

Consumption insurance with advance information

CHRISTIAN A. STOLTENBERG

Amsterdam School of Economics, University of Amsterdam and Tinbergen Institute

SWAPNIL SINGH

Center for Excellence in Finance and Economic Research (CEFER), Bank of Lithuania and School of Economics and Business, Kaunas University of Technology

This paper investigates whether assuming that households possess advance information on their income shocks helps to overcome the difficulty of standard models to understand consumption insurance in the US. As our main result, we find that the quantitative relevance of advance information crucially depends on the structure of insurance markets. For a realistic amount of advance information, a complete markets model with endogenous solvency constraints due to limited commitment explains several key consumption insurance measures better than existing models without advance information. In contrast, when advance information is integrated into a standard incomplete markets model, it affects household consumption-saving decisions too little to bridge the gap between the model and the data and can induce counterfactual correlations between current consumption growth and future income growth.

KEYWORDS. Advance information, consumption insurance, subjective expectations, endogenous borrowing constraints, limited commitment.

JEL CLASSIFICATION. D31, D52, E21.

1. INTRODUCTION

Krueger and Perri (2006) and, more recently, Broer (2013), documented that, for standard calibrations existing consumption-savings models have difficulty capturing consumption insurance of households in the United States. While standard incomplete markets models with exogenous borrowing constraints, as pioneered by Aiyagari (1994), tend to predict too little, complete markets models with endogenous solvency constraints due to limited commitment, as proposed by Alvarez and Jermann (2000), tend to yield too much consumption insurance. In this paper, we ask whether assuming that

Christian A. Stoltenberg: c.a.stoltenberg@uva.nl

Swapnil Singh: ssingh@lb.lt

We are thankful to Simon Broda, Tobias Broer, Bjoern Bruegemann, Wouter Den Haan, Jonathan Heathcote, Luigi Guiso, Patrick J. Kehoe, Sebastian Koehne, Moritz Kuhn, Tim Lee, Bartosz Maćkowiak, Plamen Nenov, Serdar Ozkan, Marcelo Pedroni, Elisabeth Proehl, Kathrin Schlaffmann, Petr Sedlacek, Vincent Sterk, Kjetil Storesletten, Arne Uhlenborff, and Sevin Yeltekin for helpful comments and discussions. Previous versions circulated under the title “Accounting for households’ perceived income uncertainty in consumption risk sharing.”

© 2020 The Authors. Licensed under the [Creative Commons Attribution-NonCommercial License 4.0](https://creativecommons.org/licenses/by-nc/4.0/). Available at <http://qeconomics.org>. <https://doi.org/10.3982/QE1169>

households know more than econometricians because they have advance information on their income shocks helps to bridge the gap between models and data.

Integrating advance information into these different insurance market structures is promising because advance information has the potential to improve the fit of both models. While advance information enhances consumption insurance with incomplete markets, we show it worsens consumption insurance with complete markets. As our main result, we find that not only the qualitative but also the quantitative relevance of advance information for consumption insurance crucially depends on the structure of insurance markets. With complete markets, advance information plays a major role in explaining consumption insurance. If markets are incomplete, however, the quantitative improvement in insurance is quantitatively not strong enough and advance information can even induce counterfactual correlations between current consumption growth and future income growth.

We consider an environment in which risk-averse households seek insurance against idiosyncratic fluctuations of their disposable income. As the new element here, we explicitly extend households' information set with signals that inform households about their income in the next period with a certain precision. The extension of households' information set is motivated by a growing literature that finds subjective expectations on future realizations of idiosyncratic risk to carry significant predictive power even when other information available to the econometrician is taken into account.¹ Even conditional on households' earnings history, [Dominitz \(1998\)](#) documented that households' reported earnings expectations are predictive for subsequent earnings realizations. Thus, households have more information than just their current earnings history to predict their future earnings. In our environment, we capture this advance information with informative signals. Correspondingly, the signals collect a wide spectrum of information relevant for future changes in disposable income that are already known to households before the actual change occurs. Examples of this type of foreknowledge are information on future performance bonuses, promotions, demotions, wage cuts, or wage rises.

Advance information affects consumption insurance in opposite ways with incomplete and complete markets. In an incomplete markets model, better informed household make more suitable consumption-saving decisions, resulting in a better allocation of risk. With complete markets and limited commitment, we show as a novel theoretical result that advance information results in higher consumption dispersion and, therefore, less consumption insurance. The mechanism for this surprising result is that more precise signals reduce the value of insurance of high-income agents because part of their future income shock is revealed early. Hence, high-income agents transfer fewer resources to low-income agents, average consumption of low-income agents decreases and consumption dispersion increases *ex ante*.

In our quantitative analysis, the main consumption insurance measures are the unconditional cross-sectional variance of household consumption and the unconditional

¹Exemplary papers are [Dominitz \(1998\)](#) and [Dominitz and Manski \(1997\)](#) for income risk, [Smith, Taylor, and Sloan \(2001\)](#) for predicting mortality risk, [Stephens \(2004\)](#) for unemployment, and [Campbell, Carruth, Dickerson, and Green \(2007\)](#) for job insecurity.

covariance between current consumption growth and income growth. Compared to the data, the cross-sectional variance of household consumption and the covariance between consumption growth and income growth with incomplete markets tend to be too large, predicting too little consumption insurance. Relative to the data, the two measures tend to be too low with complete markets, yielding too much consumption insurance.

Employing US microdata to inform the theoretical models, we find very different quantitative effects of advance information for consumption insurance in both models. With complete markets and endogenous solvency constraints, the model with advance information can explain consumption insurance better than existing models. To the best of our knowledge, it is the first model to jointly match three distinct key consumption insurance measures that are not captured without advance information: (i) the unconditional variance of household consumption in the cross-section, (ii) the covariance between current consumption growth and income growth, and (iii) the income-conditional mean of household consumption across income percentiles. Furthermore, we find that advance information does not induce counterfactual correlations between current consumption growth and future income growth.

According to the theoretical model, we quantify that advance information reduces households' mean-squared forecast error for income by approximately 12%. Hence, households know more than econometricians about their future income and there is a systematic gap between the income uncertainty as perceived by households and the income uncertainty as assessed by an econometrician. Thereby, the size of this "uncertainty gap" indirectly inferred from the theoretical model is consistent with the direct estimates of [Dominitz \(1998\)](#), who reports that accounting for households' subjective income expectations reduces the econometrician's mean-squared forecast error between 12% and 21%.

Computing allocations with incomplete markets, we find that the improvement in consumption insurance with advance information is too small to explain the measures observed in the data. Moreover, and similar to [Blundell, Pistaferri, and Preston \(2008\)](#), advance information can result in counterfactual correlations between current consumption growth and future income growth. As a consequence, we cannot convincingly quantify an uncertainty gap in the incomplete markets model.

As a methodological contribution, we develop a dynamic stochastic model with an explicit specification of the joint distribution of income and signals that is consistent with household rationality. This is a relevant task because, in a dynamic setting, the joint distribution depends on the assumed exogenous stochastic process for signals. Consistency with household rationality requires that the distributions of expected income and income realizations are aligned.² When income is persistent, we show that consistency requires nontrivial but intuitive assumptions on the stochastic process for signals. This methodological contribution is general and can be widely applied to dynamic individual decision problems beyond consumption insurance. Quantitatively, we find that households' consumption-savings decisions in the theoretical models are sensitive with respect to the assumption of household rationality.

²Empirically, [Dominitz \(1998\)](#), and more recently, [Attanasio and Augsburg \(2016\)](#) provided evidence that expected income and realized income are indeed very similar.

Related literature

We are not the first to find support for the hypothesis that households know more than econometricians about their future earnings.³ The main divergence from the existing literature is that we point out that the relevance of advance information crucially depends on the structure of insurance markets.

Our paper is closely related to [Heathcote, Storesletten, and Violante \(2014\)](#) and [Kaplan and Violante \(2010\)](#), who study the role of advance information in standard incomplete markets environments with a single nonstate contingent bond. [Heathcote, Storesletten, and Violante \(2014\)](#) considered two different type of shocks, “uninsurable shocks” and “insurable shocks.” The former shocks can be only partially smoothed while the latter type of shocks can be interpreted as perfectly forecastable and are completely insured (by construction). We consider signals on uncertain future income realizations without taking a stand a priori whether certain shocks are insurable or not. In particular, we highlight that, when households have access to state-contingent insurance possibilities (and not only a single nonstate contingent bond), perfectly forecastable shocks do not necessarily enhance but may actually restrict the degree of insurance.

[Kaplan and Violante \(2010\)](#) showed that the resulting improvement in the insurance indicators proposed by [Blundell, Pistaferri, and Preston \(2008\)](#) in a standard incomplete markets model with advance information is quantitatively not important enough to account for the indicators observed in the data. With our paper, we clarify that the quantitative effects of advance information on consumption insurance depend on the structure of insurance markets. In particular, advance information in a complete markets model with endogenous solvency constraints can very well bridge the gap to several consumption insurance and risk-sharing measures observed in the data. Relative to their paper, our methodological contribution on the joint distribution of income and signals further allows us to solve for allocations that explicitly depend on signals as an additional state variable. Thus, we can study the role of advance information for the entire cross-sectional distribution of household consumption.⁴

Our paper also draws on [Kehoe and Levine \(1993\)](#), [Alvarez and Jermann \(2000\)](#), and [Krueger and Perri \(2006, 2011\)](#) who analyze the theoretical and quantitative properties of constrained efficient allocations with limited contract enforcement. [Aiyagari \(1994\)](#) pioneered characterizing invariant distributions of consumption and assets in the standard incomplete markets model in general equilibrium. Building on these papers, [Broer \(2013\)](#) provided a thorough comparison of the quantitative implications of both consumption risk sharing models to the data. We extend the limited contract enforcement model and the standard incomplete markets model with advance information to study how households’ perceived income uncertainty—rather than the uncertainty assessed by an econometrician—affects consumption insurance of US households.

³Exemplary papers in that literature are [Cunha and Heckman \(2016\)](#), [Guvenen and Smith \(2014\)](#), [Guvenen \(2007\)](#), [Primiceri and van Rens \(2009\)](#), [Huggett, Ventura, and Yaron \(2006\)](#), [Cunha, Heckman, and Navarro \(2005\)](#).

⁴[Guvenen and Smith \(2014\)](#) studied a different type of advance information. In a life-cycle model, households have initial knowledge about their individual deterministic part of income growth while we consider households that receive signals every period about future realizations of their stochastic part of income.

Hirshleifer (1971) showed that better information makes risk-averse agents ex ante worse off if such information leads to evaporation of risks that otherwise could have been shared in a competitive equilibrium with full insurance and perfect contract enforcement. Schlee (2001) provided conditions under which better public information about idiosyncratic risk is undesirable. Like these authors, we find that better information can result in less consumption insurance. In our model, however, the negative effect relies on the importance of the limited enforceability of contracts and arises only when consumption insurance is not full but partial. If enforcement frictions are absent, information does not affect consumption allocations in the limited commitment model.

The remainder of the paper is organized as follows. In the next section, we start with a simple model to analytically show how advance information affects consumption risk sharing with limited commitment. In Section 3, we present the theoretical model that we take to the data. Section 4 describes the data and the calibration that we employ in Section 5 to study the quantitative implications of advance information for risk sharing of US households. The last section concludes.

2. A SIMPLE MODEL WITH LIMITED COMMITMENT

To understand the intuition behind the quantitative results derived later, we provide here analytical results on the effect of advance information on consumption risk sharing with limited commitment employing an illustrative example. As our main result here, we show that better information on future income realizations reduces risk sharing.

Consider a two-period, pure-exchange economy with a continuum of ex ante identical agents and a single perishable consumption good. In each period, agent i receives a stochastic labor-income endowment that can be either high, $e_h = \bar{e} + \delta_e$, or low, $e_l = \bar{e} - \delta_e$, with $\delta_e > 0$ and \bar{e} as the arithmetic mean of the income process. Both income states are equally likely and the income realizations are independent across time and agents. In the first period, each agent also receives a public signal k that informs about her income realizations in the second period.⁵ Signals are i.i.d. as well and can indicate either a high income (“good” or “high” signals) or a low income (“bad” or “low” signals) in the future. The signals’ precision κ is defined as the probability that signal and future income coincide, $\kappa = \pi(e_2 = e_j | k = e_j)$, with $j \in \{h, l\}$ and $\kappa \in [1/2, 1]$. Uninformative signals are characterized by precision $\kappa = 1/2$, perfectly informative signals by $\kappa = 1$.

The preferences of agents are given by the following expected utility function:

$$\mathbb{E}[u(c_1) + u(c_2)], \quad (1)$$

where c_1 and c_2 are consumption in the first and in the second period, respectively, $u(c)$, is increasing and strictly concave. We measure social welfare according to (1), as agents’ expected utility before any risk has been resolved.

⁵As a robustness exercise, we also consider private signals (see the accompanying Online Supplemental Appendix (Stoltenberg and Singh (2020)) for the details).

If the agents are able to commit before any endowments are realized, the efficient risk-sharing arrangement is perfect risk sharing. The commitment requirement is crucial because after observing current income an agent with a high income may have an incentive to deviate from the perfect risk-sharing agreement. To capture this rational incentive, we analyze risk-sharing possibilities with limited contract enforcement or voluntary participation. A risk-sharing arrangement is consistent with limited commitment if each agent in each possible state, after observing his first-period endowment and the signal on his future income realization, at least weakly prefers to follow the arrangement rather than to defect into autarky. For the second period, we assume that agents respect the commitments made in the first period. Otherwise, if voluntary participation were allowed in both periods, there would be no room for risk sharing because agents would always choose to consume their endowments.

