

## A note on identifying heterogeneous sharing rules

PIERRE-ANDRÉ CHIAPPORI

Department of Economics, Columbia University

JU HYUN KIM

Department of Economics, University of North Carolina at Chapel Hill

In this note, we address nonparametric identification of a collective model of household behavior in the presence of additive unobserved heterogeneity in the sharing rule. We show that the (nonstochastic part of the) sharing rule is nonparametrically identified. Moreover, under independence assumptions, individual Engel curves and the random distributions are identified except in special cases (i.e., linear Engel curves).

KEYWORDS. Sharing rule, unobserved heterogeneity, collective models, nonparametric identification.

JEL CLASSIFICATION. C14, C31, D10, D11, D13.

### 1. INTRODUCTION

The empirical estimation of collective models of household behavior has attracted much attention recently. In such models, agents have their own preferences and make Pareto efficient decisions. The econometrician can observe the household's (aggregate) demand, but not individual consumptions. The issue, then, is whether this is sufficient to identify individual demands and the decision process. Existing results distinguish two basic cases, depending on whether or not data entail price variations. If they do, Chiappori and Ekeland (2009) show that identification obtains under exclusion restrictions. Specifically, if for each agent there exists a commodity not consumed by that agent,<sup>1</sup> then generically each agent's *collective indirect utility* (which gives the agent's utility as a function of prices and incomes) can be ordinally recovered. Alternatively, Bourguignon, Browning, and Chiappori (2009; from now on BBC) consider the “cross

---

Pierre-André Chiappori: [pc2167@columbia.edu](mailto:pc2167@columbia.edu)

Ju Hyun Kim: [juhkim@email.unc.edu](mailto:juhkim@email.unc.edu)

<sup>1</sup>The choice of these “exclusive” goods plays a key role in empirical applications of the model. Two main strands can be found in the literature. One refers to consumption goods that are known to be only consumed by one person. For instance, Browning, Bourguignon, Chiappori, and Lechene (1994) use male and female clothing, while Attanasio and Lechene (2014) and Dunbar, Lewbel, and Pendakur (2013) use data on individual food consumption. Other studies, particularly those considering developing countries, may use social norms that restrict consumption of some goods for some genders (e.g., tobacco is a mostly male consumption in some countries). The second approach considers labor supply models and assumes that each spouse's leisure is the spouse's private consumption. In what follows, we are mostly referring to the first interpretation.

Copyright © 2017 The Authors. Quantitative Economics. The Econometric Society. Licensed under the Creative Commons Attribution-NonCommercial License 3.0. Available at <http://www.qeconomics.org>. DOI: 10.3982/QE414

sectional” case, in which prices are constant over the sample. Then household demand depends only on income (or total expenditures) and on one or several distribution factors—defined as variables that affect the decision process but not the budget constraint. In a framework where all commodities are privately consumed—or alternatively where utilities are separable in private consumptions—efficiency is equivalent to the existence of a “sharing rule” whereby income is split between spouses, who each independently purchase their preferred bundle. BBC show that, under similar exclusion restrictions, individual Engel curves and the sharing rule can be recovered up to an additive constant.

In practice, empirical estimation of cross sectional collective models considers equations of the form

$$\begin{aligned} q_1 &= \alpha_1(\rho(x, z)) + \eta_1, \\ q_2 &= \alpha_2(x - \rho(x, z)) + \eta_2, \\ q_i &= \alpha_{i1}(\rho(x, z)) + \alpha_{i2}(x - \rho(x, z)) + \eta_i, \quad i = 3, \dots, n. \end{aligned} \tag{1}$$

Here,  $x$  denotes income or total expenditures,  $z$  denotes a distribution factor, and  $q_i$  denotes the household demand for good  $i$ ; note that goods 1 and 2 are exclusively consumed by members 1 and 2, respectively. Moreover,  $\rho(x, z)$  denotes the sharing rule, and  $\alpha_{ix}$  ( $x = 1, 2$ ) is member  $x$ 's Engel curve for commodity  $i$ . Finally, the  $\eta$ s are independent and identically distributed (iid) random shocks. This framework is used, for instance, by [Browning et al. \(1994\)](#), [Attanasio and Lechene \(2014\)](#), and many others.

An obvious weakness of the framework just described lies in its treatment of unobserved heterogeneity. Essentially, it is captured through the  $\eta$ s, although the latter variables could (and, as we shall argue below, should) rather be interpreted as measurement errors. But a formulation like (1) imposes that the sharing rule be identical across couples; in particular, unobserved heterogeneity cannot affect the distribution of income within the household. In many contexts, the assumption may seem excessively restrictive. The intrahousehold decision process is typically complex and involves a host of factors, some of which are not observed by the econometrician. In that case, one would like to allow for unobserved heterogeneity in the decision process itself. Moreover, the extent to which (1) is compatible with heterogeneity in preferences is unclear, especially in view of the recent literature using a random utilities framework.

The goal of this note is to investigate whether this setting can be extended while preserving its main, identification properties. Specifically, we propose to replace model (1) with the following generalization: for  $i = 3, \dots, n$ ,

$$\begin{aligned} q_1 &= \alpha_1(\rho(x, z) + \varepsilon) + \eta_1, \\ q_2 &= \alpha_2(x - \rho(x, z) - \varepsilon) + \eta_2, \\ q_i &= \alpha_{i1}(\rho(x, z) + \varepsilon) + \alpha_{i2}(x - \rho(x, z) - \varepsilon) + \eta_i, \end{aligned} \tag{2}$$

where the  $\eta_i$ s are measurement errors and  $\varepsilon$  is a random shock reflecting unobserved heterogeneity (so that the actual sharing rule is the sum of a deterministic component  $\rho(x, z)$  and the random shock  $\varepsilon$ ).

We start with a motivating example showing how the structure (2) naturally emerges in specific economic problems; here, we consider the standard issue of efficient risk sharing within a group. We then give our main results. We first show that  $\rho$  can be non-parametrically identified in the neighborhood of any point  $(\bar{x}, \bar{z})$  at which  $\partial\rho/\partial z$  does not vanish. This result is fully general; it does not require specific assumptions on the joint distribution of shocks. We then consider a second problem, namely the identification of individual Engel curves and of the distributions of  $\varepsilon$  and the  $\eta$ s. The crucial assumption here is that  $\varepsilon$  is independent of the shocks  $\eta_1, \dots, \eta_n$  and these shocks are independent of each other; this assumption is natural if the  $\eta$ s are interpreted as measurement errors. Under that assumption, we show that nonparametric identification obtains except for particular cases (typically, when some of the individual Engel curves are linear). Finally, all these results only require  $n \geq 2$ ; that is, the existence of two exclusive goods is sufficient to get identification of the sharing rule, irrespective of the total number of commodities. For  $i \geq 3$ , additional, overidentifying restrictions are generated.

