To simplify the presentation, we assume that the intertemporal elasticity of substitution equals one, which gives rise to a constant dividend-to-equity ratio, a zero nominal

growth of the nominal balance sheet of the monetary authority, and $B^{\text{Fed}} = 0$. We also set a value for R_{ss}^d based on the calibration target and infer the intercept term Θ^d , which is consistent with that value.

1. Guess a stationary value for (R_{ss}^b, θ_{ss}) , the real return on loans and market tightness.
2. Given market tightness, nominal policy rates, the given long-run inflation, and R_{ss}^d , compute the liquidity yield function $\bar{\chi}$ using (9).
3. Solve banks' optimization problem for the portfolio weights $\{\bar{b}, \bar{d}, \bar{a}\}$:

$$\begin{aligned} \max_{\{\bar{b}, \bar{a}, \bar{d}\} \geq 0} \left\{ \mathbb{E} \left[R^b \bar{b} + R^m \bar{a} - R^d \bar{d} + \bar{\chi}(\bar{a}, \bar{d}, \omega) \right]^{1-\gamma} \right\}^{\frac{1}{1-\gamma}}, \\ \bar{b} + \bar{a} - \bar{d} = 1, \quad \text{and} \quad \bar{d} \leq \kappa. \end{aligned} \quad (\text{M.1})$$

4. Check whether banks' policies are consistent with steady state:
5. Compute aggregate gross equity growth as

$$E'/E = (1 + (R^b - 1)\bar{b} - (R^d - 1)\bar{d})(1 - \bar{c}).$$

6. Compute implied market tightness:

$$S^- = \int_1^{\frac{\bar{a}/\bar{d}-\rho}{(1-\rho)}} s(\omega) d\Phi \quad \text{and} \quad S^+ = \int_{\frac{\bar{a}/\bar{d}-\rho}{(1-\rho)}}^{\infty} s(\omega) d\Phi,$$

where market tightness is defined as

$$\tilde{\theta} = S^-/S^+.$$

7. If $E'/E = 1$ and $\tilde{\theta} = \theta_{ss}$, move to step 7. Otherwise, adjust the guess for R_{ss}^b and θ_{ss} and go to step 3.
8. Compute household demand for government bonds, using $R^g = R^m + \chi^+$:

$$\frac{G^h}{P} = \Theta^g (R^g)^{\epsilon^g}$$

9. Compute banks' portfolio weights on government bonds by using market clearing condition

$$\frac{G}{P} - \frac{G^h}{P} = E\bar{g}(1 - \bar{c})$$

10. Compute the nominal amount of reserves and the intercepts of the loan demand and deposit supply functions using the fact that real equity and the initial price level are normalized to one (i.e., $P = 1, E = 1$) and

$$\begin{aligned} \tilde{M}^{\text{Fed}} &= (1 - \bar{c})(\bar{a} - \bar{g})EP, \\ \Theta^b \left(\frac{1}{R^b} \right)^\epsilon &= E\bar{b}(1 - \bar{c}), \\ \Theta^d \left(\frac{1}{R^d} \right)^{-s} &= E\bar{d}(1 - \bar{c}). \end{aligned}$$

11. Compute nominal returns using definitions of real returns and transfers T^{Fed} from the Fed budget constraint:

$$\tau = (1 - \bar{c})[(i^m - \pi)\bar{m} + (i^g - \pi)\bar{g} - (i^w - i^m)\bar{w}],$$

where

$$\bar{w} = (1 - \Psi^-(\theta))S^-.$$

Let us comment on some details from the computations. To solve for the pair (R_{ss}^b, θ_{ss}) , we use the `fsolve` command in Matlab. To solve for the portfolio problem, we use the first-order conditions, which we again solve, using `fsolve`. Notice that if the capital requirement binds, there is only one portfolio variable to solve for.

To compute expectations, we use a Newton–Cotes quadrature method. Specifically, we apply the trapezoid rule with a grid of 2000 equidistant points. To specify the lower and upper boundaries of the grid, we take the shock values that guarantee 10^{-5} mass in the tails of the distribution.

M.2. Transitional Dynamics

The basic procedure to solve for transitional dynamics is to start by conjecturing an initial price level P_0 , then solve for all sequences of prices and quantities using market clearing conditions and bank problems. The price converges to the path of the price level in the stationary equilibrium. Essentially, the solution can be reduced to one equation and one unknown.

To simplify the presentation, we assume that the intertemporal elasticity of substitution equals one, giving rise to a constant dividend-to-equity ratio; a zero nominal growth of the nominal balance sheet of the monetary authority, $B_t^{\text{Fed}} = 0$; and an inelastic demand for government bonds by households.

