

SUPPLEMENT TO “PRODUCT DIFFERENTIATION,
MULTIPRODUCT FIRMS, AND ESTIMATING THE IMPACT
OF TRADE LIBERALIZATION ON PRODUCTIVITY”

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This supplement contains more information on the data and presents more details on the estimation of the multiproduct case. Finally, the conditions of invertibility of the input demand and investment functions are presented.

APPENDIX A: DATA—THE TEXTILE INDUSTRY AND THE QUOTA DATA SET

A.1. The Belgian Textile Industry

I present the structure of the different segments, subsegments, and the products in my data set in Table A.I. The different levels are important to structure the regressions and serve as additional sources of variation to identify demand parameters. The numbers in parentheses indicates the number of subsegments within a given segment, whereas the last row indicates the number of products within a given segment.

In Table A.II, I illustrate the detailed information I rely on to structure the demand shocks in my empirical model. I demonstrate the structure of the industry with the following example. I list all the products and product groups for the Mobiltech subsegment within the Technical textile segment. This structure is used as well to estimate and test the nested CES–logit demand system.

A.2. Producer Prices

As mentioned in the text, a producer price index is obtained by taking a weighted average over a representative number of products within an industry, where weights are based on sales (market shares). In the case of Belgium, the National Institute of Statistics (NIS) gathers monthly information on market relevant prices (including discounts if available) of around 2,700 representative products (an 8 digit classification (PRODCOM), where the first 4 digits indicate the NACEBELCODE). The index is constructed by using the most recent market share as weights based on sales reported in the official tax filings of the relevant companies. The relevant prices take into account both domestic and foreign markets, and for some industries both indices are reported.

An important part of the empirical analysis is the fact that I consider the EU to be the relevant market for Belgian producers. It is therefore important to verify whether some of the observed demand conditions are aligned for Belgium and the EU-15 market. I checked whether the trajectory of the producer price index (PPI) for Belgian manufacturing (excluding construction), manu-

TABLE A.I
SEGMENT STRUCTURE: NUMBER OF SUBSEGMENTS AND PRODUCTS PER SEGMENT

			Clothing (17)	
Interior (9)		Fabrics		Knitwear
Bed linen		Accessories		Accessories
Carpets		Baby & children's clothes		Babies' wear
Kitchen linen		Men's wear		Bath
Mattress ticking		Night wear & underclothing		Children's wear
Table linen		Others		Fabrics for
Terry toweling articles		Rain wear, sportswear, & leisure		Night wear
Trimming		Women's wear		Outerwear
Upholstery & furnishing fabric		Work wear & protective suits		Sportswear
Wall coverings				Stockings, tights, socks
				Underwear
19		61		36
Technical (9)		Finishing (7)		Spinning (9)
Agrotech	Carpeting	Blended aramid, polyamid, or polyacrylic		
Buildtech	Knitted fabrics	Blended artificial yarns		
Geotech	Material before spinning	Blended cotton or linen yarns		
Indutech	Nonwoven	Blended polyester yarns		
Medtech	Woven fabrics	Blended polypropylene or chlorofiber yarns		
Mobiltech	Yarns	Blended yarns		
Packtech	Specialties	Filament yarns		
Protech		Spun yarns (>85% of 1 fiber)		
Sporttech		Synthetic fibers		
231	132			84

facturing of textile products, and manufacturing of apparel is representative for the market at large. In contrast to the PPI for the manufacturing sector, producer prices for textiles and apparel products decreased in relative terms, and this was most notable in the period 1996–2000. The various PPIs for the EU-15 market as a whole follow this pattern very closely. I suppress the different graphs and tables related to this discussion and refer the reader to a previous version of this paper (De Loecker (2007)).