To the best of our knowledge, this contribution is the first to allow for heterogeneous sharing rules to identify Engel curves for each commodity and each member. Standard methods in the existing literature, including [Browning et al. \(1994\)](#) and [Attanasio and Lechene \(2014\)](#), require sharing rules to be deterministic in identifying individual Engel curves. [Matzkin and Perez-Estrada \(2011\)](#) discuss a novel result about identifying individual random utility functions in collective models. In their setting, which postulates only one private good and a composite good, each agent's consumption for the private good is identified as a sharing rule. Unfortunately, their result is not directly applicable to general collective models with multiple private goods, which we consider here.

Our work is related to but in several dimensions distinctly different from [Matzkin and Perez-Estrada \(2011\)](#) and [Lewbel and Pendakur \(2015\)](#). To focus on preference heterogeneity, [Matzkin and Perez-Estrada \(2011\)](#) assume that the bargaining power between agents is deterministic, while our method can allow unobserved heterogeneity in the bargaining power as well as in each agent's preference. Also, since their model postulates one private good, for the case where there are multiple private goods it implies that each individual's utility is a function of the total sum of the private goods consumed. Thus, their method can identify only sharing rules but not individual Engel curves for each good, while our method can identify both. [Lewbel and Pendakur \(2015\)](#) propose a new identification result for nonlinear demand systems using random coefficients models. Even though they also focus on unobserved preference heterogeneity, their result is only for the unitary model and not directly applicable to the collective model that is considered here.

The characteristic function method plays a key role in the first stage of identifying Engel curves and the distribution of shocks. This method has been widely used in the literature on stochastic deconvolution such as [Evdokimov \(2010\)](#), [Evdokimov and White \(2012\)](#), [Bonhomme and Robin \(2010\)](#), [Arellano and Bonhomme \(2012\)](#), and [Schennach and Hu \(2013\)](#), to name a few.<sup>2</sup> In particular, [Schennach and Hu \(2013\)](#) demonstrate

---

<sup>2</sup>[Evdokimov \(2010\)](#) and [Arellano and Bonhomme \(2012\)](#) take this approach to identify panel data models, and [Bonhomme and Robin \(2010\)](#) apply this method to linear factor models to decompose individual

that identifying a model with measurement errors in both dependent and independent variables can be viewed as a problem of the existence of two observationally equivalent models, one having errors only in the dependent variable and the other having errors only in independent variables. By extending their result to our model (2), we derive sufficient conditions for identification from more tractable models. Our identification result shows that the structure of the collective model allows for weaker conditions compared to the errors-in-variables models discussed in Schennach and Hu (2013).

Our proposed methods are also tied to the techniques used in the literature on identification of transformation models and of average derivative models. The transformation model is a particular case of the equation for  $q_1$  in (2), where  $\eta_1 = 0$  with probability 1. Chiappori, Komunjer, and Kristensen (2014) use partial derivatives of the distribution function of the observables to identify the functions  $\alpha_1$  and  $\rho$  for the transformation model, while we make use of average derivatives of individual demand functions in the presence of mean zero random shocks  $\eta_1, \dots, \eta_n$  to consider the system of (2). The literature on average derivative models has focused on general nonseparable models  $Y = m(X, \varepsilon)$ , where  $X \in \mathbb{R}^K$  and  $\varepsilon \in \mathbb{R}^J$  for  $K \times J \in \mathbb{R}^2$ . Altonji and Matzkin (2005) show that the average derivative  $\frac{\partial}{\partial x} E[Y | X = x]$  is identified when  $X$  and  $\varepsilon$  are independent conditionally on other observable variables. We show that in our collective models (2), the function  $\rho$  is identified up to an additive constant from average partial derivatives  $\frac{\partial}{\partial x} E[q_i | x, z]$  and  $\frac{\partial}{\partial z} E[q_i | x, z]$  for  $i = 1, 2$ .

## 2. A MOTIVATING EXAMPLE

The form (2) fits several frameworks that have recently attracted much interest. In this section, we focus on the literature on efficient risk sharing, starting with Townsend's (1994) seminal paper.<sup>3</sup> In Townsend's framework, agents are characterized by constant absolute risk aversion (CARA) preferences and share some exogenous income risk. If, for expositional simplicity, we only consider two person households, then, by a standard result of the collective literature,<sup>4</sup> any efficient intrahousehold allocation mechanism can be described as a two stage process. In the first stage, individuals agree on a sharing rule that defines how total income  $x$  will be split between them; in the second stage, and once income has been realized, members each choose their private consumption conditional on the ex post budget constraint defined by the sharing rule. If  $\rho(x)$  denotes the amount received by member 1 (so that 2 gets  $x - \rho$ ), ex post utilities are

$$u^1 = \bar{V}^1(\rho(x)), \quad u^2 = \bar{V}^2(x - \rho(x)),$$

---

earnings into permanent and transitory components. Schennach and Hu (2013) rely on characteristic functions to show that the nonparametric classical nonlinear errors-in-variables model is identified except for a few particular parametric families.

<sup>3</sup>See also Cochrane (1991), Hayashi, Altonji, and Kotlikoff (1996), Duflo and Udry (2004), and Chiappori, Samphantharak, Townsend, and Schuhlofer-Wohl (2014) among many others.

<sup>4</sup>See, for instance, Browning, Chiappori, and Weiss (2014).

where  $\bar{V}^i$  denotes  $i$ 's indirect utility. The first stage, ex ante efficient allocations therefore solve

$$\max_{\rho} E(\bar{V}^1(\rho)) + \mu E(\bar{V}^2(x - \rho))$$

for some positive Pareto weight  $\mu$ . First order conditions give the standard characterization

$$\bar{V}^{1'}(\rho(x)) = \mu \bar{V}^{2'}(x - \rho(x)).$$

With CARA preferences,  $\bar{V}^i(t) = -\exp(-\sigma_i t)$  and we have

$$\sigma_1 \exp(-\sigma_1 \rho) = \mu \sigma_2 \exp(-\sigma_2(x - \rho)).$$

Therefore,

$$\rho(x) = -\frac{1}{\sigma_1 + \sigma_2} \ln\left(\frac{\sigma_2 \mu}{\sigma_1}\right) + \frac{\sigma_2}{\sigma_1 + \sigma_2} x. \tag{3}$$

We can now introduce heterogeneity in this framework in two complementary ways. First, the Pareto weight  $\mu$  is the outcome of some ex ante bargaining process. Following a standard, collective approach, we do not explicitly specify that process; however, we assume that there exist some distribution factors  $(z_1, \dots, z_n)$  that may influence the process and, therefore, the Pareto weight  $\mu$ . The key remark here is that some (and possibly most) of these factors may be unobservable. Specifically, assume that  $z_1 = z$  is observable whereas  $(z_2, \dots, z_n)$  are not,<sup>5</sup> and that  $\mu$  is multiplicatively separable,

$$\mu = \mu(z) \gamma(z_2, \dots, z_n), \tag{4}$$

for some function  $\gamma$ . We see that the unobservable distribution factors create unobserved heterogeneity in the Pareto weight, since observationally identical households will typically exhibit different  $\mu$ . As usual, we shall treat  $\gamma(z_2, \dots, z_n)$  as a random term.