Let $c_{i,1}^j$ be first-period consumption of agents with signal $k = e_i$ and endowment e_j and $c_{i,2}^{jm}$ be second-period consumption of agents with first-period signal $k = e_i$ and endowment e_j in the first period and endowment e_m in the second period with $i, j, m \in \{l, h\}$. The incentives to deviate to autarky are represented by enforcement constraints that are given by the following expressions for high-income agents with good and bad signals:

$$u(c_{h,1}^h) + \kappa u(c_{h,2}^{hh}) + (1 - \kappa)u(c_{h,2}^{hl}) \geq u(e_{h,1}) + \kappa u(e_{h,2}) + (1 - \kappa)u(e_{l,2}) \equiv V_{h,\text{out}}^h, \quad (2)$$

$$u(c_{l,1}^h) + (1 - \kappa)u(c_{l,2}^{hh}) + \kappa u(c_{l,2}^{hl}) \geq u(e_{h,1}) + (1 - \kappa)u(e_{h,2}) + \kappa u(e_{l,2}) \equiv V_{l,\text{out}}^h \quad (3)$$

and for low-income agents with good and bad signals,

$$u(c_{h,1}^l) + \kappa u(c_{h,2}^{lh}) + (1 - \kappa)u(c_{h,2}^{ll}) \geq u(e_{l,1}) + \kappa u(e_{h,2}) + (1 - \kappa)u(e_{l,2}), \quad (4)$$

$$u(c_{l,1}^l) + (1 - \kappa)u(c_{l,2}^{lh}) + \kappa u(c_{l,2}^{ll}) \geq u(e_{l,1}) + (1 - \kappa)u(e_{h,2}) + \kappa u(e_{l,2}). \quad (5)$$

The resource feasibility constraints in the first and second period are the following:

$$\frac{1}{4}(c_{h,1}^h + c_{h,1}^l + c_{l,1}^h + c_{l,1}^l) = \frac{1}{2} \sum_{j \in \{l, h\}} e_{j,1}, \quad (6)$$

$$\frac{1}{4}[\kappa(c_{h,2}^{hh} + c_{h,2}^{lh} + c_{l,2}^{hl} + c_{l,2}^{ll}) + (1 - \kappa)(c_{h,2}^{hl} + c_{h,2}^{ll} + c_{l,2}^{hh} + c_{l,2}^{lh})] = \frac{1}{2} \sum_{j \in \{l, h\}} e_{j,2}. \quad (7)$$

An *efficient allocation* is a consumption allocation, $\{c_{i,1}^j, c_{i,2}^{jm}\}$, that maximizes ex ante utility (1), subject to the enforcement constraints (2)–(5) and the resource constraints (6)–(7).

Efficient allocations may feature either perfect risk sharing (all agents consume \bar{e} in all states), no insurance against income risk (autarky, all agents consume their endowments in all states) or partial risk sharing. Here, we focus on the empirically relevant case of partial risk sharing. As summarized in the following proposition, better public signals lead to less risk sharing and higher consumption dispersion.

PROPOSITION 1 (Information and Risk Sharing). *Consider an efficient allocation with partial risk sharing such that the enforcement constraints (2)–(3) are binding. Conditional on the income-signal pair in the first period, the consumption allocation is characterized by perfect smoothing across future income states and across periods, that is,*

$$c_{i,1}^j = c_{i,2}^{jh} = c_{i,2}^{jl} = c_i^j, \quad \forall i, j.$$

An increase in information precision has the following effects on the consumption allocation in each period:

1. *The conditional mean of consumption of high-income agents increases and the conditional mean of low-income agents decreases.*
2. *The conditional standard deviations of consumption of high-income and low-income agents increase.*
3. *The unconditional standard deviation of consumption increases.*

The proof is provided in Appendix A.1.

The proposition has two main messages: first, that conditional on an income-signal pair in the first period, efficient allocations feature perfect consumption smoothing across future income states and time periods and second—more importantly—more precise signals result in a more unequal consumption distribution when enforcement constraints matter.

To gain intuition about why better information on individual future income realizations increase consumption dispersion, consider an increase in the precision of signals. By (2) and (3), this results in an increase in the value of the outside option for high-income agents with a good signal and a decrease for agents with a bad signal. As captured by the changes in the outside option values, agents with a bad signal are more willing while the agents with a good signal are less willing to share their current high income. Thus, consumption of high-income agents spreads out and the conditional standard deviation of consumption of high-income agents increases. Thereby, the changes in the value of the outside option of high-income agents with a good signal ($V_{h,\text{out}}^h$) and with a bad signal ($V_{l,\text{out}}^h$) are symmetric:

$$\frac{\partial V_{h,\text{out}}^h}{\partial \kappa} = -\frac{\partial V_{l,\text{out}}^h}{\partial \kappa}.$$

For informative signals, the high-income agents with a good signal have a lower marginal utility of consumption and thus require more additional resources than the high-income agents with a bad signal are willing to give up. In sum, mean consumption of high-income agents increases which by resource feasibility reduces the risk-sharing possibilities for low-income agents. As a consequence, the consumption allocation becomes riskier ex ante and the unconditional standard deviation of consumption increases as well.

The main take-away from this section is that more precise signals result in a riskier consumption allocation ex ante such that the standard deviation of consumption increases. Further, better information results in higher consumption of high-income and lower consumption of low-income agents.

3. ENVIRONMENT

Our main novel result is that the quantitative importance of advance information crucially depends on the design of insurance markets. To set the stage for the quantitative exercise, we describe first how households form their income expectations when conditioning on both the current income realization and on the realization of the signal as the new element. Furthermore, we present a decentralized version of the limited commitment model from the previous section with complete markets, an infinite time horizon, endogenous solvency constraints, and persistent income shocks embedded into a production economy with capital.

Preferences and endowments

Consider an economy with a continuum of households indexed by i . Time is discrete and indexed by t from zero onward. Households have preferences over consumption streams and evaluate them conditional on the information available at $t = 0$,

$$U(\{c_t^i\}_{t=0}^\infty) = (1 - \beta)\mathbb{E}_0 \sum_{t=0}^\infty \beta^t u(c_t^i), \quad (8)$$

where the instantaneous utility function $u : \mathbb{R}_+ \rightarrow \mathbb{R}$ is strictly increasing, strictly concave, and satisfies the Inada conditions.

Household i 's disposable labor income in period t is given by $w_t y_t^i$, where w_t is the real wage per unit of effective labor and y_t^i are individual effective labor unit endowments. Effective labor unit endowments are generated by a stochastic process $\{y_t^i\}_{t=0}^\infty$, where the set of possible realizations in each period is time invariant and finite $y_t^i \in Y \equiv \{y_1, \dots, y_N\} \subseteq \mathbb{R}_{++}$, ordered. The history (y_0, \dots, y_t) is denoted by y^t . Effective labor units are independent across households and evolve across time according to a first-order Markov chain with time-invariant transition matrix P whose elements $\pi(y' = y_k | y = y_j)$ for all j, k are the conditional probabilities of next period's endowment y_k given current period endowment y_j . There is no aggregate risk, and the Markov chain induces a unique invariant distribution of income $\pi(y)$ such that the aggregate labor endowment is constant and equal to $L_t = \bar{y} = \sum_y y \pi(y)$. In the following, all relevant transition probabilities are time invariant, which is why we employ a recursive notation such that $x(x')$ denotes the value of a generic variable x in the current (future) period.

Information

Except for observing past and current endowment shocks, household i receives in each period $t \geq 0$ a public signal $k_t^i \in Y$ that informs about endowment realizations in the

next period. The signal has as many realizations as endowment states and its precision κ is captured by the time-invariant conditional probability that signal and future endowment coincide, $\kappa = \pi(y' = y_j | k = y_j)$, $\kappa \in [1/N, 1]$. Uninformative signals are characterized by precision $\kappa = 1/N$, perfectly informative signals by $\kappa = 1$. Hence, at each point in time the agents can find themselves in one of the states $s_t = (y_t, k_t)$, $s_t \in S$, where S is the Cartesian product $Y \times Y$ and $s^t = (y^t, k^t) = (s_0, \dots, s_t)$ is the history of the state.

Using the recursive notation, the conditional probabilities of future endowments y' conditional on today's state $s = (y, k)$ are denoted by $\pi(y'|s)$. The latter probabilities are given by⁶

$$\pi(y' = y_j | k = y_m, y = y_i) = \frac{\pi_{ij} \kappa \mathbf{1}_{j=m} \left(\frac{1 - \kappa}{N - 1}\right)^{1 - \mathbf{1}_{j=m}}}{\sum_{z=1}^N \pi_{iz} \kappa \mathbf{1}_{z=m} \left(\frac{1 - \kappa}{N - 1}\right)^{1 - \mathbf{1}_{z=m}}}, \tag{9}$$

where tomorrow's endowment is $y' = y_j$, today's endowment is $y = y_i$, and today's signal indicates endowment state y_m in the future, $k = y_m$; $\mathbf{1}_{j=m}$ is an indicator function that equals one if the signal and the actual realization of the endowment coincide. The formula resembles a "hit-or-miss" specification and its logic follows from Bayes' theorem. There are two independent "signals" on future endowment realizations, current endowments, and the public signal. Both signals are weighted with their precision, endowments with transition probability π_{jk} , and signals with precision κ . Intuitively, the public signal informs about future endowment shock realizations by implicitly providing advance information on future innovations to endowments.⁷

For example, with uninformative signals ($\kappa = 1/N$) the conditional probability of endowment y_j tomorrow given today's endowment y_i and given any signal k today can be computed as

$$\pi(y' = y_j | k, y = y_i) = \frac{\pi_{ij} \frac{1}{N}}{\frac{1}{N} \sum_{z=1}^N \pi_{iz}} = \pi_{ij}.$$

To derive the transition probabilities of the state $\pi(s'|s)$, we assume that signals follow an exogenous first-order Markov process with time-invariant transition probabilities $\pi(k'|k)$. Combining this assumption with (9) yields a time-invariant Markov transition matrix P_s with conditional probabilities $\pi(s'|s)$ as elements

$$\begin{aligned} \pi(s'|s) &= \pi(y' = y_k, k' = y_m | k = y_l, y = y_j) \\ &= \pi(k' = y_m | k = y_l) \pi(y' = y_k | k = y_l, y = y_j). \end{aligned} \tag{10}$$

The Markov chain induces a unique invariant distribution of the state denoted by $\pi(s)$. The formula (10) applies to any first-order Markov process for signals. To fill this degree

⁶Appendix A.2 provides details on the derivation of the formulas for the joint distribution of endowments and signals.

⁷At the end of this section, we elaborate on our modeling choices regarding the specification of signals.

of freedom, we apply a “reverse-engineering” procedure to choose the Markov process for signals such that rational expectations deliver the conditional expectations as specified in equation (9). Appendix A.3 describes the procedure in detail and analytically illustrates it with a two-state example. In general, the procedure yields signal transition probabilities that depend on the properties of the Markov process for endowments and on the precision of signals. Furthermore, when endowments are persistent, consistent signals are persistent as well. This implies that the effect of a signal realization today does not only affect endowment expectations in the next period but has long-lasting effects for future endowment expectations.

Production

A representative firm hires labor L_t and capital K_t at rental rates w_t and r_t to maximize profits. Capital depreciates at rate δ and the production of consumption goods Y_t takes place via a linear homogenous production function

$$Y_t = AF(L_t, K_t),$$

with A as a productivity parameter that is constant in the stationary equilibria that we focus on in the following. Aggregate labor endowments L_t are normalized to unity.

Endogenous solvency constraints

Following Alvarez and Jermann (2000), there is no restriction on the type of insurance contracts that can be traded but the contracts suffer from limited commitment because, in every period, agents have the option to default to autarky. Households can buy or sell state-contingent assets $a(s^t, s_{t+1})$ priced at $q(s^t, s_{t+1})$. The state-contingent asset $a(s^t, s_{t+1})$ prescribes one unit of the consumption good in state s_{t+1} to or from an agent that experiences the history s^t . Households trade the asset with financial intermediaries that live for one period and can also invest into capital. Households face state-contingent endogenous credit limits $A(s^t, s_{t+1})$ that are not “too tight,” that is, credit limits that only ensure that households have no incentive to default to autarky but do not constrain insurance contracts; otherwise,

$$a(s^t, s_{t+1}) \geq A(s_{t+1}) = \min\{a(s_{t+1}) : V[a(s_{t+1}), s_{t+1}] \geq U^{\text{Aut}}(s_{t+1})\}, \quad \forall s_{t+1}, \quad (11)$$

with U^{Aut} as the value of the outside option and $V(a, s)$ as the continuation value of a household with asset holdings a and state s (see the recursive problem (12)–(14) below). In case of defaulting to the outside option and consistent with US bankruptcy law, households lose all their assets. Further, access to financial markets is restricted. While agents can save unlimited amounts in a nonstate contingent bond with gross return R , they cannot borrow. Thus, the value of the outside option is a solution to an optimal savings problem that can be written in recursive form as follows:

$$v(a, s) = \max_{0 \leq a' \leq y + aR} \left[(1 - \beta)u(aR + y - a') + \beta \sum_{s'} \pi(s'|s)v(s', a') \right],$$

such that the value of the outside option is given by

$$U^{\text{Aut}}(s) = v(0, s).$$

Given asset holdings a , state $s = (y, k)$, and prices $w, \{q(s, s')\}$, households' problem can be written recursively as

$$V(a, s) = \max_{c, \{a'(s')\}} \left\{ (1 - \beta)u(c) + \beta \sum_{s'} \pi(s'|s) V[a'(s'), s'] \right\} \quad (12)$$

subject to a budget constraint and solvency constraints

$$c + \sum_{s'} q(s, s') a'(s') \leq wy + a, \quad (13)$$

$$a'(s') \geq A(s'), \quad \forall s'. \quad (14)$$

The results of the utility maximization problem are policy functions $c(a, s), \{a'(a, s; s')\}$. In period zero, households differ with respect to initial asset holdings and initial shocks where the heterogeneity is captured by the invariant probability measure $\Phi_{a,s}$. In an economy with one nonstate contingent asset, [Ábrahám and Cárceles-Poveda \(2010\)](#) showed that the endogenous credit limits derived according to (11) share some realistic features with credit limits observed in the Survey of Consumer Finances (SCF). As in the data, credit limits in the model become looser as labor income increases. While agents with a higher income have more incentives to default because higher income shocks lead to a higher autarky value, this does not necessarily lead to tighter credit limits. In our quantitative results, we confirm the results of [Ábrahám and Cárceles-Poveda \(2010\)](#) for a complete set of state contingent assets.

Equilibrium

The stationary recursive competitive equilibrium with solvency constraints is summarized in the following definition.