A.3. The Quota Data

The quota data come straight from the *SIGL database* constructed by the European Commission (2005) and are publicly available (<http://trade.ec.europa.eu/sigl/>). The quota data are provided using a specific product data classification, and are listed in the online SIGL database. To match this to the firm-level data, I had to map the product codes into the NACE rev.1 industry code

TABLE A.II
DETAILED SEGMENT INFORMATION: MOBILTECH

Technical Segment (<i>s</i>) > Mobiltech Subsegment (<i>g</i>)	
Sub-subsegment	Product (<i>j</i>)
Cars	Airbag fabrics
	Car boot lining materials
	Car mats
	Filtering systems
	Impregnated wovens for car tires
	Lumbar supports
	Passenger compartment lining (dashboard, door panels)
	Protection fabrics for cars, boats, airplanes
	Reinforcing fabrics for car seats
	Safety belts
	Seat covers for cars
	Textiles for sound or heat isolation
	Trimming tape for car mats
Upholstery fabrics for car seats	
Transport	Carpeting for aircraft and ships
	Upholstery fabrics for aircraft and ship seats
	Upholstery fabrics for bus and car seats
	Upholstery fabrics for caravan seats (trailers)
	Upholstery fabrics for seats in trains and trams
General	Antistatic, conductive, and heatable yarns
	Elastic tape, belts, and cord
	Labels
	Loop and eye fastenings
	Technical weaving yarns

through the PRODCOM classification. I do face the problem that the industry classification is more aggregated than the quota classification which can lead to measurement error in the quota restriction variable.

The 182 product categories used in the SIGL data base with the relevant unit of measurement (kilograms or units) can be found online at <http://trade.ec.europa.eu/sigl/products.html>. The 56 supplying countries that faced a quota at some point during the period 1994–2002 on any of the 182 product categories are Albania, Argentina, Armenia, Azerbaijan, Bangladesh, Belarus, Bosnia–Herzegovina + Croatia, Brazil, Bulgaria, Cambodia, China, Czech Republic, Egypt, Estonia, former Yugoslavian Republic of Macedonia, Georgia, Hong Kong, Hungary, India, Indonesia, Kazakstan, Kirghistan, Laos, Latvia, Lithuania, Macao, Malaysia, Malta, Moldova, Mongolia, Morocco, Nepal, North Korea, Pakistan, Peru, Philippines, Poland, Romania, Russia, Serbia and Montenegro, Singapore, Slovak Republic, Slovenia, South Korea, Sri Lanka, Syria,

Taiwan, Tajikistan, Thailand, Tunisia, Turkey, Turkmenistan, Ukraine, United Arab Emirates, Uzbekistan, and Vietnam.

Let me now use the definition of the quota protection variable qr_{it} as discussed in the main text to highlight how my measure of quota protection qr_{it} can change over time by decomposing it into the various sources. I explicitly rely on my assumptions that $a_{ect} = a_{et}$ and $a_{ct} = a_c$. A change in quota protection for a given firm can be written as

$$(A.3.1) \quad \Delta qr_{it} = \sum_{c \in J(i)} a_c \left[\sum_{e \in C} qr_{ect-1} \Delta a_{et} + \sum_{e \in A} qr_{ect} a_{et} - \sum_{e \in B} a_{et-1} qr_{ect-1} \right],$$

where I consider different sets A , B , and C that capture new, abolished, or existing quota, respectively. The full decomposition captures two additional terms related to Δqr_{ect} . However, given that qr_{ect} is a dummy variable, these terms drop out for quotas in set C , where protection is in place in both periods. Note that the product mix of a firm $J(i)$ is fixed over time, which is an assumption I maintain throughout the analysis. The set of countries captures all unique supplying countries that face a quota on a given product at a given point in time. My quota protection measure can thus change due to the entry and exit of quotas (weighted by the supplying's country production potential), and by a change in production potential of suppliers. All three terms vary significantly over time (and across firms due to differences in product mix), although the net entry term turns out to be the most important source of change. Table I clearly demonstrates the change in the number of quotas and maps directly into the last two components of the decomposition. There is quite a bit of change in the weights (a_{et}), mostly due to strong growth of a few textile intensive producing countries such as China. The last term in the decomposition shows immediately that my measure is not related to the initial market share (or quota level) and does depend on the production size of a supplier (a_{et-1}). In fact, abolishment of a quota on a small (large) country has relatively little (great) effect on my measure.