Second, we can introduce some unobserved heterogeneity in preferences; specifically, we borrow from the random utility literature by assuming that

$$V^i(t) = \varepsilon_i \bar{V}^i(t) = -\varepsilon_i \exp(-\sigma_i t),$$

where  $\varepsilon_i$  represents a shock affecting  $i$ 's preferences. The first order conditions are now

$$\varepsilon_1 \sigma_1 \exp(-\sigma_1 \rho) = \mu(z) \gamma(z_2, \dots, z_n) \varepsilon_2 \sigma_2 \exp(-\sigma_2(x - \rho)).$$

---

<sup>5</sup>The extension to several observable distribution factors is straightforward and generates additional, testable restrictions; essentially, the ratio of the partial effect of two distribution factors on the demand for commodity  $i$  does not depend on  $i$ . This “factor proportionality” property has been tested empirically in many contexts. See Bourguignon, Browning, and Chiappori (2009) and Browning, Chiappori, and Weiss (2014) for a detailed presentation of different tests, and see Attanasio and Lechene (2014) for a recent application.

Therefore,

$$\rho(x, z) = -\frac{1}{\sigma_1 + \sigma_2} \ln\left(\frac{\sigma_2 \varepsilon_2 \gamma}{\sigma_1 \varepsilon_1}\right) - \frac{1}{\sigma_1 + \sigma_2} \ln \mu(z) + \frac{\sigma_2}{\sigma_1 + \sigma_2} x \quad (5)$$

$$= \bar{\rho}(x, z) + \varepsilon, \quad (6)$$

where

$$\bar{\rho}(x, z) = -\frac{1}{\sigma_1 + \sigma_2} \ln \mu(z) + \frac{\sigma_2}{\sigma_1 + \sigma_2} x,$$

$$\text{and } \varepsilon = -\frac{1}{\sigma_1 + \sigma_2} \ln\left(\frac{\sigma_2 \varepsilon_2 \gamma}{\sigma_1 \varepsilon_1}\right).$$

Finally, individual demands for the various goods solve an ex post utility maximization problem

$$\max U^i(q_{i1}, \dots, q_{iN}) \quad \text{under} \quad \sum_j q_{ij} = \rho_j,$$

where  $\rho_1 = \rho$  and  $\rho_2 = x - \rho$ . The form (2) directly follows.

The previous setting can be generalized in different directions. One may consider a dynamic version, in which, at each period, agents allocate their current income between consumption and savings. This choice is made in the first stage; the second stage remains unchanged, but  $x$  is now interpreted as total expenditures over the period. Second, in the presence of public and private consumption, the two stage representation still holds, but first stage decisions now entail both the choice of public consumptions and the splitting of remaining income between members, to be allocated for private consumptions. Again, the second stage remains unchanged, with two twists: first,  $x$  is now the total amount spent on private consumptions by both members; second, the demands for private goods should now be conditional on the level of public expenditures (instrumented, for instance, by their price), although this conditioning disappears if one is willing to assume that preferences are separable between private and public consumptions.

In this example, the various types of unobserved heterogeneity—random preferences and unobserved distribution factors—are fully summarized by one random factor  $\varepsilon$ . While this property is clearly specific to the particular context, the example still suggests that the case under consideration is not without economic relevance.

### 3. IDENTIFYING THE SHARING RULE

Define conditional expected consumptions in the usual way:

$$Q_1(x, z) = E[q_1 | x, z] = E[\alpha_1(\rho(x, z) + \varepsilon) | x, z],$$

$$Q_2(x, z) = E[q_2 | x, z] = E[\alpha_2(x - \rho(x, z) - \varepsilon) | x, z],$$

and, for  $i \geq 3$ ,

$$\begin{aligned} Q_i(x, z) &= E[q_i | x, z] \\ &= E[\alpha_{i1}(\rho(x, z) + \varepsilon) | x, z] \\ &\quad + E[\alpha_{i2}(x - \rho(x, z) - \varepsilon) | x, z]. \end{aligned}$$

We will make use of the following assumption in our analysis.

**ASSUMPTION 1.** *The function  $Q_i$  is  $C^1$  for  $i = 1, \dots, n$ .*

**ASSUMPTION 2.** *For all  $i$ , the complement of the set of points at which  $\partial Q_i / \partial x$  and  $\partial Q_i / \partial z$  vanish is an open, dense subset.*

Our first result is the following.

**PROPOSITION 1.** *Suppose that Assumptions 1 and 2 hold. Pick any point  $(\bar{x}, \bar{z})$  such that  $\partial Q_i / \partial z(\bar{x}, \bar{z}) \neq 0$  for  $i = 1, 2$  (implying that  $\partial \rho / \partial z(\bar{x}, \bar{z}) \neq 0$ ). Then there exists an open neighborhood  $\mathcal{V}$  of  $(\bar{x}, \bar{z})$  on which the knowledge of  $Q_1$  and  $Q_2$  identifies  $\rho$  up to an additive constant.*

**PROOF.** Note that

$$\begin{aligned} \frac{\partial Q_1}{\partial x} &= \frac{\partial \rho}{\partial x} E[\alpha'_1(\rho(x, z) + \varepsilon) | x, z], \\ \frac{\partial Q_1}{\partial z} &= \frac{\partial \rho}{\partial z} E[\alpha'_1(\rho(x, z) + \varepsilon) | x, z]. \end{aligned}$$

It follows that

$$\frac{\partial Q_1 / \partial x}{\partial Q_1 / \partial z} = \frac{\partial \rho / \partial x}{\partial \rho / \partial z}. \quad (7)$$

By the same token,

$$\begin{aligned} \frac{\partial Q_2}{\partial x} &= \left(1 - \frac{\partial \rho}{\partial x}\right) E[\alpha'_2(x - \rho(x, z) - \varepsilon) | x, z], \\ \frac{\partial Q_2}{\partial z} &= -\frac{\partial \rho}{\partial z} E[\alpha'_2(x - \rho(x, z) - \varepsilon) | x, z], \end{aligned}$$

and

$$\frac{\partial Q_2 / \partial x}{\partial Q_2 / \partial z} = -\frac{1 - \partial \rho / \partial x}{\partial \rho / \partial z}. \quad (8)$$

The two equalities (7) and (8) imply that

$$\partial \rho / \partial z = \frac{1}{\frac{\partial Q_1 / \partial x}{\partial Q_1 / \partial z} - \frac{\partial Q_2 / \partial x}{\partial Q_2 / \partial z}},$$

and that

$$\frac{\partial \rho}{\partial x} = \frac{\frac{\partial Q_1 / \partial x}{\partial Q_1 / \partial z}}{\frac{\partial Q_1 / \partial x}{\partial Q_1 / \partial z} - \frac{\partial Q_2 / \partial x}{\partial Q_2 / \partial z}}.$$