DEFINITION 1. A stationary recursive competitive equilibrium with solvency constraints comprises a value function $V(a, s)$, a price system $R, w, q(s, s')$, an allocation $K, c(a, s), \{a'(a, s; s')\}$, a joint probability measure of assets and exogenous state $\Phi_{a,s}$, and endogenous credit limits $A(s')$ such that:

- (i) $V(a, s)$ is attained by the decision rules $c(a, s), \{a'(a, s; s')\}$ given $R, w, q(s, s')$.
- (ii) Endogenous credit limits are determined according to (11).
- (iii) The joint distribution of assets and state $\Phi_{a,s}$ induced by $\{a'(a, s; s')\}$ and P_s is stationary.
- (iv) No arbitrage applies

$$q(s, s') = \frac{\pi(s'|s)}{R}.$$

(v) Factor prices satisfy

$$R - 1 = AF_K(1, K) - \delta,$$

$$w = AF_L(1, K).$$

(vi) The asset market clears

$$RK = \int \sum_{s'} a'(a, s; s') \pi(s'|s) d\Phi_{a,s}.$$

Discussion

We conclude this section with a discussion of some features of the information environment outlined at the beginning of this section. With the signals, we collect a wide spectrum of information such as foreknowledge of future performance bonuses, promotions, demotions wage cuts, or wage rises. In Formula (9), we model signals with a hit-or-miss specification in the following sense. The probability that the signal indicates the correct endowment realization is κ , and $1 - \kappa$ is the probability that signal and future endowment realization differ. The latter probability is then allocated equally to all endowment states not indicated by the signal. Conditional on the signal being wrong, the transition probabilities are exclusively driven by the endowment transition probabilities $\pi(y'|y)$. For the type of information we seek to model, this is a reasonable specification, in particular when endowment shocks are persistent as in reality. To see this, suppose that an agent receives a signal that he will likely get a bonus in the next period; according to the formula, his probability to receive an endowment rise increases compared to the case without the signal. With some probability, however, he might not get the bonus. In this case, the probability to transit to a particular endowment state should no longer be affected by the signal, but rather should be solely dependent on his current endowment state. This is exactly what is captured in the formula by allocating $1 - \kappa$ equally over the states without a bonus.

An alternative to the discrete hit-or-miss signals informing on future income is to consider continuous Gaussian signals on future innovations to income. In Appendix A.4, we describe Gaussian signals in detail and provide a notion of equivalence between the two signal specifications. As one important difference relevant for computing allocations, we show that, relative to the hit-or-miss signals, the dimension of the state space relevant for computations – the number of elements in S —with discretized Gaussian signals increases by factor N . The increase in the dimension makes computing allocations very costly in particular (but not only) in a complete-markets setting, which is another point in favor of the hit-miss signal specification.

The examples in the [Introduction](#) include both private and public information. We opted for public signals and, therefore, treat subjective expectations as observable for the following reasons. First, subjective beliefs are in principle inferable—either directly in surveys or indirectly using the actions implied by an economic model. With the signals, we seek to model the predictive power of publicly available subjective income expectations as estimated in the empirical literature, and public signals serve precisely this

purpose. A further advantage of public signals, as compared to private ones, is that we can specify a fully decentralized version in which households engage in unmonitored trade of ordinary securities. Atkeson and Lucas (1992) showed that such a decentralization is not feasible with private information. Thus, with private signals we have to consider a social planner problem which is less realistic. Finally, we find that increases in signal precision with private signals have qualitatively similar effects on risk sharing in a two-period model (see the accompanying Online Supplemental Appendix).⁸

4. QUANTITATIVE EXERCISE

Before we quantitatively evaluate the implications of advance information on consumption insurance and risk sharing with complete and incomplete insurance markets, we describe the data employed in the quantitative exercise and the calibration of model parameters. Further, we explain how we measure insurance, risk-sharing, and the uncertainty gap as the difference in income uncertainty measured by econometricians and households.

4.1 *Data and calibration*

Data To quantitatively evaluate the model, one would like to employ a household panel data set with a large number of observations that contains detailed information on households' income, their consumption expenditures and their subjective expectations of future income. To the best of our knowledge, such a data set does not exist for the US. For this reason, we opt for the following strategy. To facilitate comparison with related studies, in particular to Krueger and Perri (2006) and Broer (2013), we also use as a first step the Consumer Expenditure Interview Survey (CEX) for information on households' income and consumption expenditures. Starting from the calibration used in these papers, we investigate how informative signals affect consumption inequality and insurance in the model by varying signal precision to find our preferred value for the parameter κ . Afterwards, we relate the value of the parameter to information on the predictive power of subjective expectations elicited in the special edition of the Survey of Economic Expectations (SEE) from 1993–1994 that contains information on US households' income realizations and their corresponding income expectations. Although of smaller sample size than the CEX survey, the SEE is a nationally representative sample with respondents drawn from a national probability sample of households. Due to this,

⁸It is to be noted, moreover, that public signals are more tractable than private signals, which allows us to employ realistic income processes. With private signals, we would not only have to consider occasionally binding solvency constraints but, additionally, constraints that capture households' incentives to reveal the true realization of the private signal. More than the sheer increase in occasionally binding constraints at each node each node (a, y, k) , the interaction of enforcement and truth-telling constraints results in additional challenges in the computation of allocations. For this reason, existing studies such as Broer, Kapička, and Klein (2017) focus on the theoretical implications of private information and consider stylized endowment processes with merely two states.

weighted sample moments from SEE data are relatively close to the ones obtained in CEX data.⁹

For the CEX, we follow [Krueger and Perri \(2006\)](#) and [Broer \(2013\)](#) precisely in their methodology. In particular, we decompose consumption and income inequality in between and within group inequality. Between-group inequality are differences in household income and consumption attributable to observable characteristics, such as education, region of residence, etc., and assume that households cannot insure against such characteristics. Income inequality that lacks a between group inequality component is called within group inequality. This residual measure of inequality is the focus of this paper, as it is caused by idiosyncratic income shocks; hence, depending on the insurance available against these shocks, consumption inequality will not exactly mirror income inequality.

As a measure of household consumption, we employ nondurable consumption (ND+) which also includes an estimate for service flows from housing and cars. For households' disposable income, we use after-tax labor earnings plus transfers (LEA+). Consistent with voluntary participation, we thus take the mandatory public insurance as given and focus on private insurance. LEA+ comprises the sum of wages and salaries of all household members, plus a fixed fraction of self-employment farm and nonfarm income, minus reported federal, state, and local taxes (net of refunds) and social security contributions plus government transfers.

We drop the households which: report zero or only food consumption; have a head older than 64 years or younger than 21 years; have negative or zero labor income or have negative working hours; have positive labor income but no working hours; live in a rural area or their weekly wage is below half the minimum wage; do not attend all interviews. To facilitate a comparison between households of different size, the consumption and income measures are divided by adult equivalence scales, as in [Dalaker and Naifeh \(1997\)](#).

To compute within group inequality, we follow [Krueger and Perri \(2006\)](#) and [Blundell, Pistaferri, and Preston \(2008\)](#), and regress the logs of household consumption and income on a cubic function of age and a set of dummies that include region, marital status, race, education, experience, occupation, and sex. The residuals of the regression are treated as consumption and income shock and are the objects of our study in the following.

Model parameters Our annual calibration is designed to highlight the differences between a standard limited commitment model without signals, as entertained in [Broer \(2013\)](#), and a model with informative signals. Therefore, we set a number of corresponding parameters to the same values. In particular, we consider a period utility function that exhibits constant relative risk aversion with parameter $\sigma = 1$. The discount factor β is chosen to yield an annual gross interest rate of $R = 1.025$ in general equilibrium. We

⁹For example, using Table 1 of [Dominitz \(1998\)](#), the average weekly earnings of households in the SEE is 564 dollars. For the CEX, the average current weekly earnings in 1993, using Table A2 of [Krueger and Perri \(2006\)](#) is 521 dollars. This is calculated by dividing 18,841 by 52 and then multiplying by 1.44 to correct for CPI deflation.

employ a Cobb–Douglas production function $AF(K, L)$ with a capital–production elasticity of 0.30. Given R , we choose the depreciation of the capital stock δ and the technology parameter A to yield a real wage rate of unity and an aggregate wealth-to-income ratio of 2.5 as, for example, estimated by [Kaplan and Violante \(2010\)](#) based on the Survey of Consumer Finances (SCF). With a wage rate of unity, labor income is $wy = y$, and we use the terms individual endowment and individual income interchangeably.

The standard practice in the literature is to specify the log of household income as the sum of persistent and orthogonal transitory shocks, that is, there are two innovation terms. With just one signal but two innovation terms, it remains unclear on which future innovation the signal is informative. For this reason, we employ the results provided in [Ejrnæs and Browning \(2014\)](#) to model log income of household i as an ARMA(1, 1)-process with a single innovation term that is equivalent to a persistent–transitory specification (under some conditions)

$$\ln(y_{it}) = \rho \ln(y_{it-1}) - \theta u_{it-1} + u_{it}, \quad u_{it} \stackrel{\text{i.i.d.}}{\sim} \mathcal{N}(0, \sigma_u^2), \quad (15)$$

with unconditional variance

$$\text{var}[\ln(y_{it})] = \frac{1 + \theta^2 - 2\rho\theta}{1 - \rho^2} \sigma_u^2 \quad (16)$$

and unconditional autocovariance

$$\text{cov}[\ln(y_{it}), \ln(y_{it-1})] = \rho \text{var}[\ln(y_{it})] - \theta \sigma_u^2. \quad (17)$$

The persistence parameter ρ is set to 0.9989, which is the value persistent–transitory originally estimated in [Storesletten, Telmer, and Yaron \(2004\)](#). Given a particular value of the persistence parameter, we identify θ, σ_u^2 from the cross-sectional within-group income variance and autocovariance in the CEX data as the averages of the years 1999–2003 using equations (16)–(17).¹⁰ The method proposed by [Tauchen and Hussey \(1991\)](#) is used to approximate the ARMA(1, 1) as a finite-state first-order Markov process with six distinct income states. We normalize the value of all income states such that mean income (or aggregate labor endowment) is equal to unity. For each of the six income states, there are therefore six public signals such that the joint income-signals state S is approximated by 36 states which is higher than the 14 states typically considered in related studies ([Broer \(2013\)](#), [Krueger and Perri \(2006\)](#)). The increase in the number of states leads to a numerical challenge for computing consumption allocations in general equilibrium.¹¹

¹⁰In the accompanying Online Supplemental Appendix, we explain the equivalence between the ARMA(1, 1) and the persistent-transitory log income specification. We also provide an information environment with two signals—one signal that informs about future realizations of the persistent and another signal that informs about future realizations of the transitory shock.

¹¹In the accompanying Online Supplemental Appendix, we describe our algorithm for computing allocations in the endogenous-solvency constraints model in more detail.

4.2 Insurance, risk-sharing, and uncertainty gap: Measures

Consumption insurance measures To assess the extent of consumption insurance in the data, we focus on two measures: (1) the dispersion of consumption across households and (2) the covariance between consumption and income growth. The first measure—the risk-sharing ratio RS —is defined as follows:

$$RS = 1 - \frac{\text{var}_c}{\text{var}_y}, \quad (18)$$

with $\text{var}_x = \text{var}[\ln(x)]$. As one extreme, if $\text{var}_c = \text{var}_y$, then $RS = 0$, and there is no private risk sharing against fluctuations in disposable income. On the other hand, if $\text{var}[\ln(c)] = 0$ then $RS = 1$, implying full risk sharing with respect to income shocks and the absence of consumption inequality.

The second measure focuses on the sensitivity of consumption growth to income growth and is given by the coefficient $\beta_{\Delta y}$ in the following regression equation, originally proposed by Mace (1991):

$$\Delta c_{it} = \psi + \beta_{\Delta y} \Delta y_{it} + v_t + v_{it}, \quad (19)$$

where ψ is a constant, v_t a vector of time dummies, and v_{it} a residual; Δc_{it} and Δy_{it} are the growth rates of consumption and income of individual i in period t . When the coefficient $\beta_{\Delta y}$ is zero, then consumption growth is perfectly insured against changes in income growth. The higher the coefficient, the less insurance achieved.

In Table 1, we summarize the calibrated parameters in the upper part and unconditional moments of consumption and income from the CEX data in the lower part. The value of $\beta_{\Delta y}$ is equal to 0.11 with a standard error of 0.0035; the ratio RS is $1 - \frac{\text{var}_c}{\text{var}_y} = 0.60$, which implies 40% of income shocks transfer to consumption.

To compute the two consumption insurance measures in the economic models, we employ stationarity. For the first measure, we employ the invariant distribution to calculate the cross-sectional variance of household consumption. For the second measure,

TABLE 1. Baseline parameters and CEX moments.

	Parameter	Value
σ	Risk aversion	1.0000
α	Elasticity of capital in production function	0.3000
R	Gross interest rate	1.0250
ρ	Autoregressive coefficient in ARMA(1, 1)	0.9989
θ	Dependency on past innovations in ARMA(1, 1)	0.9331
σ_u	Standard deviation of innovations in ARMA(1, 1)	0.3508
N^2	Number of income-signal states	36
var_y	Variance logged income	0.3654
var_c	Variance logged consumption	0.1462
$\beta_{\Delta y}$	Regression coefficient	0.1078
RS	Risk-sharing ratio	0.5999

we simulate the model for 300,000 time periods and discard the first 100,000 periods to ensure convergence. Then we estimate covariances of consumption and income growth using the simulated data.

Measuring the uncertainty gap To interpret the effects of an increase in information precision κ , we compute the percentage reduction of households' perceived income uncertainty $\tilde{\kappa}$ as measured by the reduction in the mean-squared forecast error resulting from conditioning expectations on signals

$$\tilde{\kappa}(\kappa) = \frac{\text{MSFE}_y - \text{MSFE}_s(\kappa)}{\text{MSFE}_y}, \quad 0 \leq \tilde{\kappa}(\kappa) \leq 1, \quad (20)$$

with

$$\text{MSFE}_y = \sum_y \pi(y) \sum_{y'} \pi(y'|y) [y' - E(y'|y)]^2, \quad (21)$$

$$\text{MSFE}_s(\kappa) = \sum_s \pi(s) \sum_{y'} \pi(y'|s) [y' - E(y'|s)]^2 \leq \text{MSFE}_y, \quad (22)$$

$\pi(s)$ is the joint invariant distribution of income and signals induced by P_s , and $E(y'|y)$, $E(y'|s)$ as the income means conditional on income only and jointly on income and signals, respectively. Thus, $\tilde{\kappa}$ captures the difference in income uncertainty as measured by an econometrician in the aggregate—ignoring the information on future shocks on the household level—and the uncertainty as perceived by households stemming from their subjective expectations. For this reason, we refer to $\tilde{\kappa}$ as the uncertainty gap. Perfectly informative signals are given by $\tilde{\kappa} = 1$ and uninformative signals (or no advance information) by $\tilde{\kappa} = 0$.