I present two examples that illustrate how the liberalization of trade occurred in the textile industry. I present the evolution of the quota level and the actual fill rate (FR) for two products imported from China and Poland, respectively.

Table A.III clearly shows the detailed level of information that is available at each point in time for each product–supplier pair. The liberalization for *bed linen* imported from Poland took place under the abolishment of the quota in 1998, whereas for *garments* from China, the increased competition came under the form of increased quota levels (by 88 percent). For both cases, the quotas were binding over their lifespan or the period that I consider.

TABLE A.III
TWO EXAMPLES OF DECREASED QUOTA PROTECTION

Year	Example 1 Garments Other Than Knitted or Crocheted Imports From China		Example 2 Bed Linen, Other Than Knitted or Crocheted Imports From Poland	
	Level (× 1000; kg)	FR (%)	Level (× 1000; kg)	FR (%)
1993	21,000	87.76	2,600	60.30
1994	21,630	99.04	2,730	96.19
1995	23,422	122.85	3,436	96.18
1996	24,125	92.92	3,787	89.14
1997	24,848	103.37	3,977	89.41
1998	25,594	109.00	Quota abolished	
1999	26,362	104.46		
2000	27,153	99.50		
2001	27,968	109.81		
2002	30,349	105.18		
2003	32,932	105.12		

Finally, in Table A.IV I report the percentage change in the quota level for the set of supplying countries for which enlarging quotas was the predominant change in protection over my sample period. The average is computed separately for products measured in kilograms and in pieces. Again I only consider quotas with fill rates above 85 percent.

TABLE A.IV
PERCENTAGE CHANGE IN AVERAGE QUOTA LEVEL (1994–2002)

Supplying Country	Products Measured In	
	Kilograms	No. of Pieces
Belarus	146	60
China	83	38
Hong Kong	62	49
India	56	127
Indonesia	90	78
Malaysia	58	66
North Korea	—	92
Pakistan	129	144
Peru	127	—
South Korea	61	69
Taiwan	36	28
Thailand	45	130
Uzbekistan	556	—
Vietnam	–92	55

APPENDIX B: MULTIPRODUCT FIRMS—THEORY AND ESTIMATION

In this appendix, I discuss the multiproduct firm extension and the role of input proportionality in obtaining an estimating equation for the entire sample of firms in my data. I provide more details on the estimating procedure when considering multiproduct–segment firms. In addition, I discuss the relationship between my estimates of productivity and the product mix of firms.

B.1. *Multiproduct Firms and Input Proportionality*

Starting from the product-specific demand structure

$$(B.1.1) \quad Q_{ijt} = Q_{st}(P_{ijt}/P_{st})^{\eta_s} \exp(\xi_{it})$$

the production function is as in the single-product case and has no product-specific shocks:

$$(B.1.2) \quad Q_{ijt} = L_{ijt}^{\alpha_l} M_{ijt}^{\alpha_M} K_{ijt}^{\alpha_k} \exp(\omega_{it} + u_{it}).$$

I now derive the estimating equation: rely on input proportionality: for every input X_{ijt} this implies that $X_{ijt} = c_{ijt}X_{it}$, where $c_{ijt} = J_{it}^{-1}$. The production function for product j of firm i is then given by

$$(B.1.3) \quad \begin{aligned} Q_{ijt} &= (c_{ijt}L_{it})^{\alpha_l} (c_{ijt}M_{it})^{\alpha_M} (c_{ijt}K_{it})^{\alpha_k} \exp(\omega_{it} + u_{it}) \\ &= J_i^{-\gamma} L_{it}^{\alpha_l} M_{it}^{\alpha_M} K_{it}^{\alpha_k} \exp(\omega_{it} + u_{it}) \\ &= J_i^{-\gamma} Q_{it}, \end{aligned}$$

where the second line uses the fact that inputs are spread across products in exact proportion to the number of products produced J_i .