Finally,

$$\frac{\partial \rho / \partial z}{\partial Q_1 / \partial z} = \frac{\partial \rho / \partial x}{\partial Q_1 / \partial x} = \frac{1}{\frac{\partial Q_1 / \partial x}{\partial Q_1 / \partial z} - \frac{\partial Q_2 / \partial x}{\partial Q_2 / \partial z}},$$

and  $\rho$  is identified up to an additive constant. In addition,  $Q_1$  and  $Q_2$  must satisfy the overidentifying restriction

$$\frac{\partial}{\partial x} \left( \frac{1}{\frac{\partial Q_1 / \partial x}{\partial Q_1 / \partial z} - \frac{\partial Q_2 / \partial x}{\partial Q_2 / \partial z}} \right) = \frac{\partial}{\partial z} \left( \frac{\frac{\partial Q_1 / \partial x}{\partial Q_1 / \partial z}}{\frac{\partial Q_1 / \partial x}{\partial Q_1 / \partial z} - \frac{\partial Q_2 / \partial x}{\partial Q_2 / \partial z}} \right),$$

which gives a partial differential equation in  $Q_1$  and  $Q_2$ . □

Note that identification obtains from the observation of only two demands, corresponding to the two exclusive goods. Other demands generate additional overidentifying restrictions, as stated by the following result.

**PROPOSITION 2.** *Suppose that Assumptions 1 and 2 hold. Then for some open neighborhood  $\mathcal{V}$  of  $(\bar{x}, \bar{z})$  such that  $\partial Q_i / \partial z(\bar{x}, \bar{z}) \neq 0$  for all  $i$ , there exists a set of overidentifying restrictions, which take the form of a system of partial differential equations (PDEs) that must be satisfied by the  $Q_i$ s for  $i \geq 3$  as follows:*

$$\begin{aligned} \frac{\partial}{\partial x} \left( \frac{\partial Q_i}{\partial x} - \frac{\partial Q_2 / \partial x}{\partial Q_2 / \partial z} \frac{\partial Q_i}{\partial z} \right) &= \frac{\partial Q_1 / \partial x}{\partial Q_1 / \partial z} \frac{\partial}{\partial z} \left( \frac{\partial Q_i}{\partial x} - \frac{\partial Q_2 / \partial x}{\partial Q_2 / \partial z} \frac{\partial Q_i}{\partial z} \right) \quad \text{and} \\ \frac{\partial}{\partial x} \left( \frac{\partial Q_i}{\partial x} - \frac{\partial Q_1 / \partial x}{\partial Q_1 / \partial z} \frac{\partial Q_i}{\partial z} \right) &= \frac{\partial Q_2 / \partial x}{\partial Q_2 / \partial z} \frac{\partial}{\partial z} \left( \frac{\partial Q_i}{\partial x} - \frac{\partial Q_1 / \partial x}{\partial Q_1 / \partial z} \frac{\partial Q_i}{\partial z} \right). \end{aligned}$$

For the proof, see the [Appendix](#).

Finally, it should be stressed that, in line with the entire collective literature, the identification of the sharing rule obtains only up to an additive constant. As is well known, the constant is welfare irrelevant; therefore, if one is interested in welfare-relevant issues, the constant may with no loss of generality be normalized to be zero. We adopt this normalization in what follows and, therefore, assume that  $\rho$  is a known function of  $(x, z)$ . Note, however, that there are issues (e.g., measures of intrahousehold inequality in consumptions) for which the constant would be relevant.

An important remark, at that point, is that the identification result just derived crucially depends on the additive separability of the error term  $\varepsilon$  in the sharing rule. Indeed, this assumption implies that the ratio  $\frac{\partial \rho / \partial x}{\partial \rho / \partial z}$  does *not* depend on the realization



of  $\varepsilon$ , which leads to a simple identification argument whereby  $\rho$  is defined by two non-stochastic PDEs. Whether this result extends to more general forms—and in particular to a nonseparable structure of the type  $\rho(x, z, \varepsilon)$ —is an open (and difficult) question.

#### 4. IDENTIFYING THE $\alpha$ S AND THE DISTRIBUTIONS

We now consider the second problem, namely the identification of individual Engel curves and the distributions of the shocks. We will need the following assumptions.

**ASSUMPTION 3.** *The random shocks  $\varepsilon, \eta_1, \dots, \eta_n$  are mutually independent, independent of expenditures and distribution factors, and  $E[\eta_k] = 0$  for  $k = 1, \dots, n$ .*

**ASSUMPTION 4.** *The term  $E[\exp\{is\eta_k\}]$  does not vanish for any  $s \in \mathbb{R}$  and  $k = 1, \dots, n$ , where  $i = \sqrt{-1}$ .*

**ASSUMPTION 5.** *The marginal distributions of  $\varepsilon$  and  $\eta_k$  for  $k = 1, \dots, n$ , admit strictly positive, finite, and differentiable density functions  $f_\varepsilon$  and  $f_{\eta_k}$ , respectively, with respect to Lebesgue measure on  $\mathbb{R}$ .*

**ASSUMPTION 6.** *The functions  $\alpha_1(\cdot)$  and  $\alpha_2(\cdot)$  are strictly increasing and three times differentiable. Their first derivatives are finite and nonvanishing.*

##### *The linear case*

We start with a very particular case, namely, linearity. One can readily see that, in that case, full identification cannot obtain. Assume, for instance, that  $\alpha_1$  and  $\alpha_2$  are linear:

$$\alpha_i(t) = m_i t + n_i.$$

The two equations become

$$\begin{aligned} q_1 &= m_1 \rho(x, z) + m_1 \varepsilon + n_1 + \eta_1, \\ q_2 &= m_2 x - m_2 \rho(x, z) - m_2 \varepsilon + n_2 + \eta_2. \end{aligned}$$

In that case, the constants  $m_1$  and  $m_2$  are identified from the knowledge of  $\rho$ . However, there is no hope to recover the distributions of  $\varepsilon$ ,  $\eta_1$ , and  $\eta_2$ .

However, the linear case is highly peculiar, and linear models tend to be systematically rejected by the data; in practice, linear demands are no longer used in the empirical literature on demand systems, and were actually never used in the estimation of collective models. Actual estimations mostly use either the almost ideal demand system (AIDS), introduced by Deaton and Muellbauer (1980), in which budget shares are linear in log income (the so-called price-independent generalized logarithmic (PIGLOG) form, so that, in our notation,  $q_i$  would be a function of  $x$  and  $x \ln x$ ), or the quadratic extension proposed by Banks, Blundell, and Lewbel (1997), which adds a term in  $x(\ln x)^2$ . For instance, Browning and Chiappori (1998), Browning, Chiappori, and Lewbel (2013), and

Attanasio and Lechene (2014) use a quadratic almost ideal demand system (QUAIDS) form, whereas Cherchye, De Rock, and Vermeulen (2012) and Dunbar, Lewbel, and Pendakur (2013) refer to an AIDS framework.<sup>6</sup>

### *The general case*

We now consider the general case. The main result, then, is the following.

