Econometrica Supplementary Material

SUPPLEMENT TO "UNBUNDLING POLARIZATION"
(*Econometrica*, Vol. 88, No. 3, May 2020, 1197–1233)

NATHAN CANEN
Department of Economics, University of Houston

CHAD KENDALL
Marshall School of Business, University of Southern California

FRANCESCO TREBBI
Vancouver School of Economics, University of British Columbia, Canadian Institute for Advanced Research, and National Bureau of Economic Research

APPENDIX A: PROOFS AND ADDITIONAL FIGURES FOR THE MODEL

A.1. *Proofs*

PROOF OF LEMMA 1: Consider first $k_t > k_t^{'}$. Given the increasing cost of exerting influence, a whip exerts the minimum amount of influence necessary to ensure a vote for k_t, provided this amount is less than or equal to $y_p^{\max}$. The minimum amount of influence is such that the member is indifferent, $u(k_t, \omega_t^i + y_t^i) = u(k_t^{'}, \omega_t^i + y_t^i)$ or $|\omega_t^i + y_t^i - k_t| = |\omega_t^i + y_t^i - k_t^{'}|$. This equality is satisfied if and only if $\omega_t^i + y_t^i = MV_t = \frac{k_t + k_t^{'}}{2}$. If $\omega_t^i \geq MV_t$, the required influence is weakly negative (absent influence, the member votes for k_t) and so no influence is exerted. If $\omega_t^i < MV_t$, a positive amount of influence, $y_t^i = MV_t - \omega_t^i > 0$ is required which increases linearly in $MV_t - \omega_t^i$. Therefore, a member is whipped if and only if their ideology is such that $MV_t - y_p^{\max} \leq \omega_t^i < MV_t$. For $k_t < k_t^{'}$, the argument is reversed: only members for which $MV_t < \omega_t^i \leq MV_t + y_p^{\max}$ are whipped. *Q.E.D.*

PROOF OF LEMMA 2: Consider the mass, $f(\theta)$, of members at some θ, each of whom has an independent signal of $\hat{\eta}_{1,t}$ due to their independent ideological shocks. The average number of Yes reports from the N members at θ is given by $\lim_{N\to\infty} \frac{f(\theta)}{N} \sum_{i=1}^{N} I(u(x_t, \theta + \delta_{1,t}^i + \hat{\eta}_{1,t}) \geq u(q_t, \theta + \delta_{1,t}^i + \hat{\eta}_{1,t}))$ where $I()$ represents the indicator function. By the law of large numbers, as $N \to \infty$, this average converges to

$$\begin{aligned} & f(\theta)E\big[I\big(u\big(x_t, \theta + \delta_t^1 + \hat{\eta}_{1,t}\big) \geq u\big(q_t, \theta + \delta_t^1 + \hat{\eta}_{1,t}\big)\big)\big] \\ & \quad = f(\theta)\Pr\big(u\big(x_t, \theta + \delta_t^1 + \hat{\eta}_{1,t}\big) \geq u\big(q_t, \theta + \delta_t^1 + \hat{\eta}_{1,t}\big)\big) \\ & \quad = f(\theta)\Pr\big(\theta + \delta_t^1 + \hat{\eta}_{1,t} \geq MV_t\big) \\ & \quad = f(\theta)\big(1 - G\big(MV_t - \theta - \hat{\eta}_{1,t}\big)\big). \end{aligned}$$

Therefore, after observing the number of Yes reports for a given θ, $\hat{\eta}_{1,t}$ is known with probability one. *Q.E.D.*

Nathan Canen: ncanen@uh.edu
Chad Kendall: chadkend@marshall.usc.edu
Francesco Trebbi: francesco.trebbi@ubc.ca