In equilibrium, total quantity demand is equal to total quantity produced. Revenue by product is then obtained by plugging the inverse demand function into $R_{ijt} = P_{ijt}Q_{ijt}$:

$$(B.1.4) \quad R_{ijt} = Q_{ijt}^{(\eta_s+1)/\eta_s} Q_{st}^{-1/\eta_s} P_{st} (\exp(\xi_{it}))^{-1/\eta_s}.$$

Combining input proportionality and the production function yields

$$(B.1.5) \quad R_{ijt} = [J_{it}^{-\gamma} L_{it}^{\alpha_l} M_{it}^{\alpha_M} K_{it}^{\alpha_k} \exp(\omega_{it} + u_{it})]^{(\eta_s+1)/\eta_s} Q_{st}^{-1/\eta_s} P_{st} (\exp(\xi_{it}))^{-1/\eta_s}$$

and summing over all products firm i produces generates an expression for a firm's total revenue,

$$(B.1.6) \quad \begin{aligned} R_{it} &= \sum_j R_{ijt} \\ &= J_{it}^{(1-\gamma)((\eta_s+1)/\eta_s)} \\ &\quad \times [L_{it}^{\alpha_l} M_{it}^{\alpha_M} K_{it}^{\alpha_k} \exp(\omega_{it} + u_{it})]^{(\eta_s+1)/\eta_s} Q_{st}^{-1/\eta_s} P_{st} (\exp(\xi_{it}))^{-1/\eta_s}. \end{aligned}$$

Taking logs and redefining parameters appropriately, the estimating equation is then given by

$$(B.1.7) \quad \tilde{r}_{it} = \beta_{np}np_i + \beta_l l_{it} + \beta_m m_{it} + \beta_k k_{it} + \beta_s q_{st} + \omega_{it}^* + \xi_{it}^* + u_{it}.$$

Finally, so that firms active in multiple segments are included in the sample, I consider the same expression but simply expand the term on q_{st} to $\sum_s s_{is}\beta_s q_{st}$, capturing the idea that a multisegment firm faces different demand elasticities in each segment, where the demand shifter q_{st} is weighted by the importance of each segment in a firm's total output as measured by the share of products s_{is} :

$$(B.1.8) \quad \tilde{r}_{it} = \beta_{np}np_i + \beta_l l_{it} + \beta_m m_{it} + \beta_k k_{it} + \sum_s s_{is}\beta_s q_{st} + \omega_{it}^* + \xi_{it}^* + u_{it}.$$

The assumptions on both unobservables then formulate the identification conditions which are discussed in detail in Section 4.¹

Measuring Segment-Level Output

I turn to the importance of observing firm-specific product information and how it allows for more flexible substitution patterns across products and product groups. At the cost of assuming proportionality of revenue across segments based on the number of products, the introduction of a firm's product mix has three different empirical advantages. First of all, it enables me to construct segment-specific demand shifters (Q_{st}) to identify segment-specific substitution parameters. Second, as described in the data section (Section 3.2), it is crucial to construct firm-specific protection measure (qr_{it}), which varies greatly by quota product categories. Finally, the detailed segment structure is used to proxy for unobserved demand shocks.

The number of products produced by a firm J_{it} is used to create segment-specific demand shifters which are consistent with the demand system. I apply Klette and Griliches' (1995) insight to construct total demand for a segment s and measure it by a (market share) weighted average of deflated revenue,

$$(B.1.9) \quad q_{st} = \sum_{i=1}^{N_s} ms_{ist} \tilde{r}_{ist},$$

where N_s is the number of firms in segment s , ms_{ist} is the market share of firm i in segment s , and \tilde{r}_{ist} is (log) deflated revenue of firm i in segment s . The two

¹Alternatively separate product specific shocks could enter the error term as well. Following the same steps gives rise to a similar estimating equation where the product effects enter in a non linear fashion. I deal with the demand error term ξ_{it}^* extensively in the main text (under Section 4).

terms on the right hand side are typically not observed and I construct them using J_{it} as

$$(B.1.10) \quad r_{ist} = \ln\left(R_{it} \frac{J_{ist}}{J_{it}}\right),$$

$$(B.1.11) \quad ms_{ist} = \frac{R_{ist}}{\sum_i R_{ist}},$$

where J_{ist} and J_{it} denote the number of products of firm i in segment s and in total, respectively. Now it suffices to apply a firm-specific weight to each segment, using $s_{ist} = \frac{J_{ist}}{J_{it}}$, to obtain a firm-specific total demand shifter ($\sum_s s_{ist} q_{st}$).