**PROPOSITION 3.** *Suppose that Assumptions 1–6 hold and that there exist four  $C^2$  functions  $(\alpha_1, \alpha_2, \bar{\alpha}_1, \bar{\alpha}_2)$  and six random variables  $(\varepsilon, \eta_1, \eta_2, \bar{\varepsilon}, \bar{\eta}_1, \bar{\eta}_2)$  such that the random variables  $(q_1, q_2)$  and  $(\bar{q}_1, \bar{q}_2)$ , where*

$$\begin{aligned} q_1(x, z) &= \alpha_1(\rho(x, z) + \varepsilon) + \eta_1, \\ q_2(x, z) &= \alpha_2(x - \rho(x, z) - \varepsilon) + \eta_2, \end{aligned} \tag{9}$$

and

$$\begin{aligned} \bar{q}_1(x, z) &= \bar{\alpha}_1(\rho(x, z) + \bar{\varepsilon}) + \bar{\eta}_1, \\ \bar{q}_2(x, z) &= \bar{\alpha}_2(x - \rho(x, z) - \bar{\varepsilon}) + \bar{\eta}_2, \end{aligned} \tag{10}$$

have the same distribution for all  $(x, z) \in \mathcal{V}$ , where  $\mathcal{V}$  is some open neighborhood of  $(\bar{x}, \bar{z})$  such that  $\partial Q_i / \partial z(\bar{x}, \bar{z}) \neq 0$  for all  $i$ . Then either  $\alpha_1 = \bar{\alpha}_1$  and  $\alpha_2 = \bar{\alpha}_2$  or  $\alpha_1, \alpha_2, \bar{\alpha}_1$  and  $\alpha_2$  are linear.

The detailed proof is given in the [Appendix](#). Here we provide a sketch of the proof. Note, first, that the result is local; as such, the only assumption needed on the support of the conditioning variables is that it contains the neighborhood under consideration.

Our proof consists of two steps. In the first step, we show that our identification problem can reduce to considering two observationally equivalent systems, one having no  $\eta_1$  and the other having no  $\eta_2$ . We then in the second step derive a sufficient condition for identification of the reduced models. Our result and proof in the first step are similar to [Schennach and Hu \(2013\)](#), while those in our second step are different. Note that our model is a *system* consisting of two equations of  $q_1$  and  $q_2$  each of which involves an unknown Engel curve, while [Schennach and Hu \(2013\)](#) consider a single equation. Technically, our second step formulates differential equations from the joint distribution function of the household demand using change of variables. We find that our sufficient condition for identification, which requires nonlinearity of individual Engel curves, is weaker than theirs due to the more intricate structure of the collective model than the errors-in-variables models in [Schennach and Hu \(2013\)](#).

<sup>6</sup>Other nonlinear forms can, however, be found in the literature. For instance, [Browning et al. \(1994\)](#) use a log-quadratic demand function. See [Browning, Chiappori, and Weiss \(2014\)](#) for an overview.

## 5. CONCLUSION

Much work remains to be done. Our key assumptions for identification, including additivity and mutual independence of unobserved heterogeneity terms, are restrictive. It would be important for wider applicability to relax these assumptions or to develop an alternative framework that does not rely on these assumptions. One possibility is using a random coefficient model to capture the general heterogeneity. An interesting approach would be to extend the result of [Lewbel and Pendakur \(2015\)](#) to collective models, which consider generalized random coefficient models for unitary models. We leave this work for future research.

## APPENDIX

**PROOF OF PROPOSITION 2.** Consider  $i \geq 3$ . From

$$Q_i(x, z) = E[\alpha_{i1}(\rho(x, z) + \varepsilon) | x, z] + E[\alpha_{i2}(x - \rho(x, z) - \varepsilon) | x, z],$$

we get

$$\begin{aligned} \frac{\partial Q_i}{\partial x} &= \frac{\partial \rho}{\partial x} E[\alpha'_{i1}(\rho(x, z) + \varepsilon) | x, z] + \left(1 - \frac{\partial \rho}{\partial x}\right) E[\alpha'_{i2}(x - \rho(x, z) - \varepsilon) | x, z], \\ \frac{\partial Q_i}{\partial z} &= \frac{\partial \rho}{\partial z} E[\alpha'_{i1}(\rho(x, z) + \varepsilon) | x, z] - \frac{\partial \rho}{\partial z} E[\alpha'_{i2}(x - \rho(x, z) - \varepsilon) | x, z]. \end{aligned}$$

Denote

$$\begin{aligned} a_{i1}(x, z) &= E[\alpha'_{i1}(\rho(x, z) + \varepsilon) | x, z], \\ a_{i2}(x, z) &= E[\alpha'_{i2}(x - \rho(x, z) - \varepsilon) | x, z]. \end{aligned} \tag{11}$$

Then

$$\begin{aligned} a_{i1}(x, z) &= \frac{\partial Q_i}{\partial x} - \frac{\partial Q_2 / \partial x}{\partial Q_2 / \partial z} \frac{\partial Q_i}{\partial z}, \\ a_{i2}(x, z) &= \frac{\partial Q_i}{\partial x} - \frac{\partial Q_1 / \partial x}{\partial Q_1 / \partial z} \frac{\partial Q_i}{\partial z}. \end{aligned}$$

Since the gradient of  $a_{i1}(x, z)$  should be collinear to that of  $\rho$  from (11), by (7) and (8),

$$\frac{\partial}{\partial x} \left( \frac{\partial Q_i}{\partial x} - \frac{\partial Q_2 / \partial x}{\partial Q_2 / \partial z} \frac{\partial Q_i}{\partial z} \right) = \frac{\partial Q_1 / \partial x}{\partial Q_1 / \partial z} \frac{\partial}{\partial z} \left( \frac{\partial Q_i}{\partial x} - \frac{\partial Q_2 / \partial x}{\partial Q_2 / \partial z} \frac{\partial Q_i}{\partial z} \right),$$

and by the same token,

$$\frac{\partial}{\partial x} \left( \frac{\partial Q_i}{\partial x} - \frac{\partial Q_1 / \partial x}{\partial Q_1 / \partial z} \frac{\partial Q_i}{\partial z} \right) = \frac{\partial Q_2 / \partial x}{\partial Q_2 / \partial z} \frac{\partial}{\partial z} \left( \frac{\partial Q_i}{\partial x} - \frac{\partial Q_1 / \partial x}{\partial Q_1 / \partial z} \frac{\partial Q_i}{\partial z} \right). \quad \square$$

**PROOF OF PROPOSITION 3.** The proof is in two stages.

STAGE 1. Note, first, that since  $\rho$  is known, we change variables and consider  $(\rho, y)$  instead of  $(x, z)$ , where  $y = x - \rho$ .