 https://doi.org/10.3982/ECTA16756

PROOF OF LEMMA 3: Consider $x_t > q_t$. Let $G_{1+2}()$ denote the cdf of $\delta^i_{1,t} + \delta^i_{2,t}$ (with corresponding pdf, $g_{1+2}()$). For a given $\tilde{MV}_{2,t}$, the number of votes for x_t from a given party's members is known with probability one due to independent idiosyncratic shocks and a continuum of members. To see this fact, consider the continuum of party p's members located at each θ, each with independent shocks, $\delta^i_{1,t}$ and $\delta^i_{2,t}$. With N voters at θ, the average number of votes from these members is given by $\lim_{N\to\infty} \frac{f(\theta)}{N}\sum_{i=1}^{N} I(\theta^i + \delta^i_{1,t} + \delta^i_{2,t} \geq \tilde{MV}_{2,t} \pm y^{\max}_p)$, where the sign with which $y^{\max}_p$ enters depends upon the direction that party p whips. By the law of large numbers, as $N \to \infty$, this average converges to

$$f(\theta)E\big[I\big(\theta + \delta^1_t + \delta^2_t \geq \tilde{MV}_{2,t} \pm y^{\max}_p\big)\big] = f(\theta)\Pr\big(\theta + \delta^1_t + \delta^2_t \geq \tilde{MV}_{2,t} \pm y^{\max}_p\big)$$
$$= f(\theta)\big(1 - G_{1+2}\big(\tilde{MV}_{2,t} \pm y^{\max}_p - \theta\big)\big).$$

Using this fact, the number of votes for x_t from party D's members is given by $Y_D(\tilde{MV}_{2,t}) = N_D[\int_{-\infty}^{\infty}(1 - G_{1+2}(\tilde{MV}_{2,t} - \theta \pm y^{\max}_D))f_D(\theta)\,d\theta]$. The corresponding expression for party R is $Y_R(\tilde{MV}_{2,t}) = N_R[\int_{-\infty}^{\infty}(1 - G_{1+2}(\tilde{MV}_{2,t} - \theta \pm y^{\max}_R))f_R(\theta)\,d\theta]$. The total number of votes for x_t is then given by $Y(\tilde{MV}_{2,t}) \equiv Y_D(\tilde{MV}_{2,t}) + Y_R(\tilde{MV}_{2,t})$.

$Y(\tilde{MV}_{2,t})$ is strictly decreasing in x_t. To see this, consider the votes from party D's members, $Y_D(\tilde{MV}_{2,t})$:

$$\frac{\partial Y_D(\tilde{MV}_{2,t})}{\partial x_t} = \frac{1}{2}\frac{\partial}{\partial \tilde{MV}_{2,t}} N_D\left[\int_{-\infty}^{\infty}\big(1 - G_{1+2}\big(\tilde{MV}_{2,t} - \theta \pm y^{\max}_D\big)\big)f_D(\theta)\,d\theta\right]$$
$$= -\frac{N_D}{2}\int_{-\infty}^{\infty} g_{1+2}\big(\tilde{MV}_{2,t} - \theta \pm y^{\max}_D\big)f_D(\theta)\,d\theta. \tag{A.1}$$

(A.1) is strictly less than zero given that that ideological shocks are unbounded, independent of the (finite) amount or direction of whipping. The same is true of the derivative of $Y_R(\tilde{MV}_{2,t})$, ensuring $Y(\tilde{MV}_{2,t})$ strictly decreases in x_t for $x_t > q_t$. For $x_t < q_t$, we have $Y_D(\tilde{MV}_{2,t}) = N_D[\int_{-\infty}^{\infty} G_{1+2}(\tilde{MV}_{2,t} - \theta \pm y^{\max}_D)f_D(\theta)\,d\theta]$ and $Y_R(\tilde{MV}_{2,t}) = N_R[\int_{-\infty}^{\infty} G_{1+2}(\tilde{MV}_{2,t} - \theta \pm y^{\max}_R)f_R(\theta)\,d\theta]$ so that $Y(\tilde{MV}_{2,t})$ increases in x_t. Since for $q_t < \theta^m_p$, we must have $x_t > q_t$ and for $q_t > \theta^m_p$ we must have $x_t < q_t$; we see that the number of votes for x_t strictly decreases the closer it gets to the proposing party's ideal point. *Q.E.D.*

PROOF OF PROPOSITION 1: For $q_t = \theta^m_D$, clearly $x^{\text{count}}_t = x^{\text{no count}}_t = \theta^m_D$ are the unique optimal alternative policies because party D can do no better than its ideal point.