This approach potentially introduces measurement error by forcing firm-segment revenues to be proportional to the share of the number of products sold in a given segment. However, as long as the proportionality is not violated in some systematic way across products and segments, it is not expected to bias my estimates in any specific way. This measurement error is only present for firms active in multiple segments. I verify the robustness of my results by estimating the model on single-segment firms only, where this measurement error is absent.

Finally, as discussed in the main text, I consider the EU-15 as the relevant market and hence my measure of total output or total segment output needs to incorporate imports. In the empirical implementation, I consider a measure of total output that captures total domestic production in the EU and total imports. For the latter, I do not have firm-level data for all firms that produce in the EU textile market; I simply rely on Belgian producers having a constant share of EU production. I thereby let quotas impact all producers equally to reflect the institutional feature that quotas are allocated across EU members in a relatively constant fashion over time. The constant share assumption does not imply that the market share of Belgian producers is constant. It is exactly the change in quota over time that impacts the amount imported (Q_t^{IMP}) and therefore the amount supplied by EU producers (Q_t^{EU}), of which Belgian producers hold a fixed share. More precisely, I rely on $Q_t = Q_t^{\text{EU}} + Q_t^{\text{IMP}}$, which I can then write as $Q_t = ((a^{\text{Bel}})^{-1} Q_t^{\text{Bel}}) + Q_t^{\text{IMP}}$ as an aggregate demand shifter, since I assume $Q_t^{\text{Bel}} = a^{\text{Bel}} Q_t^{\text{EU}}$. In terms of my empirical model, this means that I require $\ln((a^{\text{Bel}})^{-1} Q_t^{\text{Bel}} + Q_t^{\text{IMP}})$ to estimate the demand parameter. This shows that the constant share of Belgian producers is part of the constant term. In this sense, variation over time (and segments) of EU demand is picked up by the variation in demand for Belgian producers and imports. The change in quota protection over time impacts the market share of Belgian producers by increasing the amount imported, which potentially reduces the share of EU producers and, therefore, proportionally reduces that of Belgian producers.

Input Proportionality

Note that extending the input proportionality is somewhat restricted due to the structure of production. Given that input prices do not vary across various products, and that production is identical across products, the only reason to violate this condition is for multisegment firms who face potentially different demand elasticities.

Not imposing input proportionality implies that a share c_{ijt} captures the firm-product-specific share of a given input (L, M, K) used to produce product j at firm i at time t , where $\sum_{j \in J(i)} c_{ijt} = 1$ and $c_{ijt}^h = c_{ijt}$ for $h = \{L, M, K\}$. The last two conditions rule out cost synergies and imply a constant share across various inputs for a given product produced at a given firm at time t . The production function at the product level is therefore simply given by

$$(B.1.12) \quad Q_{ijt} = (c_{ijt})^\gamma Q_{it},$$

where γ denotes the returns to scale parameter. Following the same steps as before, and relying on the demand system, I obtain a similar expression, except for an additional term that captures potential variation in production shares across firms:

$$(B.1.13) \quad \tilde{r}_{it} = \beta_{np} n p_i + \beta_l l_{it} + \beta_m m_{it} + \beta_k k_{it} + \beta_s q_{st} \\ + \ln \left[\sum_{j \in J(i)} (c_{ijt})^{\gamma((\eta_s+1)/\eta_s)} \right] + \omega_{it}^* + \xi_{it}^* + u_{it}.$$

Given that I assume a common production function across products (and therefore segments) combined with one given input price across products, it is optimal for a firm to simply spread production factors equally over its products. In other words, the marginal revenue is the same everywhere and the marginal cost (i.e., ω_{it}) is assumed to be firm specific. However, when allowing a firm to sell in different segments, although segmented, it will want to spread its input in a different way across its products, simply because it will sell different amounts in the different segments due to the different elasticities of demand.