The first stage is similar to [Schennach and Hu \(2013\)](#). Consider the four models

M1:

$$q_1 = \alpha_1(\rho + \varepsilon) + \eta_1,$$

$$q_2 = \alpha_2(y - \varepsilon) + \eta_2,$$

M2:

$$q_1 = \bar{\alpha}_1(\rho + \bar{\varepsilon}) + \bar{\eta}_1,$$

$$q_2 = \bar{\alpha}_2(y - \bar{\varepsilon}) + \bar{\eta}_2,$$

and

M3:

$$\bar{q}_1 = \alpha_1(\rho + \varepsilon) + \tilde{\eta}_1,$$

$$\bar{q}_2 = \alpha_2(y - \varepsilon),$$

M4:

$$\bar{q}_1 = \bar{\alpha}_1(\rho + \bar{\varepsilon}),$$

$$\bar{q}_2 = \bar{\alpha}_2(y - \bar{\varepsilon}) + \tilde{\eta}_2,$$

where all random variables are mutually independent.

LEMMA 1. *There exist two distinct observationally equivalent models M1 and M2 if and only if there exist two distinct observationally equivalent models M3 and M4.*

PROOF. As in [Schennach and Hu \(2013\)](#), the joint characteristic functions  $\Phi_{(q_1, q_2)}(s_1, s_2)$  of  $q_1$  and  $q_2$  are written in M1 and M2 as follows: under M1,

$$\begin{aligned} \Phi_{(q_1, q_2)}(s_1, s_2) &= E[e^{i(s_1(\alpha_1(\rho + \varepsilon) + \eta_1) + s_2(\alpha_2(y - \varepsilon) + \eta_2))}] \\ &= \Phi_{\eta_1}(s_1)\Phi_{\eta_2}(s_2)E[e^{is_1\alpha_1(\rho + \varepsilon)}e^{is_2\alpha_2(y - \varepsilon)}], \end{aligned}$$

while under M2,

$$\Phi_{(q_1, q_2)}(s_1, s_2) = \Phi_{\bar{\eta}_1}(s_1)\Phi_{\bar{\eta}_2}(s_2)E[e^{is_1\bar{\alpha}_1(\rho + \bar{\varepsilon})}e^{is_2\bar{\alpha}_2(y - \bar{\varepsilon})}].$$

For observationally equivalent M1 and M2,

$$\Phi_{\eta_1}(s_1)\Phi_{\eta_2}(s_2)E[e^{is_1\alpha_1(\rho + \varepsilon)}e^{is_2\alpha_2(y - \varepsilon)}] = \Phi_{\bar{\eta}_1}(s_1)\Phi_{\bar{\eta}_2}(s_2)E[e^{is_1\bar{\alpha}_1(\rho + \bar{\varepsilon})}e^{is_2\bar{\alpha}_2(y - \bar{\varepsilon})}],$$

and so

$$\frac{\Phi_{\eta_1}(s_1)}{\Phi_{\bar{\eta}_1}(s_1)}E[e^{is_1\alpha_1(\rho + \varepsilon)}e^{is_2\alpha_2(y - \varepsilon)}] = \frac{\Phi_{\bar{\eta}_2}(s_2)}{\Phi_{\eta_2}(s_2)}E[e^{is_1\bar{\alpha}_1(\rho + \bar{\varepsilon})}e^{is_2\bar{\alpha}_2(y - \bar{\varepsilon})}].$$

Take  $\frac{\Phi_{\eta_1}(s_1)}{\Phi_{\eta_1}(s_1)}$  and  $\frac{\Phi_{\tilde{\eta}_2}(s_2)}{\Phi_{\eta_2}(s_2)}$  to be the characteristic functions of  $\tilde{\eta}_1$  in M3 and of  $\tilde{\eta}_2$  in M4, respectively; therefore, the conclusion.  $\square$

STAGE 2. We now show that if  $\alpha_1 \neq \bar{\alpha}_1$  and  $\alpha_2 \neq \bar{\alpha}_2$ , M3 and M4 cannot be observationally equivalent unless the  $\alpha$ s are linear.

Noting that the joint distribution of  $\bar{q}_1$  and  $\bar{q}_2$  is the same under M3 and M4, for any  $(t_1, t_2) \in \mathbb{R}^2$ , start with

$$\begin{aligned} G(t_1, t_2, y, \rho) &= \Pr[\alpha_1(\rho + \varepsilon) + \tilde{\eta}_1 \leq t_1, \alpha_2(y - \varepsilon) \leq t_2] \\ &= \Pr[\bar{\alpha}_1(\rho + \bar{\varepsilon}) \leq t_1, \bar{\alpha}_2(y - \bar{\varepsilon}) + \tilde{\eta}_2 \leq t_2]. \end{aligned}$$

Here the function  $G$  is not necessarily observed. Let  $a_i$  be the inverse of  $\alpha_i$ . We first have that

$$\begin{aligned} G(t_1, t_2, y, \rho) &= \Pr[\alpha_1(\rho + \varepsilon) + \tilde{\eta}_1 \leq t_1, y - \varepsilon \leq a_2(t_2)] \\ &= \Pr[\alpha_1(\rho + \varepsilon) + \tilde{\eta}_1 \leq t_1, y - a_2(t_2) \leq \varepsilon] \\ &= \int_{y-a_2(t_2)}^{+\infty} F_{\tilde{\eta}_1}(t_1 - \alpha_1(\rho + \varepsilon)) f_\varepsilon(\varepsilon) d\varepsilon \end{aligned}$$

and, in particular,

$$\frac{\partial G(t_1, t_2, y, \rho)}{\partial y} = -F_{\tilde{\eta}_1}(t_1 - \alpha_1(\rho + y - a_2(t_2))) f_\varepsilon(y - a_2(t_2)).$$

Also

$$\begin{aligned} G(t_1, t_2, y, \rho) &= \Pr[\bar{\alpha}_1(\rho + \bar{\varepsilon}) \leq t_1, \bar{\alpha}_2(y - \bar{\varepsilon}) + \tilde{\eta}_2 \leq t_2] \\ &= \Pr[\bar{\varepsilon} \leq \bar{a}_1(t_1) - \rho, \tilde{\eta}_2 \leq t_2 - \bar{\alpha}_2(y - \bar{\varepsilon})] \\ &= \int_{-\infty}^{\bar{a}_1(t_1) - \rho} F_{\tilde{\eta}_2}(t_2 - \bar{\alpha}_2(y - \bar{\varepsilon})) f_{\bar{\varepsilon}}(\bar{\varepsilon}) d\bar{\varepsilon} \end{aligned}$$

and, in particular,

$$\frac{\partial G(t_1, t_2, y, \rho)}{\partial \rho} = -F_{\tilde{\eta}_2}(t_2 - \bar{\alpha}_2(y - \bar{a}_1(t_1) + \rho)) f_{\bar{\varepsilon}}(\bar{a}_1(t_1) - \rho).$$

Therefore,

$$\begin{aligned} \frac{\partial^2 G(t_1, t_2, y, \rho)}{\partial y \partial \rho} &= \alpha'_1(\rho + y - a_2(t_2)) f_{\tilde{\eta}_1}(t_1 - \alpha_1(\rho + y - a_2(t_2))) f_\varepsilon(y - a_2(t_2)) \\ &= \bar{\alpha}'_2(y - \bar{a}_1(t_1) + \rho) f_{\tilde{\eta}_2}(t_2 - \bar{\alpha}_2(y - \bar{a}_1(t_1) + \rho)) f_{\bar{\varepsilon}}(\bar{a}_1(t_1) - \rho), \end{aligned} \tag{12}$$

where the first expression depends on  $y - a_2(t_2)$  and the second depends on  $\rho - \bar{a}_1(t_1)$ .