5. QUANTITATIVE RESULTS

The objective of this section is to establish our main result, which is that the quantitative importance of advance information depends on the structure of insurance markets. First, we employ the complete markets model with endogenous solvency constraints (*ESC* model) presented in Section 3 to quantify advance information through the lens of this model. We further discuss how the value for advance information inferred from the economic model relates to direct estimates of the predictive power of subjective expectations and study the implications of the quantified amount of advance information for various “overidentifying restrictions.” Second, we study the effects of advance information when markets are incomplete and solvency constraints are exogenous, as in a standard incomplete markets model (*SIM*). Comparing the quantitative effects of advance information for the two different structure of insurance markets, we find that advance information plays a quantitatively important role for consumption inequality and insurance with complete markets, and a relatively small role with incomplete markets.

5.1 Consumption insurance with complete markets

In this section, we analyze how advance information affects consumption insurance in the *ESC* model.

TABLE 2. Risk sharing, insurance, and advance information.

Risk-Sharing Ratio, RS			Regression Coefficient, $\beta_{\Delta y}$		
$\tilde{\kappa} = 0.00$ (0.07)	$\tilde{\kappa} = 0.124$ (0.44)	Data	$\tilde{\kappa} = 0.00$ (0.07)	$\tilde{\kappa} = 0.116$ (0.43)	Data
0.94	0.60	0.60	0.01	0.11	0.11

Note: ESC model. Risk-sharing ratio and regression coefficient in the data and in the model for different values of $\tilde{\kappa}$. Values for κ in parentheses. Uninformative signals, $\tilde{\kappa} = 0.00$, $\kappa = 0.07$, and $N = 14$. Informative signals, $N = 6$.

5.1.1 *Quantifying advance information* To discipline the only free parameter $\tilde{\kappa}$, we choose the parameter such that household consumption in the invariant distribution of the model matches the the risk-sharing ratio (18) and the regression coefficient of current consumption growth with respect to income growth (19). In general, we therefore expect to pin down two values for the reduction in households' perceived income uncertainty $\tilde{\kappa}$ that yield consumption insurance measures in the model which are consistent with the values of the two measures observed in the CEX.

Our main quantitative findings are summarized in Table 2. The left panel displays how informative signals impact on the risk-sharing ratio. With uninformative signals, $\tilde{\kappa} = 0$, consumption is with a risk-sharing ratio of $RS = 0.94$ too little dispersed across households compared to the data with $RS = 0.60$.¹² Consistent with the third part of Proposition 1, the risk-sharing ratio decreases in the precision of signals or equivalently in $\tilde{\kappa}$. For $\tilde{\kappa} = 0.124$, the risk-sharing ratio in the model is reconciled with the ratio of 0.60 observed in the data.

The right panel of Table 2 shows how the uncertainty gap affects the regression coefficient $\beta_{\Delta y}$. While, in the absence of informative signals, consumption growth is well guarded against changes in income with a coefficient of 0.01, the sensitivity of consumption increases with the size of the uncertainty gap. For $\tilde{\kappa} = 0.116$, the model matches the regression coefficient observed in the data. In that sense, both measures—the risk-sharing ratio and the insurance coefficient—are jointly explained by the model for an uncertainty gap of 12% (rounded). This result is remarkable: in general, the two measures have to coincide only in the extreme cases when risk sharing (and insurance) is either perfect or absent.¹³

Why is consumption insurance so sensitive with respect to advance information? With complete markets and endogenous solvency constraints, advance information affects

¹²With uninformative signals and $N = 6$, consumption allocations are characterized by (almost) perfect risk sharing. To avoid this result, we use a finer income grid with $N = 14$ states as a baseline in the case of uninformative signals. This implies that the number of income states with informative signals ($N = 6$) is smaller than in the case of uninformative signals ($N = 14$). Thus, the latter case is no longer nested as a special case of informative signals.

¹³As robustness exercises, we study in the accompanying Online Supplemental Appendix how the quantified amount of advance information changes if the risk-sharing ratio and the regression coefficient in the data are different from the baseline estimates displayed in Table 1. In this Appendix, we also investigate how advance information affects the sensitivity of consumption growth with respect to income increases and decreases.

consumption allocation through two channels. First, advance information affects the prices of state-contingent assets $\pi(s'|s)/R$ by affecting $\pi(s'|s) = \pi(y'|s)\pi(k'|k)$ because R is calibrated to a constant. When signal precision increases, the conditional probabilities of states indicated by the signals (not indicated by the signals) increase (decrease), leading to asset prices spreading out and consumption becoming riskier ex-ante. Second, more precise signals spread out outside option values; this spreading out is reflected in tighter and more dispersed endogenous credit limits for high- and low-income households, which also decreases consumption insurance.

To disentangle the quantitative importance of the two channels, we run the following decomposition exercise. We fix the credit limits at levels that apply to uninformative signals and then compute the resulting risk-sharing ratio for $\tilde{\kappa} = 0.124$ in the stationary equilibrium (the results for the regression coefficient are very similar). With this experiment, we isolate the first channel of advance information. We find that the second channel dominates. To be more precise, 76% of the total decrease in the risk-sharing ratio with advance information can be attributed to the change in credit limits. Thus, the endogenous credit limits' dependency on advance information is the driving force behind the quantitatively important role of advance with complete markets.

External validity: Advance information in the model versus direct estimates In the Spring and Fall of 1993, households in the Survey of Economic Expectations (SEE) were asked to report their actual earnings realizations in 1993 and their weekly earnings expectations for 1994. In 1994, the respondents were asked again for their actual earnings realizations. Thus, the data contains expectations and corresponding realizations as well as the earnings realizations at the time when the expectations are reported. These characteristics make the data particularly useful for testing the external validity of the amount of advance information that we infer from the economic model.

Dominitz (1998) employed this data to elicit households' earnings expectations and their predictive power as follows. In Spring and Fall 1993, he compares the resulting mean-squared forecast errors in two best-linear predictor OLS-regressions. In the first regression, he employs only earnings realizations in 1993 (Spring or Fall) and a set of observable variables. The second regression conditions on the same variables as the first but additionally on the subjective earnings mean of each individual. In total, he runs 4 regressions—two using the Spring earnings and subjective expectations (1 year ahead) and two with the Fall earnings and expectations in 1993 (6 months ahead).

Dominitz (1998) finds that reported subjective expectations yield additional predictive value for both the Spring and Fall data (see his Table 7 on p. 385). With the Spring data, conditioning not only on earnings realizations in 1993 but additionally on the subjective earnings mean for 1994 decreases the mean-squared forecast error for the 1994 earnings realizations in his OLS-regression by 0.118. For the Fall data, the mean-squared forecast error is reduced by 0.214. Given that we employ 1 year ahead earnings forecasts in the model, the values of $\tilde{\kappa}_1 = 0.124$ and $\tilde{\kappa}_2 = 0.116$ we indirectly infer using the theoretical model are consistent with the relevant direct evidence of 0.118 stemming from the Spring 1993 forecast.

Summing up, we find that the risk-sharing and the insurance measure can be jointly explained when households' perceived income uncertainty is reduced by 12%. In the following, we set information precision to this value, and analyze the model's performance for various "overidentifying restrictions."

5.1.2 Overidentifying restrictions The goal of this section is to further test the model with the amount of advance information that was quantified in the previous section. Throughout this section, we compare the standard model without signals to the case of informative signals.

Consumption–income growth correlations with advance information To test for advance information in the data, [Blundell, Pistaferri, and Preston \(2008\)](#) employed household panel data to estimate correlations of current consumption growth $\Delta c_{i,t} = \log(c_t^i) - \log(c_{t-1}^i)$ with future income growth $\Delta y_{i,t+j} = \log(y_{t+j}^i) - \log(y_{t+j-1}^i)$ for $j \geq 1$, with i as household index. Through the lens of a standard incomplete markets model, they argue that if there was advance knowledge of income shocks, the correlation in the data should be significantly different from zero because consumption should have adjusted before the shock has occurred.¹⁴ [Blundell, Pistaferri, and Preston \(2008\)](#) did not find support for advance information because they estimated correlations of current consumption with future income growth that are not significantly different from zero with p -values larger than 25%.

The complete markets model with advance information is consistent with the evidence provided in [Blundell, Pistaferri, and Preston \(2008\)](#). As reported in the first column of Table 3, the correlation of current consumption growth with future income growth is not significantly different from zero for the standard model with $\tilde{\kappa} = 0$. This pattern does not change for informative signals. As displayed in the second column for $\tilde{\kappa} = 0.116$, only the correlation of current income growth and current consumption growth is significantly different from zero. In line with the evidence, the correlations of current consumption growth with future income growth in the model are not significantly different

TABLE 3. Income and consumption growth regressions.

	$\tilde{\kappa} = 0.00$ (0.07)	$\tilde{\kappa} = 0.116$ (0.43)	Data
$\beta_{\Delta y_i}$	0.01 [0.00]	0.11 [0.05]	0.11 [0.00]
β_2	−0.00 [0.17]	0.01 [0.89]	– –

Note: ESC-model. Regression coefficients and their p values for the regression equation $\Delta c_{it} = \beta_0 + \beta' \Delta y_i + \epsilon_{it}$, with $\beta = [\beta_{\Delta y_i}, \beta_2]'$ and $\Delta y_i = [\Delta y_{it}, \Delta y_{it+1}]'$ in the model and in the data according to (19). p -values are reported in square brackets. Values for κ in parentheses. Uninformative signals, $\tilde{\kappa} = 0.00$, $\kappa = 0.07$, and $N = 14$. Informative signals, $N = 6$.

¹⁴[Güvenen and Smith \(2014\)](#) considered households with initial knowledge about their individual deterministic part of income growth. This type of advance information does not result in the counterfactual consumption–income growth correlations in a SIM-model.

from zero with p -values larger than 89%.¹⁵ Thus, advance information in the *ESC* model does not induce counterfactual correlations of current consumption growth with future income growth.¹⁶

The logic for this result can be rationalized within the limited commitment endowment economy presented in Section 2. As summarized in Proposition 1, one key feature of the (constrained) efficient allocation is that, conditional on a particular income-signal pair in the first period, consumption is perfectly smoothed across both income states in the second period but also across time, decoupling consumption from future income realizations. Thus, the planner encourages high-income agents with binding enforcement constraints to transfer resources today in exchange for perfect insurance of income shocks in the future. When signals become more precise, the outside option becomes more attractive for agents with a high income, and it becomes more difficult for the planner to generate transfer to less fortunate agents. The efficient way to facilitate these transfers is to continue to promise consumption smoothing across time and states, and to increase the level of consumption for high-income agents but not to break up the decoupling of consumption from future income realizations. Consequently, current consumption remains decoupled from future income realizations, and current consumption growth is not correlated with future income growth even when signals become more precise.

Income-conditional distribution of consumption We also investigate whether advance information improves the fit of the model's income-conditional consumption distribution to the data. We start with the stationary distribution of income implied by Tauchen and Hussey (1991)'s procedure and compute for each income state the conditional mean and standard deviation of consumption. In the CEX data, we employ the exact same income percentiles as in the model to slice the data and to compute the two conditional moments accordingly. For our calibration with six income states, these are the following percentiles: [17th, 33th, 50th, 67th, 83th]. For example, households with a high income represent the top 17% of income earners in the CEX. With uninformative signals, the income-conditional moments are quite far away from the data because risk-sharing is, with a coefficient of 0.94, too high. To facilitate a fair comparison, we employ in case of uninformative signals an endowment economy with $N = 6$ but without capital that yields a risk-sharing ratio of 0.83.¹⁷

In Figure 1, we plot the conditional mean of log consumption in the data, for uninformative signals and for informative signals with precision $\tilde{\kappa} = 0.116$. In the absence of signals, the average consumption of low-income households is too high compared

¹⁵The CEX is a revolving panel in which households drop out after 1 year. For each household, the CEX contains only information of household consumption and income at two different points in time. For this reason, we can neither estimate correlations of current consumption with future income growth nor employ the estimators to measure consumption responses to transitory and persistent shocks, as proposed by Blundell, Pistaferri, and Preston (2008).

¹⁶This result also applies for more precise signals. In an endowment economy, we compute p -values larger than 60% even when information precision is higher than $\kappa = 0.95$.

¹⁷Alternatively, using a model with uninformative signals and the possibility to return from autarky, as in Broer (2013), yields similar conditional consumption moments and are available on request.

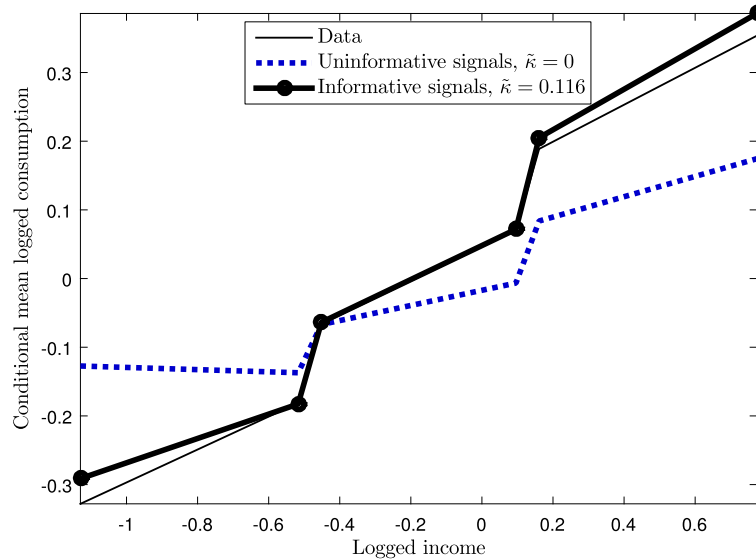


FIGURE 1. *ESC* model. Conditional mean of logged consumption with respect to logged income for different precisions of signals. The x -axis captures the log income and y -axis represents the conditional mean of log consumption. Income steps represent percentiles: [17th, 33th, 50th, 67th, 83th]. Solid line captures the conditional means for the years 1999–2003 in the CEX.

to the data while the consumption of high-income agents is too low. Further, average consumption is constant for the two low-income groups in the absence of information. In the data, however, average consumption is increasing for all income states. First, advance information results in more dispersed household consumption, which resembles the logic of the first part of Proposition 1: average consumption of low-income households decreases while consumption of high-income households increases. Further, the conditional mean of consumption with informative signals is now increasing in income over all incomes states. Taken together, the conditional mean of consumption is tracked in an almost perfect way for informative signals over all six income groups.