This has implications for my empirical analysis when allowing for segment-specific elasticities combined with multisegment firms. For multisegment firms, imposing input proportionality may lead to the inclusion of the deviation away from the J_i^{-1} weight in the error term. Therefore, I verified whether my results are sensitive to excluding multisegment firms and my results are robust to this scenario. This observation is not surprising given that my approach already controls for product-specific input share differences in addition to proxying for productivity; put differently, if the input shares at the product level are uncorrelated with any of the inputs, conditional on a function of materials (or investment), capital, and quota protection, then my estimates are not affected. In addition, the quota protection measure qr_{it} also picks-up changes in c_{ijt} due

to changes in protection; therefore, the coefficients of the production function are not biased.

B.2. Estimation Details for Multiproduct–Multisegment Producers

The estimating equation is now given by

$$(B.2.1) \quad \tilde{r}_{it} = \beta_{np}np_i + \beta_l l_{it} + \beta_m m_{it} + \beta_k k_{it} + \sum_s \beta_s s_{is} q_{st} + \omega_{it}^* + \xi_{it}^* + u_{it}.$$

The exact same steps apply as in the main text, where the details depend again on the choice between a static or a dynamic input to control for productivity. I show the steps for the static input approach, while giving up on identification of the labor coefficient in a first stage and allowing the material demand function to depend on the number of products produced. The first stage is then given by

$$(B.2.2) \quad \tilde{r}_{it} = \phi_{np,t}(l_{it}, m_{it}, k_{it}, qr_{it}, q_{st}, D) + \varepsilon_{it},$$

where $\phi_{np,t}(\cdot) = \beta_{np}np_i + \beta_l l_{it} + \beta_m m_{it} + \beta_k k_{it} + \sum_s \beta_s s_{is} q_{st} + \delta D + \tau qr_{it} + h_t(\cdot)$. Under the same law of motion on productivity as before, I obtain estimates of the production and demand parameters as before. Focusing on the additional moment conditions for the multiproduct/segment case,

$$(B.2.3) \quad E \left\{ v_{it+1} \begin{pmatrix} np_i \\ s_{i1} q_{1t} \\ \vdots \\ s_{iS} q_{St} \end{pmatrix} \right\} = 0,$$

the coefficient β_{np} is identified from the moment condition that productivity shocks are not correlated with the number of products produced. The latter is the assumption made throughout this paper that the product mix is fixed over time, due to data constraints, and hence should not react to unexpected shocks in the productivity process. The moment conditions on the demand parameters β_s are as before, where now I have to take into account that firms can sell in multiple segments. The estimation procedure is very similar for the investment approach, where the main difference is that the segment demand parameters are estimated in a first stage.

The estimate of β_{np} throughout the various specifications is around -0.045 and is always highly significant. The term on the number of products np_i in my empirical model is not of direct interest. It enters the estimating equation due to the aggregation of production across products to the firm level. It is because of the availability of detailed product-level information matched to the production data that I can estimate a more flexible demand system and incorporate the product-specific quota information. To use these data, I followed a standard assumption in empirical work—either explicitly or implicitly

TABLE B.I

THE RELATIONSHIP BETWEEN THE NUMBER OF PRODUCTS AND MEASURES OF PRODUCTIVITY

Productivity Measure	Quota Restriction (qr_{it})	Number of Products (np_i)	R^2
Productivity (ω_{it})	-0.089 (0.023)	0.106 (0.006)	0.18
Measured productivity (ω_{it}^{st})	-0.153 (0.020)	-0.015 (0.006)	0.04

stated—that the production function is identical across all products produced. To explore the reduced form relationship between a firm’s productivity level and the number of products it produces, I consider the following descriptive OLS regression without controls:

$$(B.2.4) \quad \widehat{\omega}_{it} = \mu_0 + \mu_1 qr_{it} + \mu_2 np_i + e_{it}.$$