In the case of no whip count, and $q_t < \theta^m_D$ so that $x_t > q_t$, we can rewrite party D's expected utility as

$$EU^{\text{no count}}_D(q_t, x_t) = \left(1 - \Phi\left(\frac{MV_t - \hat{MV}_{R,R}}{\sigma}\right)\right)\big(u\big(x_t, \theta^m_D\big) - u\big(q_t, \theta^m_D\big)\big) + u\big(q_t, \theta^m_D\big) - C_b.$$

The derivative with respect to x_t is given by

$$\left(1 - \Phi\left(\frac{MV_t - \hat{MV}_{R,R}}{\sigma}\right)\right)u_x\big(x_t, \theta^m_D\big) - \frac{1}{2\sigma}\phi\left(\frac{MV_t - \hat{MV}_{R,R}}{\sigma}\right)\big(u\big(x_t, \theta^m_D\big) - u\big(q_t, \theta^m_D\big)\big),$$

where $\phi()$ denotes the pdf of the standard Normal distribution. At $x_t = q_t$, the derivative is strictly positive given $q_t < \theta_D^m$ and the fact that $\hat{MV}_{R,R}$ is finite. At $x_t = \theta_D^m$, it is strictly negative given $u(q_t, \theta_D^m) < 0$. Together these facts ensure an interior solution, which we now show is unique. Any interior solution must satisfy the first-order condition,

$$\begin{aligned}&\left(1-\Phi\left(\frac{MV_t^{\text{no count}}-\hat{MV}_{R,R}}{\sigma}\right)\right)u_x\left(x_t^{no\ \text{count}},\theta_D^m\right)\\&\quad-\frac{1}{2\sigma}\phi\left(\frac{MV_t^{\text{no count}}-\hat{MV}_{R,R}}{\sigma}\right)\left(u\left(x_t^{no\ \text{count}},\theta_D^m\right)-u\left(q_t,\theta_D^m\right)\right)=0. \qquad \text{(A.2)}\end{aligned}$$

Defining $z_t^{\text{no count}} \equiv \frac{MV_t^{\text{no count}}-\hat{MV}_{R,R}}{\sigma}$, we can rewrite the first-order condition as

$$\frac{1-\Phi\left(z_t^{\text{no count}}\right)}{\phi\left(z_t^{\text{no count}}\right)}=\frac{1}{2\sigma}\frac{u\left(x_t^{\text{no count}},\theta_D^m\right)-u\left(q_t,\theta_D^m\right)}{u_x\left(x_t^{\text{no count}},\theta_D^m\right)}. \qquad \text{(A.3)}$$

The left-hand side of (A.3) is the inverse hazard rate of a standard Normal distribution and so is strictly decreasing in $z_t^{\text{no count}}$ (and, therefore, $x_t^{\text{no count}}$ since $x_t^{\text{no count}}$ strictly increases in $z_t^{\text{no count}}$). The sign of the derivative of the right-hand side with respect to $x_t^{\text{no count}}$ is given by $u_x(x_t^{\text{no count}}, \theta_D^m)^2 - u_{xx}(x_t^{\text{no count}}, \theta_D^m)(u(x_t^{\text{no count}}, \theta_D^m) - u(q_t, \theta_D^m))$ which is strictly positive because $u_{xx}(x_t^{\text{no count}}, \theta_D^m) < 0$ and $u(x_t^{\text{no count}}, \theta_D^m) > u(q_t, \theta_D^m)$. Thus, the right-hand side is strictly increasing in $x_t^{\text{no count}}$. Together, these facts guarantee a unique solution, $x_t^{\text{no count}} \in (q_t, \theta_D^m)$.[1]

In the case of a whip count and and $q_t < \theta_D^m$, we can rewrite the party's expected utility:

$$\begin{aligned}&EU_D^{\text{count}}(q_t,x_t)\\&\quad=\Pr(\eta_{1,t}\geq\underline{\eta}_{1,t})\left(\Pr(x_t\text{ wins}|\eta_{1,t}\geq\underline{\eta}_{1,t})\left(u\left(x_t,\theta_D^m\right)-u\left(q_t,\theta_D^m\right)\right)+u\left(q_t,\theta_D^m\right)-C_b\right)\\&\qquad+\Pr\left(\eta_{1,t}<\underline{\eta}_{1,t}\right)u\left(q_t,\theta_D^m\right)\\&\quad=\Pr(\eta_{1,t}\geq\underline{\eta}_{1,t},x_t\text{ wins})\left(u\left(x_t,\theta_D^m\right)-u\left(q_t,\theta_D^m\right)\right)-\Pr\left(\eta_{1,t}\geq\underline{\eta}_{1,t}\right)C_b+u\left(q_t,\theta_D^m\right)\\&\quad=\int_{\underline{\eta}_{1,t}}^{\infty}\left(1-\Phi\left(\frac{MV_t-\hat{MV}_{R,R}-\eta}{\sigma_\eta}\right)\right)\frac{1}{\sigma_\eta}\phi\left(\frac{\eta}{\sigma_\eta}\right)d\eta\left(u\left(x_t,\theta_D^m\right)-u\left(q_t,\theta_D^m\right)\right)\\&\qquad-\left(1-\Phi\left(\frac{\underline{\eta}_{1,t}}{\sigma_\eta}\right)\right)C_b+u\left(q_t,\theta_D^m\right).\end{aligned}$$

[1]The second-order condition at $x_t^{\text{no count}}$ is also easily checked, but must be satisfied given that marginal expected utility is increasing at $x_t = q_t$, decreasing at $x_t = \theta_D^m$ and the solution is unique.

Taking the derivative with respect to x_t yields:[2]

$$\begin{aligned}
&\frac{dEU_D^{\text{count}}(q_t, x_t)}{dx_t} \\
&= -\frac{d\underline{\eta}_{1,t}}{dx_t}\frac{1}{\sigma_\eta}\phi\left(\frac{\underline{\eta}_{1,t}}{\sigma_\eta}\right)\left(1-\Phi\left(\frac{MV_t-\hat{M}V_{R,R}-\underline{\eta}_{1,t}}{\sigma_\eta}\right)\right)\left(u\left(x_t,\theta_D^m\right)-u\left(q_t,\theta_D^m\right)\right) \\
&\quad -\frac{1}{2\sigma_\eta^2}\int_{\underline{\eta}_{1,t}}^{\infty}\phi\left(\frac{MV_t-\hat{M}V_{R,R}-\eta}{\sigma_\eta}\right)\phi\left(\frac{\eta}{\sigma_\eta}\right)d\eta\left(u\left(x_t,\theta_D^m\right)-u\left(q_t,\theta_D^m\right)\right) \\
&\quad +\frac{1}{\sigma_\eta}u_x\left(x_t,\theta_D^m\right)\int_{\underline{\eta}_{1,t}}^{\infty}\left(1-\Phi\left(\frac{MV_t-\hat{M}V_{R,R}-\eta}{\sigma_\eta}\right)\right)\phi\left(\frac{\eta}{\sigma_\eta}\right)d\eta \\
&\quad +\frac{1}{\sigma_\eta}\frac{d\underline{\eta}_{1,t}}{dx_t}\phi\left(\frac{\underline{\eta}_{1,t}}{\sigma_\eta}\right)C_b \\
&= \frac{1}{\sigma_\eta}u_x\left(x_t,\theta_D^m\right)\int_{\underline{\eta}_{1,t}}^{\infty}\left(1-\Phi\left(\frac{MV_t-\hat{M}V_{R,R}-\eta}{\sigma_\eta}\right)\right)\phi\left(\frac{\eta}{\sigma_\eta}\right)d\eta \\
&\quad -\frac{1}{2\sigma_\eta^2}\int_{\underline{\eta}_{1,t}}^{\infty}\phi\left(\frac{MV_t-\hat{M}V_{R,R}-\eta}{\sigma_\eta}\right)\phi\left(\frac{\eta}{\sigma_\eta}\right)d\eta\left(u\left(x_t,\theta_D^m\right)-u\left(q_t,\theta_D^m\right)\right), \qquad \text{(A.4)}
\end{aligned}$$

where the second equality uses the fact that $\underline{\eta}_{1,t}$ satisfies

$$\left(1-\Phi\left(\frac{MV_t-\hat{M}V_{R,R}-\underline{\eta}_{1,t}}{\sigma_\eta}\right)\right)\left(u\left(x_t,\theta_D^m\right)-u\left(q_t,\theta_D^m\right)\right)=C_b. \qquad \text{(A.5)}$$