As displayed in Table 4, the mean-squared deviations of the conditional mean of consumption between model and data are approximately 34 times as large in the absence of signals than for $\tilde{\kappa} = 0.116$; for $\tilde{\kappa} = 0.124$, the mean deviations are 4.5 times higher than for $\tilde{\kappa} = 0.116$ but still over 7 times lower than in the standard model. Moreover, the spread between average consumption of high- and low-income households in the CEX data of 0.68 is perfectly captured by signals with $\tilde{\kappa} = 0.116$.

Figure 2 shows the income-conditional standard deviation of consumption in the data, for uninformative and for informative signals with precision $\tilde{\kappa} = 0.116$. The second part of Proposition 1 suggests that more precise signals increase the income-conditional standard deviations of consumption. Following the intuition from the simple model, advance information in the *ESC*-model indeed results in a higher conditional standard deviation for all income groups. In particular, information leads to an increase in consumption dispersion conditional on a high income; with uninformative signals,

TABLE 4. Conditional moments of consumption.

	$\tilde{\kappa} = 0$ (0.17)	$\tilde{\kappa} = 0.116$ (0.43)	$\tilde{\kappa} = 0.124$ (0.44)	Data
$MSE[E(\log(c) y)]^R$	34.15	1	4.66	–
$\Delta E[\log(c) y]$	0.30	0.68	0.80	0.68
$MSE[STD(\log(c) y)]^R$	3.47	1	0.91	–
$\frac{STD[\log(c) y_{\max}]}{STD[\log(c) y_{\min}]}$	0	0.38	0.36	0.95

Note: ESC-model. In Rows 1 and 3, the table provides the mean squared deviations of model and data for the conditional means and standard deviations of consumption expressed relative to signals with $\tilde{\kappa} = 0.116$ (0.43), $MSE[E(\log(c)|y)]^R$, and $MSE[STD(\log(c)|y)]^R$; for example, the entry 34.15 means that the mean squared deviations for the income-conditional mean of consumption of model and data are 34 times larger without signals than in case of signals with $\tilde{\kappa} = 0.116$ (0.43). Row 2 displays the spread of average consumption of households with the lowest (y_{\min}) and the highest income realization (y_{\max}), $\Delta E[\log(c)|y] = E[\log(c)|y_{\max}] - E[\log(c)|y_{\min}]$. Row 4 shows the ratio of the conditional standard deviation of consumption of households with the lowest and the highest income realization. Uninformative signals, $\tilde{\kappa} = 0$, endowment economy with $N = 6$.

the standard deviation is zero while with informative signals it is positive and increasing in signal precision. With advance information, the conditional standard deviation is tracked reasonably well for low- and middle-income earners, however, the distance to the data increases for the high-income groups.

As can be seen in the last two rows of Table 4, there is also measurable improvement in fit for the conditional standard deviation of consumption. Relative to the model without signals, the mean-square error is 3.5 times smaller for the model with signals

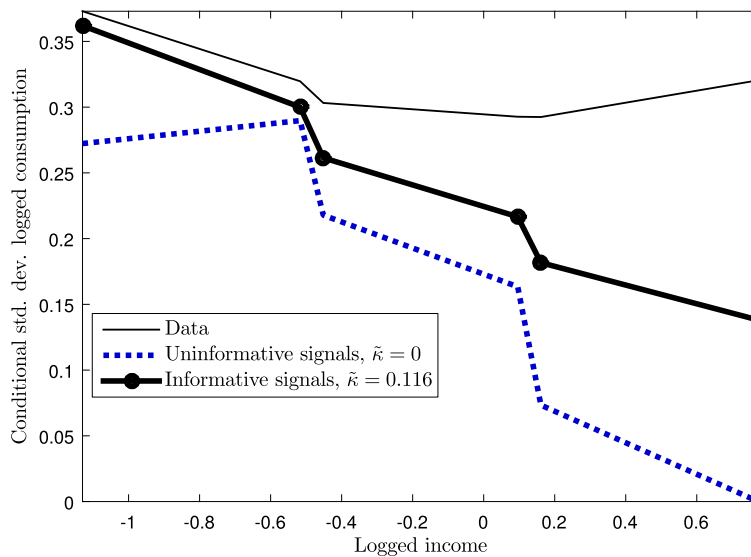


FIGURE 2. ESC model. Conditional standard deviation of logged consumption with respect to logged income for different precisions of the signals. The x-axis captures the logged income and the y-axis the conditional standard deviation of logged consumption. Income steps represent percentiles: [17th, 33th, 50th, 67th, 83th]. Solid line captures the conditional standard deviations for the years 1999–2003 in the CEX.

of precision $\tilde{\kappa} = 0.116$, and approximately 4 times smaller in case of $\tilde{\kappa} = 0.124$. Furthermore, the ratio of the conditional standard deviations for households with the highest and lowest income realizations increases from 0 in the standard model to 0.4 with advance information. This increase is, however, too small to capture the ratio of almost 1 observed in the CEX. Overall, the model with advance information can also better track the conditional standard deviation of consumption in the data but the improvement is not as striking as for the conditional mean.

5.2 Consumption insurance with incomplete markets

In this section, we study how advance information consumption insurance in a standard incomplete markets (*SIM*) model.

5.2.1 Environment While preferences and endowments are as described in Section 3, households in the standard incomplete markets economy can only trade in a single non-state contingent bond with gross return R and face an exogenous borrowing limit \bar{a} . There are no enforcement frictions, and we directly focus on stationary allocations. The model we consider is similar to Huggett (1993) and relies on a market structure with a continuum of households, as in Aiyagari (1994). Given asset holdings a , state $s = (y, k)$, and an interest rate R , households' problem can be written recursively as

$$V(a, s) = \max_{c, a'} \left[(1 - \beta)u(c) + \beta \sum_{s'} \pi(s'|s)V(a', s') \right]$$

subject to a budget and a borrowing constraint

$$\begin{aligned} c + a' &\leq wy + Ra, \\ a' &\geq -\bar{a}. \end{aligned}$$

Here, households differ with respect to initial asset holdings and initial shocks where the heterogeneity is captured by the probability measure $\Psi_{a,s}$. The state space is given by $M = A \times S$, where $A = [-\bar{a}, \infty)$.

The stationary recursive competitive equilibrium is summarized in the following definition.

DEFINITION 2. A stationary recursive competitive equilibrium in the standard incomplete markets economy comprises a value function $V(a, s)$, prices R, w , an allocation $c(a, s), a'(a, s)$, K a joint probability measure of assets and the state $\Psi_{a,s}$, and an exogenous borrowing limit \bar{a} such that:

- (i) $V(a, s)$ is attained by the decision rules $c(a, s), a'(a, s)$ given R .
- (ii) The joint distribution of assets and state $\Psi_{a,s}$ induced by $a'(a, s)$ and P_s is stationary.
- (iii) Factor prices satisfy

$$\begin{aligned} R - 1 &= AF_K(1, K) - \delta, \\ w &= AF_L(1, K). \end{aligned}$$

(iv) The bond market clears

$$\int a'(a, s) d\Psi_{a,s} = K.$$

Households are restricted to trading a single nonstate contingent asset. This implies that the distinction between public or private information is irrelevant in the *SIM*-model.

5.2.2 Quantitative results As emphasized by [Blundell, Pistaferri, and Preston \(2008\)](#) and [Kaplan and Violante \(2010\)](#), in a *SIM* model, better information on future income realizations allows households to improve on their consumption-savings decisions, and risk sharing improves. Thus, better information has a positive effect by improving individual decisions, which is referred to as the [Blackwell \(1953\)](#) effect of information. For generating the quantitative results, we employ for the common parameters the parameter values listed in [Table 1](#). [Wolff \(2011\)](#) finds that 19% of all US households are borrowing constrained. For this reason, we choose an exogenous borrowing limit \bar{a} to yield in equilibrium 19% of borrowing-constrained households with uninformative signals.

Consumption insurance Quantitatively, we find that risk-ratios improve monotonically in information precision, but this improvement is too small to capture the risk-sharing ratio of 0.60 observed in the data even for very informative signals. In the absence of signals, the model implies that households share about 40% of all fluctuations in their after-tax income. As an extreme case, if information precision amounts to $\kappa = 0.99$ —corresponding to a reduction of income uncertainty $\tilde{\kappa}$ of 97%—the risk-sharing ratio reaches 0.51. Thus, the increase in risk sharing by better information is quantitatively too small to capture the insurance observed in CEX data.

The simulation results for the regression coefficient displayed in [Table 5](#) confirm the findings from the first consumption insurance measure. For the standard case of uninformative signals, current consumption growth reacts with a coefficient of 0.32 too sensitively to changes in current income. With better information, the sensitivity decreases to 0.29 for $\tilde{\kappa} = 0.21$ as the upper value estimated by [Dominitz \(1998\)](#). Even for a very high $\tilde{\kappa} = 0.97$, the coefficient $\beta_{\Delta y_t}$ is, with a value of 0.17, too high as compared to the data.

TABLE 5. Income and consumption growth regressions.

	$\tilde{\kappa} = 0.00$ (0.17)	$\tilde{\kappa} = 0.12$ (0.43)	$\tilde{\kappa} = 0.21$ (0.54)	$\tilde{\kappa} = 0.76$ (0.9)	Data
$\beta_{\Delta y_t}$	0.32 [0.00]	0.31 [0.00]	0.29 [0.00]	0.20 [0.01]	0.11 0.00
β_2	-0.03 [0.74]	-0.01 [0.91]	0.01 [0.94]	0.12 [0.10]	- -

Note: *SIM* model. In the table, we provide regression coefficients and their *p* values for the regression $\Delta c_t^i = \beta_0 + \beta' \Delta y^i + \epsilon_t^i$, with $\beta = [\beta_{\Delta y_t}, \beta_2]'$ and $\Delta y^i = [\Delta y_t^i, \Delta y_{t+1}^i]'$. *p*-values are reported in square brackets. Values for κ in parentheses. Uninformative signals, $\tilde{\kappa} = 0.00$, $\kappa = 0.17$, and $N = 6$. Informative signals, $N = 6$.

Why is consumption insurance not strongly responding to advance information? With incomplete markets, more precise signals have two opposite effects on consumption insurance. On the one hand, households can better predict their future income, which allows them to make better informed consumption-savings decisions, resulting in better consumption insurance. On the other hand, households face less income uncertainty, which reduces their incentives for precautionary savings and tends to decrease consumption insurance. The quantitative results imply that the first effect dominates the second, but the overall effect of advance information on consumption insurance is weaker than in the complete markets model.

With complete markets, the change in the credit limits as a result of advance information is the driving force. In the incomplete markets model, by contrast, the borrowing limits are not only exogenously given but independent from advance information. As a thought experiment, consider $\tilde{\kappa} = 0.118$, as in [Dominitz \(1998\)](#). For the calibrated value of \bar{a} that generates 19% of households in equilibrium which are borrowing-constrained with uninformative signals, the risk-sharing ratio equals 0.43. Now, suppose that the borrowing limit is relaxed by 100%, which corresponds to the average change in credit limits with complete markets. Then the risk-sharing ratio increases to 0.61. This result suggests that the missing feedback from advance information to borrowing limits is indeed the reason why consumption insurance is not sensitive with respect to advance information in the incomplete markets model.

Consumption–income growth correlations with advance information As signals become informative, the *SIM* model predicts that current consumption growth is counterfactually correlated with future income growth. For uninformative signals and informative signals with precisions below $\tilde{\kappa} = 0.76$, current consumption growth is uncorrelated with income growth one period ahead on a 10% significance level (see the first three columns). However, the regression coefficient of current consumption with current income growth of 0.20 is still too high, as compared to the 0.11 estimated in the data. From $\tilde{\kappa} = 0.76$ onward, the correlation between current consumption growth and one-period ahead income growth is statistically significantly different from zero and with a coefficient of $\beta_2 = 0.12$ also economically significant (see the fourth column). For $\tilde{\kappa} = 0.76$ as the highest value for $\tilde{\kappa}$ that yields no counterfactual correlation of current consumption with future income growth, the risk-sharing ratio equals 0.50, which however is short compared to the 0.60 observed in the data.

The logic behind the nonzero correlation between current consumption and future income growth in the *SIM* model can be rationalized as follows. Better information reduces the uncertain income fluctuations households want to insure, and thus decreases households' incentives for precautionary savings. Before the income shock realizes, households' consumption today reacts to the part of the future income shock that is known in advance, and future consumption reacts less to the income shock when it actually realizes. When signals are precise enough, today's consumption growth becomes correlated with future income growth.

In line with earlier findings of [Kaplan and Violante \(2010\)](#), we conclude that advance information cannot reconcile risk-sharing ratios or regression coefficients in a standard

incomplete markets model with the measures observed in the data. We find that the picture changes when we rather employ a complete markets model with endogenous solvency constraints. Here, advance information on future income shocks can bridge the gap between model and data. Correspondingly, we can indirectly infer households' advance information by matching the risk-sharing ratio or, alternatively, by capturing the sensitivity of household consumption to income changes. The resulting quantified amount of advance information with complete markets is consistent with direct estimates stemming from survey data on subjective earnings expectations.

6. CONCLUSIONS

In this paper, we have studied the relevance of advance information on future income shocks for understanding consumption insurance in the US. When households possess advance information on their future income shocks, there is a disconnect between the uncertainty as assessed by an econometrician and income uncertainty as perceived by households. As a main novel result, we have found that the importance of advance information crucially depends on the design of insurance markets.