Define

$$A(t_1, t_2, y, \rho) = \frac{\partial^2 G(t_1, t_2, y, \rho)}{\partial y \partial \rho}.$$

Then from (12), there exist functions  $B$  and  $C$  such that (s.t.)

$$A(t_1, t_2, y, \rho) = B(t_1, y - a_2(t_2), \rho)$$

and

$$A(t_1, t_2, y, \rho) = C(\bar{a}_1(t_1) - \rho, y, t_2).$$

Therefore, there exist a function  $D$  s.t.

$$\begin{aligned} A(t_1, t_2, y, \rho) &= D(\bar{a}_1(t_1) - \rho, y - a_2(t_2)) \\ &= D(T, Y), \end{aligned}$$

where

$$\begin{aligned} T &= \bar{a}_1(t_1) - \rho, \\ Y &= y - a_2(t_2). \end{aligned}$$

Also, note that

$$a_2(t_2) = y - Y \quad \Rightarrow \quad t_2 = \alpha_2(y - Y),$$

so that

$$\begin{aligned} D(T, Y) &= \bar{\alpha}'_2(y - T) f_{\tilde{\eta}_2}(t_2 - \bar{\alpha}_2(y - T)) f_{\bar{e}}(T) \\ &= \bar{\alpha}'_2(y - T) f_{\tilde{\eta}_2}(\alpha_2(y - Y) - \bar{\alpha}_2(y - T)) f_{\bar{e}}(T). \end{aligned}$$

If we consider the change in variable

$$(t_1, t_2, y, \rho) \rightarrow (Y, T, y, \rho),$$

then the function  $D$  only depends on  $(Y, T)$ ,

$$\frac{\partial D(T, Y)}{\partial y} = 0 \quad \Rightarrow \quad \frac{\partial (\bar{\alpha}'_2(y - T) f_{\tilde{\eta}_2}(\alpha_2(y - Y) - \bar{\alpha}_2(y - T)))}{dy} = 0,$$

or

$$\begin{aligned} 0 &= \bar{\alpha}''_2(y - T) f_{\tilde{\eta}_2}(\alpha_2(y - Y) - \bar{\alpha}_2(y - T)) \\ &\quad + \bar{\alpha}'_2(y - T) (\alpha'_2(y - Y) - \bar{\alpha}'_2(y - T)) f'_{\tilde{\eta}_2}(\alpha_2(y - Y) - \bar{\alpha}_2(y - T)). \end{aligned}$$

At any point where  $f_{\tilde{\eta}_2}$  does not vanish,

$$\frac{f'_{\tilde{\eta}_2}(\alpha_2(y - Y) - \bar{\alpha}_2(y - T))}{f_{\tilde{\eta}_2}(\alpha_2(y - Y) - \bar{\alpha}_2(y - T))} = - \frac{\bar{\alpha}''_2(y - T)}{\bar{\alpha}'_2(y - T)} \frac{1}{\alpha'_2(y - Y) - \bar{\alpha}'_2(y - T)}$$

or

$$\frac{f'_{\tilde{\eta}_2}(\alpha_2(u) - \bar{\alpha}_2(v))}{f_{\tilde{\eta}_2}(\alpha_2(u) - \bar{\alpha}_2(v))} = -\frac{\bar{\alpha}_2''(v)}{\bar{\alpha}_2'(v)} \frac{1}{\alpha_2'(u) - \bar{\alpha}_2'(v)}, \quad (13)$$

where

$$\begin{aligned} u &= y - Y = a_2(t_2), \\ v &= y - T = y - (\bar{a}_1(t_1) - \rho). \end{aligned}$$

Define

$$\phi(\cdot) = \frac{f'_{\tilde{\eta}_2}(\cdot)}{f_{\tilde{\eta}_2}(\cdot)}.$$

Then

$$(\alpha_2'(u) - \bar{\alpha}_2'(v))\phi(\alpha_2(u) - \bar{\alpha}_2(v)) = -\frac{\bar{\alpha}_2''(v)}{\bar{\alpha}_2'(v)}.$$

Differentiating in  $v$  yields

$$\alpha_2'(u)\bar{\alpha}_2'(v)\phi' - [\bar{\alpha}_2'(v)]^2\phi' + \bar{\alpha}_2''(v)\phi = \frac{d}{dv} \left( \frac{\bar{\alpha}_2''(v)}{\bar{\alpha}_2'(v)} \right),$$

and we can eliminate  $\alpha_2'(u)$  between these equations,

$$\phi^2 = \phi' + \phi \frac{1}{\bar{\alpha}_2''(v)} \frac{d}{dv} \left( \frac{\bar{\alpha}_2''(v)}{\bar{\alpha}_2'(v)} \right), \quad (14)$$

and  $\frac{1}{\bar{\alpha}_2''(v)} \frac{d}{dv} \left( \frac{\bar{\alpha}_2''(v)}{\bar{\alpha}_2'(v)} \right)$  cannot depend on  $v$ . Thus,

$$\frac{d}{dv} \left( \frac{\bar{\alpha}_2''(v)}{\bar{\alpha}_2'(v)} \right) = K\bar{\alpha}_2''(v), \quad \text{which gives}$$

$$\frac{\bar{\alpha}_2''(v)}{\bar{\alpha}_2'(v)} = K\bar{\alpha}_2'(v) + L$$

for some constants  $K$  and  $L$ . This ordinary differential equation has two types of solutions. One is that  $\bar{\alpha}_2'$  is constant,

$$\bar{\alpha}_2'(v) = -\frac{L}{K} \Rightarrow \bar{\alpha}_2(v) = -\frac{L}{K}v + K',$$

where  $K'$  is an integration constant, and thus  $\bar{\alpha}_2$  is linear.

The second is such that

$$\bar{\alpha}_2'(v) = L \frac{e^{Lv-CL}}{K - Ke^{Lv-CL}},$$

where  $C$  is an integration constant; finally,  $\bar{\alpha}_2(v)$  must be of the form

$$\bar{\alpha}_2(v) = \frac{1}{k} \log(1 - le^{Lv}) + k' \quad (15)$$

for some parameters  $k$ ,  $l$ ,  $L$ , and  $k'$ .