Table B.I demonstrates how both variables help explain productivity with a relatively high R^2 of 0.18. The coefficient on the quota restriction is as discussed in Section 5.2 (in the standard two-stage approach case) and multiproduct firms have a productivity premium of 10 percent, so the coefficient is very precisely estimated. The last row contrasts the correlations by relying on the standard revenue based productivity estimate. I find a much higher coefficient on the quota protection variable (-0.16) and a negative coefficient on the number of products. The coefficient on the number of products in this specification contains the correlation of np_i both with productivity, and with prices and scale, leading to a negative but very small coefficient of -0.015 . To interpret the negative correlation, I run the same regression of standard productivity, and include productivity and the term $x_{it}(\alpha/|\eta|)$ to isolate the correlation between p_{it} and np_i . The coefficients on the number of products is -0.11 and, given my structure, this coefficient captures the cross-sectional correlation of price and the number of products. This negative correlation is consistent with more productive firms having (on average) more products and lower prices holding input use fixed, implying that firms with higher prices have, on average, a lower number of products.

These regressions show the importance of correcting for unobserved prices so as to establish the correct correlations between productivity and the number of products. In fact, the positive correlation between np_i and ω_i is a standard prediction in various theoretical work on multiproduct firms (Bernard, Redding, and Schott (2009), Mayer, Melitz, and Ottaviano (2009)). However, the underlying mechanism cannot be identified in my application due to the inability to observe output and input by product (for a given firm), and therefore lies beyond the scope of this paper.

APPENDIX C: INVERTIBILITY CONDITIONS IN LP AND OP

I briefly discuss the invertibility conditions in the LP and OP framework in my application. I also discuss the role of quota protection in the underlying investment model.

C.1. *Invertibility of LP in Imperfect Competition*

Appendix A of Levinsohn and Petrin (2000) assumed that firms operate in a competitive environment and that output and input prices are given. This implied that intermediate input demand is a monotonic increasing function of productivity, and allowed them to invert the productivity shock and proceed as in Olley and Pakes (1996). Models of imperfect competition on the output market do not satisfy those assumptions and the proof depends on the specific degree of competition. For example, so as to use the LP procedure, Levinsohn and Melitz (2006) assumed that more productive firms do not set disproportionately higher markups than the less productive firms.

However, given the constant markup assumption of the CES demand system, this condition still holds. That is, firms with different productivity charge different prices, but all charge the same markup over cost, therefore preserving the relationship between input demand and productivity. Both DeSouza (2006) and De Loecker (2007) provided more details on the exact formal conditions for monotonicity and hence under which conditions the material input demand proxy is valid under imperfect competition. The crucial part of this proof is to realize that under monopolistic competition and CES preferences, markups do not differ across firms and therefore do not violate the monotonicity of material input demand in productivity (conditional on the fixed inputs of production).

C.2. *Invertibility of the Extended OP Model and the Role of Quota Protection*

My model allows for the quota restriction variable qr_{it} to affect a firm's optimal investment decision. Formally, the investment policy function is given by

$$(C.2.1) \quad i_{it} = i_t(k_{it}, \omega_{it}, qr_{it}),$$

where I suppress the product dummies. To apply the Olley and Pakes methodology, investment has to be increasing in productivity. The proof in Pakes (1994) turns out to readily apply to this setting under the plausible assumption that qr_{it} is an exogenous state variable with known support $[0, 1]$. Note that if protection did not vary across products and therefore across firms, it would be subsumed in the t subscript of the policy function $i_t(\cdot)$. Formally, denote $k_t^* = (k_t, qr_t)$ in Pakes (1994, p. 246) and use k_t^* for k_t in the proof everywhere, and proceed from there to show invertibility. In other words, I assume that conditional on a firm's capital stock and protection level, higher productivity

firms still invest more. Note that given my demand system, markups do not change and do not have implications for the conditions of invertibility.