With complete markets and endogenous solvency constraints, the model with advance information can explain consumption insurance better than existing models. For a realistic amount of advance information, the model jointly matches three distinct key consumption insurance measures that are not captured without advance information: (i) the unconditional variance of households consumption in the cross-section, (ii) the covariance of current consumption growth and income growth and (iii) the income-conditional mean of household consumption for six income groups. In a standard incomplete markets model, advance information affects households' consumption-saving decisions too little to bridge the gap to the data. Moreover—and in contrast to a complete-markets model—advance information can induce counterfactual correlations between current consumption and future income growth.

With their recent paper, [Heathcote, Storesletten, and Violante \(forthcoming\)](#) contributed to a lively debate on the optimal progressivity of taxes in the United States. One of the main arguments in favor of a progressive tax system is that it helps to insure idiosyncratic earnings uncertainty when private insurance is limited. Thereby, a higher tax progressivity reduces the earnings risk after taxes. Computing the optimal tax progressivity requires a precise estimate for households' earnings uncertainty. In particular, if there is a systematic uncertainty gap, as suggested in this paper, and income uncertainty is actually lower than what is typically considered, less tax progressivity might be more desirable than conventional wisdom suggests.

APPENDIX

A.1 Proof of Proposition 1

The first-order conditions for agents with a low income and a high signal in the first period are

$$\frac{u'(c_{h,1}^l)}{4} - \frac{\lambda_1^{rs}}{4} = 0, \quad \kappa \frac{u'(c_{h,2}^{lh})}{4} - \kappa \frac{\lambda_2^{rs}}{4} = 0, \quad (1 - \kappa) \frac{u'(c_{h,2}^{ll})}{4} - (1 - \kappa) \frac{\lambda_2^{rs}}{4} = 0.$$

Dividing through by κ and $(1 - \kappa)$ implies that $c_{h,1}^l, c_{h,2}^{lh}, c_{h,2}^{ll}$ have the identical marginal effect on social welfare. Thus, as long as the amount of resources is identical in both periods, we get $\lambda_1^{rs} = \lambda_2^{rs}$, and thus $c_{h,1}^l = c_{h,2}^{ll} = c_{h,2}^{lh} = c_h^l$. The first-order conditions for consumption of agents with a high income and a high signal in the first period can be written

$$\begin{aligned} \frac{u'(c_{h,1}^h)}{4} + \lambda_h^{h,pc} u'(c_{h,1}^h) - \frac{\lambda^{rs}}{4} &= 0, \\ \kappa \frac{u'(c_{h,2}^{hh})}{4} + \lambda_h^{h,pc} \kappa u'(c_{h,2}^{hh}) - \kappa \frac{\lambda^{rs}}{4} &= 0, \\ (1 - \kappa) \frac{u'(c_{h,2}^{hl})}{4} + \lambda_h^{h,pc} (1 - \kappa) u'(c_{h,2}^{hl}) - (1 - \kappa) \frac{\lambda^{rs}}{4} &= 0. \end{aligned}$$

It follows that $c_{h,1}^h = c_{h,2}^{hl} = c_{h,2}^{hh} = c_h^h$. In a similar way, we get $c_{l,1}^h = c_{l,2}^{hl} = c_{l,2}^{hh} = c_l^h, c_{l,1}^l = c_{l,2}^{ll} = c_{l,2}^{lh} = c_l^l$, and consumption of high-income agents follows directly from the binding participation constraints

$$\begin{aligned} 2u(c_h^h) &= u(e_{h,1}) + \kappa u(e_{h,2}) + (1 - \kappa)u(e_{l,2}) \\ &\geq 2u(c_l^h) = u(e_{h,1}) + (1 - \kappa)u(e_{h,2}) + \kappa u(e_{l,2}). \end{aligned}$$

1. The conditional mean of consumption of high-income agents is $c^h = (c_h^h + c_l^h)/2$ such that the derivative of it with respect to κ is

$$\frac{\partial c^h}{\partial \kappa} = \frac{u(e_{h,2}) - u(e_{l,2})}{2} \left(\frac{1}{u'(c_h^h)} - \frac{1}{u'(c_l^h)} \right) \geq 0.$$

From resource feasibility, it follows immediately that the conditional mean of consumption for low-income agents decreases in κ .

2. The conditional mean of consumption of high-income agents increases because c_h^h increases by more than c_l^h decreases. Thus, the conditional mean increases by less than c_h^h such that $(c_h^h - c^h)^2$ and $(c_l^h - c^h)^2$ increase and, therefore, also the conditional standard deviation of consumption of high-income agents. For low-income agents, there are two cases, either both enforcement constraints are slack or the enforcement constraints of low-income agents with a high signal bind (for sufficiently high precision). In the first case, the conditional standard deviation is zero because consumption of low-income agents is independent from signal realizations, that is, $c_h^l = c_l^l = c^l$. In the second case, it follows from the enforcement constraints that c_h^l is increasing in κ . From the first part, we get that the conditional mean of consumption of low-income agents decreases which implies that c_l^l decreases by more than c_h^l increases such that also the conditional standard of consumption for low-income agents increases in this case.

3. The unconditional mean of consumption in both periods equals $\bar{e} = (e_h + e_l)/2$ such that the unconditional variance is

$$\frac{1}{4} [(c_h^h - \bar{e})^2 + (c_l^h - \bar{e})^2 + (c_h^l - \bar{e})^2 + (c_l^l - \bar{e})^2].$$

The first two terms increase in κ because c_h^h increases by more than c_l^h decreases (see the first part), irrespectively whether c_l^h is larger or smaller than \bar{e} . When enforcement constraints of low-income agents are all slack, $c_l^l = c_l^l = c^l$ decreases in κ (see the first part) such that the last two term collapse and increase in κ . Mean consumption of high-income agents is always larger than the income mean: enforcement constraints of high-income agents bind, for uninformative signals, $c_l^h = c_h^h = c^h > \bar{e}$, and increases in κ further increase c^h . Thus, $(c_h^l + c_l^l)/2 < \bar{e}$, and only $c_h^l > \bar{e}$ is possible. When enforcement constraints of low-income agents with a high signal bind, their consumption increases in κ . However, only if $c_h^l < 0$, one of the last terms can decrease when κ increases. From the previous part, we get that c_l^l decreases by more than c_h^l increases such that the sum of the two last terms increases even when $c_h^l < 0$, and as a result the unconditional standard deviation of consumption increases in κ .

A.2 Joint distribution of endowments and signals: Formulas

In this subsection, we explain how to derive the formulas (9) and (10) stated in the main text.

We start with the derivation of the conditional probability of future endowments. Using the general formula for calculating conditional probabilities, we receive

$$\pi(y' = y_j | k = y_m, y = y_i) = \frac{\pi(y' = y_j, k = y_m, y = y_i)}{\pi(k = y_m, y = y_i)}.$$

The conditional probability can be simplified using the identity

$$\sum_{z=1}^N \pi(y' = y_z | k = y_m, y = y_i) = 1$$

to replace the denominator with the following expression:

$$\pi(k = y_m, y = y_i) = \sum_{z=1}^N \pi(y' = y_z, k = y_m, y = y_i).$$

The joint probability in the numerator is

$$\pi(y' = y_j, k = y_m, y = y_i) = \pi_{ij} \kappa^{\mathbf{1}_{m=j}} \left(\frac{1 - \kappa}{N - 1} \right)^{1 - \mathbf{1}_{m=j}},$$

where π_{ij} is the Markov transition probability for moving from endowment i to endowment z . For all endowment states that are not indicated by the signal, $j \neq m$, we assume here that their probability of occurrence conditional on the signal is identical and, therefore, equals $(1 - \kappa)/(N - 1)$. For the conditional probability of endowments, the general

formula can then be written as

$$\pi(y' = y_j | k = y_m, y = y_i) = \frac{\pi_{ij} \kappa^{1_{m=j}} \left(\frac{1 - \kappa}{N - 1} \right)^{1 - 1_{m=j}}}{\sum_{z=1}^N \pi_{iz} \kappa^{1_{m=z}} \left(\frac{1 - \kappa}{N - 1} \right)^{1 - 1_{m=z}}}, \quad (\text{A-1})$$

which resembles (9) in the main text. For example, with two equally likely persistent endowment states, the conditional probability of receiving a low endowment y_l in the future conditional on a high signal $k = y_h$ and a low endowment today is given according to (A-1) by

$$\pi(y' = y_l | k = y_h, y = y_l) = \frac{(1 - \kappa) \pi_{11}}{(1 - \kappa) \pi_{11} + (1 - \pi_{11}) \kappa}.$$

The joint transition probability $\pi(s' | s) = \pi(y', k' | k, y)$ can be computed by combining the conditional probability of income with an assumption on the signal process. With signals following an exogenous first-order Markov process, the conditional probability $\pi(y', k' | k, y)$ is given by

$$\pi(y' = y_j, k' = y_l | k = y_m, y = y_i) = \pi_{ml} \frac{\pi_{ij} \kappa^{1_{m=j}} \left(\frac{1 - \kappa}{N - 1} \right)^{1 - 1_{m=j}}}{\sum_{z=1}^N \pi_{iz} \kappa^{1_{m=z}} \left(\frac{1 - \kappa}{N - 1} \right)^{1 - 1_{m=z}}}, \quad \forall k', \quad (\text{A-2})$$

with $\pi(k' = y_l | k = y_m)$ as the Markov signal transition probabilities that are consistent with household rationality as further explained in the following section.

A.3 Choosing the signal process

In this section, we explain our procedure to reverse engineer consistent Markov signal processes, that is, signal processes that yield the conditional probabilities in equation (9) under rational expectations. As first step, we define our notion of consistent signal processes. Afterwards, we analytically characterize consistent Markov signal processes to deliver the following main messages. First, when the Markov transition matrix of endowments is symmetric consistency requires that the signals follow the same stochastic process as endowments and the consistent signal transition probabilities are independent of signal precision. Second, in case of a nonsymmetric transition matrix for endowments, the consistent Markov of signals is in general not identical to the endowment transition matrix and depends on the precision of signals.

We define consistent signal processes as follows.

DEFINITION 3 (Consistent Signal Processes). Consider the conditional probabilities $\pi(y' | s)$ and $\pi(s' | s)$ as defined in equations (9) and (10) for a given Markov signal transition matrix P_k whose elements are the transition probabilities $\pi(k' | k)$. A Markov signal process is consistent if the following two consistency requirements are satisfied:

– *Consistency requirement I*: The marginal distribution of the joint invariant distribution of endowments and signals, $\pi(s) = \pi(y, k)$, with respect to endowments equals the invariant distribution of endowments $\pi(y)$,

$$\hat{\pi}(y) = \sum_{k \in Y} \pi(s) = \sum_{k \in Y} \pi(y, k) \doteq \pi(y).$$

– *Consistency requirement II*: The conditional distribution of endowments $\pi(y'|y)$ follows from integrating $\pi(y'|s) = \pi(y'|y, k)$ with respect to signals,

$$\hat{\pi}(y'|y) = \sum_{k \in Y} \pi(y'|s) \pi(k|y) = \sum_{k \in Y} \pi(y'|y, k) \pi(k|y) \doteq \pi(y'|y),$$

with $\pi(k|y)$ as the probability of signal k conditional on endowment y ,

$$\pi(k|y) = \frac{\pi(k, y)}{\sum_k \pi(k, y)}.$$

Essentially, the two consistency requirements demand that households' subjective endowment transitions equal the actual endowment transitions. In the following, we deliver our two main messages in this section.

A.3.1 Symmetric endowment transition matrix In this subsection, we consider a symmetric endowment transition matrix and show that if and only if signals follow the same stochastic process as endowments, the two consistency requirements are satisfied. Furthermore, the consistent signal transition matrix is identical independent from signal precision. Consider an endowment process with two values $y_l < y_h$ and a symmetric transition matrix given by

$$P = \begin{bmatrix} p & 1-p \\ 1-p & p \end{bmatrix},$$

with $0 < p < 1$, rows represent the present endowment state and columns represent the future endowment states. For $p = 0.5$, endowments follow an i.i.d. process as a special case of a symmetric endowment transition matrix.

PROPOSITION 2. *Consider a Markov endowment process with transition matrix P and informative signals with $\kappa \in (0.5, 1)$.*

(i) *If signals follow the same stochastic process as endowments, then both consistency requirements are satisfied.*

(ii) *Consider a Markov process for signals with transition matrix \tilde{P} ,*

$$\tilde{P} = \begin{bmatrix} \tilde{p} & 1-\tilde{p} \\ 1-\tilde{p} & \tilde{p} \end{bmatrix}$$

and $0 < \tilde{p} < 1$, $\tilde{p} \neq p$. Then Consistency Requirement II is violated.

PROOF. (i) When signals follow the same transition probabilities as endowments, the transition probabilities of s can be computed and are then summarized in the transition matrix P_s . For example, the probability of a low endowment and a low signals conditional on a low endowment and signal is

$$\pi(y' = y_l, k' = y_l | k = y_l, y = y_l) = p \frac{\kappa p}{(1 - \kappa)(1 - p) + p\kappa}.$$

The unique stationary distribution corresponding to the transition matrix P_s is given by

$$\pi(y, k) = \begin{bmatrix} \pi(y_l, k_l) \\ \pi(y_l, k_h) \\ \pi(y_h, k_l) \\ \pi(y_h, k_h) \end{bmatrix} = \begin{bmatrix} \kappa p - \frac{p}{2} - \frac{\kappa}{2} + \frac{1}{2} \\ \frac{\kappa}{2} + \frac{p}{2} - \kappa p \\ \frac{\kappa}{2} + \frac{p}{2} - \kappa p \\ \kappa p - \frac{p}{2} - \frac{\kappa}{2} + \frac{1}{2} \end{bmatrix}.$$

Adding the first two and last two rows show that Consistency Requirement I is satisfied. Further, the probabilities of signals conditional on endowments can be computed from the invariant distribution. For example, the probability of a low signal conditional on a low endowment can be computed as

$$\pi(k = y_l | y = y_l) = \frac{\kappa p - \frac{p}{2} - \frac{\kappa}{2} + \frac{1}{2}}{\kappa p - \frac{p}{2} - \frac{\kappa}{2} + \frac{1}{2} + \frac{\kappa}{2} + \frac{p}{2} - \kappa p} = 2\kappa p - \kappa - p + 1.$$

To check for the Consistency Requirement II, we consider present endowment $y = y_l$ and future endowment $y' = y_l$ (the other transitions can be computed in the same way and are omitted here)

$$\begin{aligned} \hat{\pi}(y' = y_l | y = y_l) &= \sum_{k \in Y} \pi(y' = y_l | y = y_l, k) \pi(k | y = y_l) \\ &= \frac{\kappa p}{\kappa p + (1 - \kappa)(1 - p)} (2\kappa p - \kappa - p + 1) \\ &\quad + \frac{p(1 - \kappa)}{\kappa(1 - p) + p(1 - \kappa)} (\kappa + p - 2\kappa p) \\ &= p, \end{aligned}$$

which is also satisfied. From the other side, for the transition from low endowment today to low endowment in the future, Requirement II calls for

$$p \doteq \sum_{k \in Y} \pi(y' = y_l | y = y_l, k) \hat{\pi}(k | y = y_l),$$

which has as unique solution $\hat{\pi}(k = y_l | y = y_l) = 2\kappa p - \kappa - p + 1$ which completes the proof of part (i).