1. Establish a finite period $T \in \mathbb{N}$ for convergence to steady state, a convergence criterion ε , and an initial value for aggregate real equity E_0 .
2. Guess an initial price level P_0 .
3. Set $t = 0$.
4. Given E_0, P_0 and level of nominal reserves set by the monetary authority \tilde{M}^{Fed} , we can obtain an implied level of real reserve holdings:

$$\tilde{m}_0 \equiv \frac{\tilde{M}^{\text{Fed}}}{\beta P_0 E_0}.$$

5. Compute

$$\tilde{g}_0 = \frac{\frac{G}{P} - \frac{G^h}{P}}{E_0(1 - \bar{c})}.$$

6. Denote $\tilde{a}_0 = \tilde{m} + \tilde{g}$.
7. Find (R_1^m, R_1^b) that solves

$$\bar{a}_0 - \tilde{a}_0 = 0,$$

$$\beta E_0(1 + \bar{d}_0 - (\tilde{m}_0 + \tilde{g}_0)) = \Theta_0^b \left(\frac{1}{1 + r_{t+1}^b} \right)^\epsilon,$$

where \bar{a}_0, \bar{d}_0 satisfy

$$(\bar{a}_0, \bar{d}_0) = \arg \max_{\bar{b}, \bar{a}, \bar{d} \leq \kappa} \left\{ \mathbb{E} \left[R_1^b (1 + \bar{d}_0 - \bar{a}) + R_1^m \bar{a} - R_1^d \bar{d} + \bar{\chi}(\bar{a}, \bar{d}, \omega) \right]^{1-\gamma} \right\}^{\frac{1}{1-\gamma}}$$

and $\bar{\chi}$ follows (9).

Given i^m and R_1^m , compute inflation between period 0 and 1 as

$$\pi_1 = \left(\frac{1 + i^m}{R_1^m} \right) - 1.$$

8. Given π_1 and P_0 , compute next-period price $P_1 = (1 + \pi_1)P_0$.

9. Compute next-period equity using the law of motion

$$E_1 = (1 + (R^b - 1)\bar{b} - (R^d - 1)\bar{d})(1 - \bar{c})E_0.$$

10. Repeat steps 4–9 for $t = 1, \dots, T$.

11. Compute criteria for convergence of $z = P_{T+1} - P_0$. Notice that if there is steady-state inflation, this condition for convergence is replaced by $z = P_{T+1} - P_0(1 + \pi_{ss})^T$.

12. If $|z| < \varepsilon$, exit algorithm. Otherwise, adjust P_0 and go to step 4.

REFERENCES

- DEL NEGRO, M., G. EGGERTSSON, A. FERRERO, AND N. KIYOTAKI (2017): “The Great Escape? A Quantitative Evaluation of the Fed’s Liquidity Facilities,” *American Economic Review*, 107 (3), 824–857. [42]
- DRECHSLER, I., A. SAVOV, AND P. SCHNABL (2017): “The Deposits Channel of Monetary Policy,” *The Quarterly Journal of Economics*, 132 (4), 1819–1876. [41]
- FRIEDMAN, B. M., AND K. N. KUTTNER (1993): “Why Does the Paper-Bill Spread Predict Real Economic Activity?” in *Business Cycles, Indicators and Forecasting*, ed. by J. H. Stock and M. W. Watson. University of Chicago Press. [46]
- GILCHRIST, S., AND E. ZAKRAJŠEK (2012): “Credit Spreads and Business Cycle Fluctuations,” *American Economic Review*, 102 (4), 1692–1720. [46]
- LAGOS, R., AND R. WRIGHT (2005): “Unified Framework for Monetary Theory and Policy Analysis,” *Journal of Political Economy*, 113 (3), 463–484. [9]
- LAGOS, R., G. ROCHETEAU, AND R. WRIGHT (2017): “Liquidity: A New Monetarist Perspective,” *Journal of Economic Literature*, 55 (2), 371–440. [8,9]
- NAGEL, S. (2016): “The Liquidity Premium of Near-Money Assets,” *The Quarterly Journal of Economics*, 131 (4), 1927–1971. [42,48]
- STOCK, J. H., AND M. W. WATSON (1989): “New Indices of Coincident and Leading Economic Indicators,” in *NBER Macroeconomics Annual*, Vol. 4. [46]

Co-editor Giovanni L. Violante handled this manuscript.

Manuscript received 20 August, 2018; final version accepted 26 February, 2021; available online 4 June, 2021.