This rules out beliefs that future quotas affect current investment decisions. From the firm's point of view, current protection is all that matters, and this variable follows an exogenous process over time, whereby qr_{it} is a sufficient statistic for future values of quota protection. I illustrate the restriction I impose on the beliefs of future quotas using a simplified investment model. I highlight the trade-off an empirical researcher faces between sample size (and hence efficiency and precision) and allowing for a more general setting whereby firms, investments depend on the entire path of quotas, where firms know their entire time profile.

I consider a simplified version of the model introduced in the main text by only having i.i.d. demand shocks such that $\xi_{it} = \tilde{\xi}_{it}$; therefore, quotas only enter the model through the law of motion productivity, $\omega_{it} = g(\omega_{it-1}, qr_{it-1}) + v_{it}$. Furthermore, for simplicity, I consider single product producers.

I depart from the main assumption in the text by allowing for the complete future path of quotas, to impact investment, $i_{it} = i_t(k_{it}, \omega_{it}, qr_{it}, qr_{it+1}, \dots, qr_{it+A})$, where A is the last period firms have information about quota levels. The latter shows immediately that we run into trouble with this specifications by not restricting A at all. Consider the simple linear investment model

$$(C.2.2) \quad i_{it} = \psi_1 k_{it} + \psi_2 \omega_{it} + \sum_{a=0}^A \psi_{3a} qr_{it+a},$$

where $a = 0, 1, \dots, A$ indexes the future periods. Revisiting the revenue generating production function, I obtain an estimating equation where I use the inverse investment equation to control for unobserved productivity,

$$(C.2.3) \quad r_{it} = \beta_l l_{it} + \beta_m m_{it} + \beta_s q_{st} + \psi_2^* \left((\beta_k - \psi_1) k_{it} + i_{it} - \sum_{a=0}^A \psi_{3a} qr_{it+a} \right) + \varepsilon_{it},$$

TABLE C.I

IMPACT OF RELAXING THE ROLE OF QUOTA IN INVESTMENT^a

Horizon (A)	β_l	β_m	No. of Observations	R^2	F -Test
0	0.22	0.64	985	0.95	
1	0.22	0.64	985	0.94	0.74
2	0.22	0.63	854	0.94	0.64
3	0.23	0.62	722	0.94	0.44
4	0.23	0.60	583	0.94	0.36

^aThe F -test is on the joint significance of the coefficients on the future quota variables, and I report the p -value of the null hypothesis that $\sum_{a=1}^A \psi_{3a} = 0$ for various A .

where $\varepsilon_{it} = u_{it} + \xi_{it}^*$ and ψ_2^* captures both the productivity coefficient ψ_2 and the demand parameter as before, that is, $\psi_2^* = (\frac{\eta+1}{\eta})\psi_2^{-1}$ since $\omega_{it}^* \equiv \omega_{it}(\frac{\eta+1}{\eta})$ and $\xi_{it}^* \equiv \xi_{it}|\eta|^{-1}$. The restriction imposed in the main text is to exclude all terms where $a > 0$, and therefore not to include future protection values. This is a result of modeling the quota protection variable as an exogenous evolving Markov process where the current value qr_{it} is a sufficient statistic for future values. To check the importance of my restriction, $a = 0$, I run (C.2.3) on my data by adding additional future quota variables and checking how the variable input coefficients and their standard errors change, as well as the (adjusted) R^2 . In addition, I check whether the future values of quotas, $a > 0$, are jointly significant while letting A increase. Table C.I presents the results.

Table C.I is not a formal test, but directly supports my maintained assumption for the investment model, throughout the main text. Future quota values do not change my estimated coefficients at all, but do reduce the sample period significantly and increase the standard errors of my estimates, while not adding any explanatory power to the investment model. In this case, the standard error on the labor and materials coefficients goes up by around 30 percent. More importantly, such a specification reduces the sample over which I can estimate the full model and analyze the productivity effects from trade liberalization. The results are similar when running this procedure on the full model with multiproduct firms and all demand variables as described in Sections 2 and 4. Finally, the above problem is not a concern for the material demand proxy, since only current quotas affect the optimal material demand, through residual demand, but future values of quotas do not.

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