(ii) The general symmetric transition matrix for signals \tilde{P} results in a joint transition matrix for signals and endowments \tilde{P}_s and in a unique invariant distribution for endowment and signals $\tilde{\pi}(y, k)$ with a unique conditional probability $\tilde{\pi}(k = y_l | y = y_l)$. If and only if $\tilde{p} = p$, it is $\tilde{\pi}(k = y_l | y = y_l) = \hat{\pi}(k = y_l | y = y_l) = 2\kappa p - \kappa - p + 1$. Thus, Requirement II is violated for $\tilde{p} \neq p$. Requirement I is satisfied because $\sum_k \tilde{\pi}(y_l, k) = 1/2 = \sum_k \tilde{\pi}(y_h, k)$ for any $0 < \tilde{p} < 1$. \square

As an immediate implication of the proposition, i.i.d. signals violate Requirement II when endowments are persistent.

A.3.2 Nonsymmetric endowment transition matrix We continue our analysis with considering the case of nonsymmetric endowment transitions. As before, we consider a two-state endowment process but now the endowment transition matrix is more general and given by

$$P_g = \begin{bmatrix} p_{11} & 1 - p_{11} \\ 1 - p_{22} & p_{22} \end{bmatrix},$$

where rows represent the present endowment state and columns represent the future endowment states, $0 < p_{11}, p_{22} < 1$, and $p_{11} \neq p_{22}$.

PROPOSITION 3. Consider a Markov endowment process with transition matrix P_g and informative signals with $\kappa \in (0.5, 1)$.

(i) The transition matrix for signals that satisfies Consistency Requirement II is

$$P_k = \begin{bmatrix} p_{11}^k & 1 - p_{11}^k \\ 1 - p_{22}^k & p_{22}^k \end{bmatrix},$$

with $p_{11}^k = p_{22}(1 - \kappa) + \kappa p_{11}$, $p_{22}^k = p_{11}(1 - \kappa) + \kappa p_{22}$.

(ii) Signals that follow the transition P_k also satisfy Consistency Requirement I.

PROOF. (i) To satisfy the second consistency requirement, the following two equations must be satisfied:

$$p_{11} = \sum_{k \in Y} \pi(y' = y_l | y = y_l, k) \pi(k | y = y_l), \tag{A-3}$$

$$p_{22} = \sum_{k \in Y} \pi(y' = y_h | y = y_h, k) \pi(k | y = y_h). \tag{A-4}$$

The logic of the proof is to use (A-3)–(A-4) to solve for the two signal transition probabilities p_{11}^k, p_{22}^k . From the terms on the right-hand side of the two equations, only the conditional probabilities $\pi(k | y = y_l), \pi(k | y = y_h)$ are functions of the signal transition probabilities; the conditional probabilities $\pi(y' = y_l | y = y_l, k), \pi(y' = y_h | y = y_h, k) \pi(k | y = y_h)$

are independent from the signal transition probabilities. The former probabilities are computed as follows. For given p_{11}^k, p_{22}^k , solve first for the joint invariant distribution of endowments and signals $\pi(y, k)$ and then use this invariant distribution to derive the conditional probabilities $\pi(k|y = y_l), \pi(k|y = y_h)$ as functions of p_{11}^k and p_{22}^k . The resulting expressions for the conditional probabilities are in closed form but rather tedious and not reported here. Substituting these expressions in (A-3) and (A-4) and solving for p_{11}^k, p_{22}^k eventually gives

$$p_{11}^k = p_{22}(1 - \kappa) + \kappa p_{11}, \quad p_{22}^k = p_{11}(1 - \kappa) + \kappa p_{22}$$

if the following two regularity conditions hold:

$$p_{22}(1 - \kappa) + \kappa p_{11} < 1, \quad p_{11}(1 - \kappa) + \kappa p_{22} < 1,$$

which are satisfied for $\kappa \in [0.5, 1]$ and $0 < p_{11}, p_{22} < 1$.

(ii) The invariant distribution of income (y_l, y_h) is given by

$$\pi(y_l, y_h) = \left(\frac{1 - p_{22}}{2 - p_{11} - p_{22}}, \frac{1 - p_{11}}{2 - p_{11} - p_{22}} \right).$$

Start with the joint invariant distribution of endowments and signals, substitute for p_{11}^k, p_{22}^k with the solutions found in (i) to receive the following expression for the joint distribution $\pi(y, k)$:

$$\pi(y, k) = \begin{bmatrix} \pi(y_l, k_l) \\ \pi(y_l, k_h) \\ \pi(y_h, k_l) \\ \pi(y_h, k_h) \end{bmatrix} = \begin{bmatrix} \frac{(1 - p_{22})(\kappa + p_{11} - 2\kappa p_{11} - 1)}{2 - p_{11} - p_{22}} \\ \frac{(1 - p_{22})(\kappa + p_{11} - 2\kappa p_{11})}{2 - p_{11} - p_{22}} \\ \frac{(1 - p_{11})(\kappa + p_{22} - 2\kappa p_{22} - 1)}{2 - p_{11} - p_{22}} \\ \frac{(1 - p_{11})(\kappa + p_{22} - 2\kappa p_{22})}{2 - p_{11} - p_{22}} \end{bmatrix}.$$

Adding the first two and the last two rows produces $\pi(y_l, y_h)$ such that the first consistency requirement is satisfied as well. \square

The results summarized in the proposition generalize the findings for symmetric transitions. The signal transition matrix depends in general on the precision of signals. Only when the income transition is symmetric, the transition probabilities for signals are independent of κ and are given by the corresponding income transition probabilities.

Unlike in the case of a symmetric endowment transition, i.i.d. signals now neither satisfy the first nor the second consistency requirement. The rationale why now also the first requirement is violated is as follows. Without loss of generality, consider $p_{11} > p_{22}$ such that the ergodic distribution is characterized by $\pi(y_l) > \pi(y_h)$. With $p_{11} > p_{22}$, a larger fraction of households with a low income should receive a low signal than households with a high income receive a high signal. For i.i.d. signals, the fractions are equal.

TABLE A1. Consistency requirements with i.i.d. signals and signals consistent with household rationality as summarized in Definition 3. Signal precision $\kappa = 0.99$. ARMA(1, 1)-income process with $N = 6$, $\theta = 0.9331$, $\rho = 0.9989$, and $\sigma_u = 0.3508$.

	Consistency Requirements	
	I, $\max(\hat{\pi}(y) - \pi(y))$	II, $\max(\hat{\pi}(y' y) - \pi(y' y))$
IID signals	0.0115	0.1620
Household-rational signals	$3.70e-16$	$2.11e-16$

As a consequence, households underestimate the fraction of people with a low income and over estimate the fraction of households with a high income.

For $N > 2$, we apply a numerical procedure. For each κ , we use the $N^2 - N$ restrictions imposed by Consistency Requirement II to solve for the transition probabilities p_{ij}^k . Then we check whether the first consistency requirement is satisfied given the probabilities p_{ij}^k . In Table A1, we compare i.i.d. signals to the signal transition consistent household rationality using the endowment process employed for computing the quantitative results in the main text for $\kappa = 0.99$ as an extreme case. As displayed in the first row of the table, i.i.d. signals fail both consistency requirements. The inconsistency following from i.i.d. signals is not negligible. On average, i.i.d. signals imply a perceived transition that differs from the true transition by 16%. When we compute signal transition probabilities according to the numerical procedure, the second requirement is satisfied by construction, while the first requirement—in line with the second part of Proposition 3—is satisfied, too.

A.4 Gaussian signals

In this section, we consider Gaussian signals that inform on future innovations to income. First, we derive an equivalent news representation to the noise specification with signals following the work of Chahrour and Jurado (2018). Second, we describe how the news representation can be discretized. Afterwards, we set up the relevant state space for computations and find it is of a higher dimension than in case of the hit-or-miss specification considered in the main text. Despite of this difference, the uncertainty gap as the reduction in the mean-squared forecast error of future income resulting from conditioning probabilities on informative signals offers a notion of equivalence between the two signal specifications.

Gaussian signals: Noise and news Consider the following ARMA(1, 1)-process for logged income used to generate the quantitative results in the main text

$$\ln(y_{it}) = \rho \ln(y_{it-1}) - \theta u_{it-1} + u_{it}, \quad u_{it} \stackrel{\text{i.i.d.}}{\sim} (0, \sigma_u^2),$$

with unconditional variance

$$\text{var}[\ln(y_{it})] = \frac{1 + \theta^2 - 2\rho\theta}{1 - \rho^2} \sigma_u^2. \quad (\text{A-5})$$

Households receive a noisy signal s_{it} on future innovations to income

$$s_{it} = u_{it+1} + v_{it}, \quad v_{it} \stackrel{\text{i.i.d.}}{\sim} (0, \sigma_v^2)$$

such that

$$E(u_{it+1}|s_{it}) = s_{it} \frac{\text{cov}(u_{it+1}, s_{it})}{\text{var}(s_{it})} = \frac{\sigma_u^2}{\sigma_u^2 + \sigma_v^2} s_{it} = \tau s_{it},$$

with $\text{cov}(u_{it+1}, s_{it})$ as the unconditional covariance between signals and future innovations to income and τ as the Kalman gain. Income expectations conditional on all available information at time t including the signal s_{it} are therefore given by

$$E_t[\ln(y_{it+1})] = \rho \ln(y_{it}) - \theta u_{it} + E(u_{it+1}|s_{it}) = \rho \ln(y_{it}) - \theta u_{it} + \tau s_{it}. \quad (\text{A-6})$$

Following [Chahrouh and Jurado \(2018\)](#), we can equivalently represent the noise model specified above as an advance information (or news) model in which income is composed of two orthogonal parts

$$\ln(y_{it}) = d_{it-1} + b_{it},$$

with d_{it-1} as the component of period t 's income already known in period $t - 1$ and b_{it} as the income component revealed in period t . One can show that the two orthogonal components d_{it-1} , b_{it} should follow the same type of stochastic process as income—an ARMA(1, 1)—with identical parameters ρ , θ . Thus, the advance information model is given by

$$\ln(y_{it}) = d_{it-1} + b_{it}, \quad (\text{A-7})$$

$$d_{it} = \rho d_{it-1} + \epsilon_{it}^d - \theta \epsilon_{it-1}^d, \quad \epsilon_{it}^d \stackrel{\text{i.i.d.}}{\sim} \mathcal{N}(0, \sigma_{\epsilon^d}^2), \quad (\text{A-8})$$

$$b_{it} = \rho b_{it-1} + \epsilon_{it}^b - \theta \epsilon_{it-1}^b, \quad \epsilon_{it}^b \stackrel{\text{i.i.d.}}{\sim} \mathcal{N}(0, \sigma_{\epsilon^b}^2). \quad (\text{A-9})$$

To pin down the parameters $\sigma_{\epsilon^d}^2$, $\sigma_{\epsilon^b}^2$, we need two conditions. The first condition renders the conditional income expectations in the noise and news representation equivalent. Using equation (A-6) and equations (A-7)–(A-9), yields the following expression:

$$\begin{aligned} E_t(d_{it} + b_{it+1}) &= \rho \ln(y_{it}) + \epsilon_{it}^d - \theta \epsilon_{it-1}^d - \theta \epsilon_{it}^b \stackrel{!}{=} \rho \ln(y_{it}) - \theta u_{it} + \tau s_{it} \\ \Leftrightarrow \epsilon_{it}^d - \theta \epsilon_{it-1}^d - \theta \epsilon_{it}^b &\stackrel{!}{=} \tau s_{it} - \theta u_{it}. \end{aligned}$$

Equalize the variances of the left-hand and right-hand side to eventually get the following condition:

$$(1 + \theta^2) \sigma_{\epsilon^d}^2 + \theta^2 \sigma_{\epsilon^b}^2 \stackrel{!}{=} \frac{\sigma_u^4}{\sigma_u^2 + \sigma_v^2} + \theta^2 \sigma_u^2. \quad (\text{A-10})$$

Unconditional moments of income are directly observed in the data and should not be affected by introducing news or noise. Thus, the second condition is a consistency

condition that requires, for example, that the unconditional variance of income in (A-5) is unaffected by the introducing advance information on a period-to period basis which results in the following:

$$\begin{aligned} \text{var}(d_{it-1}) + \text{var}(b_{it}) &\stackrel{!}{=} \text{var}[\ln(y_{it})] \\ \Leftrightarrow \frac{1 + \theta^2 - 2\rho\theta}{1 - \rho^2} (\sigma_{\epsilon^d}^2 + \sigma_{\epsilon^b}^2) &\stackrel{!}{=} \frac{1 + \theta^2 - 2\rho\theta}{1 - \rho^2} \sigma_u^2. \end{aligned} \quad (\text{A-11})$$