Now, if the  $\alpha$ s are linear, M3 and M4 are obviously observationally equivalent:

$$\bar{q}_1 = \alpha_1 \rho + \alpha_1 \varepsilon + \tilde{\eta}_1,$$

$$\bar{q}_2 = \alpha_2 y - \alpha_2 \varepsilon,$$

and

$$\bar{q}_1 = \bar{\alpha}_1 \rho + \bar{\alpha}_1 \bar{\varepsilon},$$

$$\bar{q}_2 = \bar{\alpha}_2 y - \bar{\alpha}_2 \bar{\varepsilon} + \tilde{\eta}_2.$$

Check the second case. Under (15),

$$\frac{1}{\bar{\alpha}_2'(v)} \frac{d}{dv} \left( \frac{\bar{\alpha}_2''(v)}{\bar{\alpha}_2'(v)} \right) = -k,$$

and (14) becomes

$$\phi^2 = \phi' - k\phi,$$

which gives either  $\phi = 0$  or

$$\phi(X) = \frac{k}{Ce^{-kX} - 1},$$

where  $C$  is an integration constant. Then

$$\frac{f'_{\tilde{\eta}_2}(X)}{f_{\tilde{\eta}_2}(X)} = \frac{k}{Ce^{-kX} - 1}$$

defines  $f_{\tilde{\eta}_2}$  up to two integration constants. Finally, (13) gives

$$\frac{k}{Ce^{-k(\alpha_2(u) - (1/k) \log(1 - le^{Lv}) - k')} - 1} = \frac{L}{e^{Lv - CL} - 1} \frac{1}{\alpha_2'(u) - L \frac{e^{Lv - CL}}{K - Ke^{Lv - CL}}}$$

or

$$\alpha_2'(u) = L \frac{e^{Lv - CL}}{K - Ke^{Lv - CL}} + \frac{L}{e^{Lv - CL} - 1} \frac{1}{k} (Ce^{-k(\alpha_2(u) - (1/k) \log(1 - le^{Lv}) - k')} - 1).$$

Differentiating in  $v$ ,

$$0 = d \left( L \frac{e^{Lv - CL}}{K - Ke^{Lv - CL}} + \frac{L}{e^{Lv - CL} - 1} \frac{1}{k} \left( C \exp \left( -k \left( \alpha - \frac{1}{k} \log(1 - le^{Lv}) - k' \right) \right) - 1 \right) \right) / dv,$$



gives

$$\frac{KL^2e^{-CL} + L^2ke^{-CL}}{CKL^2e^{kk'}(e^{-CL} - 1)} = e^{-k\alpha_2(u)},$$

implying that  $\alpha_2(u)$  is constant, a contradiction.

We conclude that M3 and M4 cannot be observationally equivalent unless the  $\alpha$ s are linear.  $\square$

#### REFERENCES

- Altonji, J. G. and R. L. Matzkin (2005), “Cross section and panel data estimators for non-separable models with endogenous regressors.” *Econometrica*, 73, 1053–1102. [204]
- Arellano, M. and S. Bonhomme (2012), “Identifying distributional characteristics in random coefficients panel data models.” *Review of Economic Studies*, 79, 987–1020. [203]
- Attanasio, O. and V. Lechene (2014), “Efficient responses to targeted cash transfers.” *Journal of Political Economy*, 122 (1), 178–222. [201, 202, 203, 205, 210]
- Banks, J., R. Blundell, and A. Lewbel (1997), “Quadratic Engel curves and consumer demand.” *The Review of Economics and Statistics*, 79 (4), 527–539. [209]
- Bonhomme, S. and J.-M. Robin (2010), “Generalized nonparametric deconvolution with an application to earnings dynamics.” *Review of Economic Studies*, 77, 491–533. [203]
- Bourguignon, F., M. Browning, and P.-A. Chiappori (2009), “Efficient intra-household allocations and distribution factors: Implications and identification.” *Review of Economic Studies*, 76, 503–528. [201, 205]
- Browning, M., F. Bourguignon, P.-A. Chiappori, and V. Lechene (1994), “Income and outcomes: A structural model of intra household allocation.” *Journal of Political Economy*, 102, 1067–1096. [201, 202, 203, 210]
- Browning, M. and P.-A. Chiappori (1998), “Efficient intra-household allocations: A general characterization and empirical tests.” *Econometrica*, 66, 1241–1278. [209]
- Browning, M., P.-A. Chiappori, and A. Lewbel (2013), “Estimating consumption economies of scale, adult equivalence scales, and household bargaining power.” *Review of Economic Studies*, 80, 1267–1303. [209]
- Browning, M., P.-A. Chiappori, and Y. Weiss (2014), *Economics of the Family*. Cambridge University Press, Cambridge. [204, 205, 210]
- Cherchye, L., B. De Rock, and F. Vermeulen (2012), “Married with children: A collective labor supply model with detailed time use and intrahousehold expenditure information.” *American Economic Review*, 102 (7), 3377–3405. [210]
- Chiappori, P.-A. and I. Ekeland (2009), “The microeconomics of efficient group behavior: Identification.” *Econometrica*, 77, 763–799. [201]

- Chiappori, P.-A., I. Komunjer, and D. Kristensen (2014), "Nonparametric identification and estimation of transformation models." *Journal of Econometrics*, 188, 22–39. [204]
- Chiappori, P.-A., K. Samphantharak, R. Townsend, and S. Schuhlofer-Wohl (2014), "Heterogeneity and risk sharing in village economies." *Quantitative Economics*, 5, 1–27. [204]
- Cochrane, J. H. (1991), "A simple test of consumption insurance." *Journal of Political Economy*, 99 (5), 957–976. [204]
- Deaton, A. and J. Muellbauer (1980), *Economics and Consumer Behavior*. Cambridge University Press, Cambridge. [209]
- Duflo, E. and C. Udry (2004), "Intrahousehold resource allocation in Cote d'Ivoire: Social norms, separate accounts and consumption choices." NBER Working Paper No. 10498. [204]
- Dunbar, G. R., A. Lewbel, and K. Pendakur (2013), "Children's resources in collective households: Identification, estimation, and an application to child poverty in Malawi." *American Economic Review*, 103 (1), 438–471. [201, 210]
- Evdokimov, K. (2010), "Identification and estimation of a nonparametric panel data model with unobserved heterogeneity." Working Paper. [203]
- Evdokimov, K. and H. White (2012), "Some extensions of a lemma of Kotlarski." *Econometric Theory*, 28, 925–932. [203]
- Hayashi, F., J. Altonji, and L. Kotlikoff (1996), "Risk-sharing between and within families." *Econometrica*, 64 (2), 261–294. [204]
- Lewbel, A. and K. Pendakur (2015), "Unobserved preference heterogeneity in demand using generalized random coefficients." Working Paper. [203, 211]
- Matzkin, R. L. and J. Perez-Estrada (2011), "Heterogeneous collective demand: Exploring differential generosity within elderly couples." Working Paper. [203]
- Schennach, S. M. and Y. Hu (2013), "Nonparametric identification and semiparametric estimation of classical measurement error models without side information." *Journal of the American Statistical Association*, 108, 177–186. [203, 204, 210, 212]
- Townsend, R. (1994), "Risk and insurance in village India." *Econometrica*, 62, 539–591. [204]

---

Co-editor Rosa Matzkin handled this manuscript.

Manuscript received 9 December, 2013; final version accepted 8 February, 2016; available online 20 March, 2017.