Using the two conditions (A-10) and (A-11), the two parameters can be computed as

$$\sigma_{\epsilon^d}^2 = \frac{\sigma_u^4}{\sigma_v^2 + \sigma_u^2} = \frac{\sigma_u^2}{\sigma_v^2 + \sigma_u^2} \sigma_u^2 = \tau \sigma_u^2$$

and

$$\sigma_{\epsilon^b}^2 = \frac{\sigma_u^2 \sigma_v^2}{\sigma_v^2 + \sigma_u^2} = (1 - \tau) \sigma_u^2$$

such that

$$\sigma_u^2 = \sigma_{\epsilon^d}^2 + \sigma_{\epsilon^b}^2.$$

Discretization and state space Consider finite-state Markov approximations of b_{it}, d_{it} —using, for example, the method described in [Tauchen and Hussey \(1991\)](#)—with the two time invariant and finite sets of possible realizations containing N elements each, that is, $b_{it} \in B \equiv \{b_1, \dots, b_N\}$ and $d_{it} \in D \equiv \{d_1, \dots, d_N\}$.¹⁸ Furthermore, the Markov approximation results in transition matrixes P_b, P_d whose elements are the time-invariant transition probabilities, $\pi(b'|b)$ and $\pi(d'|d)$, respectively. Logged income is given by the sum of a particular pair ($d = d_i, b = b_j$) such that also the set of possible income realizations is time invariant and finite but of dimension N^2 , where d is today's income component that is known already since the previous period. This is not yet the whole state vector. Instead the whole state vector comprises current income as well as the known part of next period's income, d' , resulting in the state given by $s_g = (y, d' = d_k)$ with S_g as the set of time-invariant realizations of s_g comprising N^3 elements, with subscript g for Gaussian signals. Thus, the dimension of the state space in case of Gaussian signals exceeds the dimension of the state space of the hit-or-miss specification from the main text by factor N . The conditional income probability with Gaussian signals is denoted by $\pi(y'|s_g)$, instead of $\pi(y'|s)$ in the baseline, and given by

$$\begin{aligned} \pi(y'|s_g) &= \pi(d' = d_l, b' = b_m | d = d_i, b = b_j, d' = d_k) \\ &= \begin{cases} 0 & \text{if } d_l \neq d_k, \\ \pi(b' = b_m | b = b_j) & \text{if } d_l = d_k. \end{cases} \end{aligned} \quad (\text{A-12})$$

¹⁸The number of elements of B, D can in principle be freely chosen but for comparability we choose the number of elements in B and A to be the same as the number of income states as in the main body of the paper. In case that income is either known completely one period in advance or logged income consists exclusively of the risky component, the approximation of income boils down to the approximation of the ARMA(1, 1) in the main text with N states.

Correspondingly, the transition probability of the state $\pi(s'_g|s_g)$ reads as follows:

$$\begin{aligned}\pi_g(s'_g|s_g) &= \pi(d' = d_l, b' = b_m, d'' = d_n | d = d_i, b = b_j, d' = d_k) \\ &= \begin{cases} 0 & \text{if } d_l \neq d_k, \\ \pi(d' = d_n | d = d_k) \pi(b' = b_m | b = b_j) & \text{if } d_l = d_k, \end{cases} \end{aligned} \quad (\text{A-13})$$

with d'' denotes the income component two periods in the future that is known already one period before; the formulas use that the processes for d and b are independent and that due to the Markov property $\pi(d' = d_n | d = d_k) = \pi(d'' = d_n | d' = d_k)$.

Mean-squared forecast errors and notion of equivalence The dimension and the objects in the state vector of the hit-or-miss and the Gaussian signal specification are different. Nevertheless, we can establish a notion of equivalence between the two specifications using the uncertainty gap. To serve that goal, we first define the uncertainty gap for Gaussian signals analogue to (20) as follows:

$$\tilde{\kappa}_g(\tau) = \frac{\text{MSFE}_y - \text{MSFE}_{s,g}(\tau)}{\text{MSFE}_y}, \quad 0 \leq \tilde{\kappa}_g(\tau) \leq 1, \quad (\text{A-14})$$

with MSFE_y as the forecast error defined in (21) and the mean-squared forecast error conditioning additionally on Gaussian signals $\text{MSFE}_{s,g}$ given by

$$\text{MSFE}_{s,g}(\tau) = \sum_{s_g} \pi(s_g) \sum_{y'} \pi(y'|s_g) [y' - \text{E}(y'|s_g)]^2 \leq \text{MSFE}_y,$$

with $\pi(s_g)$ as the joint invariant distribution of the state induced by (A-13), conditional income probabilities $\pi(y'|s_g)$ as defined in (A-12), and $\text{E}(y'|s_g)$ as the conditional income mean with Gaussian signals. The hit-or-miss and the Gaussian-signal specification are equivalent with respect to their income forecast errors if and only if

$$\text{MSFE}_{s,g}(\tau) = \text{MSFE}_s(\kappa) \Leftrightarrow \tilde{\kappa}_g(\tau) = \tilde{\kappa}(\kappa). \quad (\text{A-15})$$

Consider the income specification with $N = 6$ income states as described in Section 4. In our baseline specification, the number of income-signals states is 36 while with Gaussian signals the number of states is 216. The increased size of the state space with Gaussian signals makes computing allocations (in particular but not only in the complete-markets model) prohibitively expensive. For this reason, we cannot compare the two signal specifications with respect to their resulting consumption allocations. Nevertheless, we can compare the specifications using the notion of equivalence as summarized in (A-15). The notion of equivalence is illustrated in Figure A1 which displays the uncertainty gap $\tilde{\kappa}$ as function of signal precision $\kappa \in [1/6, 1]$ for the hit-or-miss specification (left panel) and as a function of the Kalman gain $\tau \in [0, 1]$ for the Gaussian signals (right panel). As can be seen in the figure, for each value of κ there is a unique value τ that results in the same value of the uncertainty gap (or income mean-squared forecast error). For example, the 12% reduction in perceived income uncertainty by signals with precision $\kappa = 0.43$ can be equivalently achieved by Gaussian signals with a Kalman gain of $\tau = 0.14$. Given $\sigma_u = 0.35$, this Kalman gain corresponds to a noise variance of $\sigma_v^2 = 0.76$.

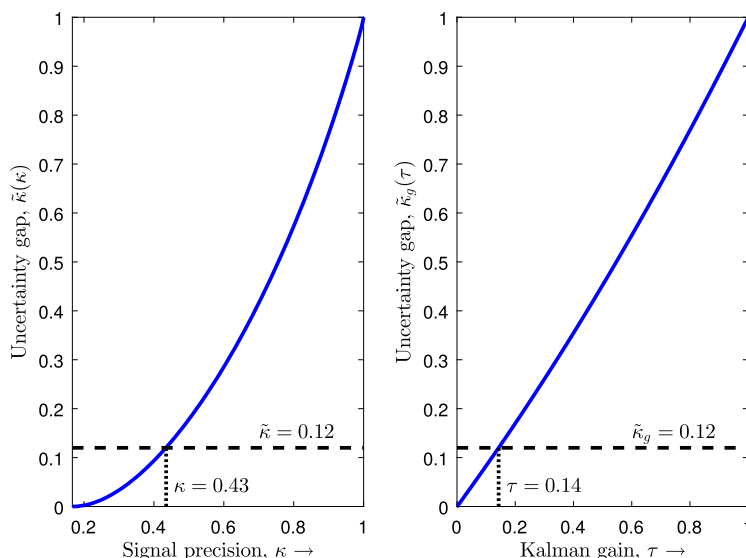


FIGURE A1. Equivalence of hit-or-miss and Gaussian signals. Left panel displays the uncertainty gap as a function of signal precision, $\tilde{\kappa}(\kappa)$, for the hit-or-miss specification as defined in (20). Right panel displays the uncertainty gap as a function of the Kalman gain, $\tilde{\kappa}_g(\tau)$, for the Gaussian-signals specification as defined in (A-14). The dashed horizontal line indicates an uncertainty gap of 0.12.

REFERENCES

- Ábrahám, Á. and E. Cárceles-Poveda (2010), “Endogenous trading constraints with incomplete asset markets.” *Journal of Economic Theory*, 145 (3), 974–1004. [681]
- Aiyagari, S. R. (1994), “Uninsured idiosyncratic risk and aggregate saving.” *Quarterly Journal of Economics*, 109 (3), 659–684. [671, 674, 694]
- Alvarez, F. and U. J. Jermann (2000), “Efficiency, equilibrium, and asset pricing with risk of default.” *Econometrica*, 68 (4), 775–798. [671, 674, 680]
- Atkeson, A. and R. E. Lucas (1992), “On efficient distribution with private information.” *Review of Economic Studies*, 59 (3), 427–453. [683]
- Attanasio, O. and B. Augsburg (2016), “Subjective expectations and income processes in rural India.” *Economica*, 83 (331), 416–442. [673]
- Blackwell, D. (1953), “Equivalent comparisons of experiments.” *The Annals of Mathematical Statistics*, 24 (2), 265–272. [695]
- Blundell, R., L. Pistaferri, and I. Preston (2008), “Consumption inequality and partial insurance.” *American Economic Review*, 98 (5), 1887–1921. [673, 674, 684, 690, 691, 695]
- Broer, T. (2013), “The wrong shape of insurance? What cross-sectional distributions tell us about models of consumption smoothing.” *American Economic Journal: Macroeconomics*, 5 (4), 107–140. [671, 674, 683, 684, 685, 691]

- Broer, T., M. Kapička, and P. Klein (2017), “Consumption risk sharing with private information and limited enforcement.” *Review of Economic Dynamics*, 23, 170–190. [683]
- Campbell, D., A. Carruth, A. Dickerson, and F. Green (2007), “Job insecurity and wages.” *The Economic Journal*, 117 (518), 544–566. [672]
- Chahrour, R. and K. Jurado (2018), “News or noise? The missing link.” *American Economic Review*, 108 (7), 1702–1736. [705, 706]
- Cunha, F., J. Heckman, and S. Navarro (2005), “Separating uncertainty from heterogeneity in life cycle earnings.” *Oxford Economic Papers*, 57 (2), 191–261. [674]
- Cunha, F. and J. Heckman (2016), “Decomposing trends in inequality in earnings into forecastable and uncertain components.” *Journal of Labor Economics*, 34 (S2), S31–S65. [674]
- Dalaker, J. and M. Naifeh (1998), “Poverty in the United States: 1997.” Current Population Reports, Series P60-201, US Bureau of the Census. [684]
- Dominitz, J. (1998), “Earnings expectations, revisions, and realizations.” *Review of Economics and Statistics*, 80 (3), 374–388. [672, 673, 684, 689, 695, 696]
- Dominitz, J. and C. F. Manski (1997), “Using expectations data to study subjective income expectations.” *Journal of the American Statistical Association*, 92 (439), 855–867. [672]
- Ejrnaes, M. and M. Browning (2014), “The persistent–transitory representation for earnings processes.” *Quantitative Economics*, 5 (3), 555–581. [685]
- Guvenen, F. (2007), “Learning your earning: Are labor income shocks really very persistent?” *American Economic Review*, 97 (3), 687–712. [674]
- Guvenen, F. and A. A. Smith (2014), “Inferring labor income risk and partial insurance from economic choices.” *Econometrica*, 82 (6), 2085–2129. [674, 690]
- Heathcote, J., K. Storesletten, and G. L. Violante (2014), “Consumption and labor supply with partial insurance: An analytical framework.” *American Economic Review*, 104 (7), 1–52. [674]
- Heathcote, J., K. Storesletten, and G. L. Violante (2017), “Optimal tax progressivity: An analytical framework.” *Quarterly Journal of Economics*, 132 (4), 1693–1754. [697]
- Hirshleifer, J. (1971), “The private and social value of information and the reward to inventive activity.” *American Economic Review*, 61 (4), 561–574. [674, 675]
- Huggett, M. (1993), “The risk-free rate in heterogeneous-agent incomplete-insurance economies.” *Journal of Economic Dynamics and Control*, 17 (5–6), 953–969. [694]
- Huggett, M., G. Ventura, and A. Yaron (2006), “Human capital and earnings distribution dynamics.” *Journal of Monetary Economics*, 53 (2), 265–290. [674]
- Kaplan, G. and G. L. Violante (2010), “How much consumption insurance beyond self-insurance?” *American Economic Journal: Macroeconomics*, 2 (4), 53–87. [674, 685, 695, 696]

- Kehoe, T. J. and D. K. Levine (1993), “Debt-constrained asset markets.” *Review of Economic Studies*, 60 (4), 865–888. [674]
- Krueger, D. and F. Perri (2006), “Does income inequality lead to consumption inequality? Evidence and theory.” *Review of Economic Studies*, 73 (1), 163–193. [671, 674, 683, 684, 685]
- Krueger, D. and F. Perri (2011), “Public versus private risk sharing.” *Journal of Economic Theory*, 146 (3), 920–956. [674]
- Mace, B. J. (1991), “Full insurance in the presence of aggregate uncertainty.” *Journal of Political Economy*, 99 (5), 928–956. [686]
- Primiceri, G. E. and T. van Rens (2009), “Heterogeneous life-cycle profiles, income risk and consumption inequality.” *Journal of Monetary Economics*, 56 (1), 20–39. [674]
- Schlee, E. E. (2001), “The value of information in efficient risk-sharing arrangements.” *American Economic Review*, 91 (3), 509–524. [675]
- Smith, V. K., D. H. Taylor, and F. A. Sloan (2001), “Longevity expectations and death: Can people predict their own demise?” *American Economic Review*, 91 (4), 1126–1134. [672]
- Stephens, M. (2004), “Job loss expectations, realizations, and household consumption behavior.” *Review of Economics and Statistics*, 86 (1), 253–269. [672]
- Stoltenberg, C. A. and S. Singh (2020), “Supplement to ‘Consumption insurance with advance information.’” *Quantitative Economics Supplemental Material*, 11, <https://doi.org/10.3982/QE1169>. [675]
- Storesletten, K., C. I. Telmer, and A. Yaron (2004), “Cyclical dynamics in idiosyncratic labor market risk.” *Journal of Political Economy*, 112 (3), 695–717. [685]
- Tauchen, G. and R. Hussey (1991), “Quadrature-based methods for obtaining approximate solutions to nonlinear asset pricing models.” *Econometrica*, 59 (2), 371–396. [685, 691, 707]
- Wolff, E. N. (2011), “Recent trends in household wealth, 1983–2009: The irresistible rise of household debt.” *Review of Economics and Institutions*, 2 (1), Article 4. [695]

Co-editor Peter Arcidiacono handled this manuscript.

Manuscript received 6 July, 2018; final version accepted 7 December, 2019; available online 24 January, 2020.