Consider the limit as $C_b \to 0$. From (A.5), we can see that, provided x_t is bounded away from q_t so that $u(x_t, \theta_D^m) - u(q_t, \theta_D^m) > 0$ (which we subsequently confirm), we must have $\underline{\eta}_{1,t} \to -\infty$ as $C_b \to 0$. But, as $\underline{\eta}_{1,t} \to -\infty$, the party always continues to pursue the bill after the first aggregate shock. In this case, the optimal alternative policy is identical to the case of no whip count. Formally,

$$\begin{aligned}
\lim_{\underline{\eta}_{1,t}\to-\infty}\frac{dEU_D^{\text{count}}(q_t, x_t)}{dx_t} &= \frac{1}{\sigma_\eta}u_x\left(x_t,\theta_D^m\right)\int_{-\infty}^{\infty}\left(1-\Phi\left(\frac{MV_t-\hat{M}V_{R,R}-\eta}{\sigma_\eta}\right)\right)\phi\left(\frac{\eta}{\sigma_\eta}\right)d\eta \\
&\quad -\frac{1}{2\sigma_\eta^2}\int_{-\infty}^{\infty}\phi\left(\frac{MV_t-\hat{M}V_{R,R}-\eta}{\sigma_\eta}\right) \\
&\quad \times\phi\left(\frac{\eta}{\sigma_\eta}\right)d\eta\left(u\left(x_t,\theta_D^m\right)-u\left(q_t,\theta_D^m\right)\right)
\end{aligned}$$

[2]The necessary conditions for applying the Leibniz integral rule with an infinite bound are satisfied. Specifically, the integrand and its partial derivative with respect to x_t are both continuous functions of x_t and η, and it is possible to find integrable functions of η that bound the integrand and it's partial derivative with respect to x_t.

$$= u_x(x_t, \theta_D^m)\left(1 - \Phi\left(\frac{MV_t - \hat{M}V_{R,R}}{\sigma}\right)\right)$$
$$- \frac{1}{2\sigma}\phi\left(\frac{MV_t - \hat{M}V_{R,R}}{\sigma}\right)\left(u(x_t, \theta_D^m) - u(q_t, \theta_D^m)\right), \quad \text{(A.6)}$$

where the equality follows from the fact that the convolution of two standard Normal distributions is a Normal distribution with the sum of the variances, and using $\sigma^2 = 2\sigma_\eta^2$. Comparing (A.6) with (A.2), we can see immediately that, in the limit, the first-order condition for the whip and no whip cases are identical, and it therefore follows that x_t^{count} is unique and interior as in the no whip case. This fact ensures that $u(x_t, \theta_D^m) - u(q_t, \theta_D^m) > 0$ in the limit, confirming that we must have $\underline{\eta}_{1,t} \to -\infty$ as $C_b \to 0$.

We now show that x_t^{count} is unique and interior for strictly positive C_b. From (A.4), we see that $\frac{dEU_D^{\text{count}}(q_t, x_t)}{dx_t}$ is strictly positive at $x_t = q_t$ and strictly negative at $x_t = \theta_D^m$, ensuring an interior optimum, x_t^{count} which must satisfy the first-order condition[3]

$$\frac{\int_{\underline{\eta}_{1,t}}^{\infty}\left(1 - \Phi\left(\frac{MV_t^{\text{count}} - \hat{M}V_{R,R} - \eta}{\sigma_\eta}\right)\right)\phi\left(\frac{\eta}{\sigma_\eta}\right)d\eta}{\frac{1}{2\sigma_\eta}\int_{\underline{\eta}_{1,t}}^{\infty}\phi\left(\frac{MV_t^{\text{count}} - \hat{M}V_{R,R} - \eta}{\sigma_\eta}\right)\phi\left(\frac{\eta}{\sigma_\eta}\right)d\eta} = \frac{\left(u(x_t^{\text{count}}, \theta_D^m) - u(q_t, \theta_D^m)\right)}{u_x(x_t^{\text{count}}, \theta_D^m)}. \quad \text{(A.7)}$$

As in the case of no whip count, the right-hand side of (A.7) strictly increases in x_t^{count}. It remains to show that, in the limit as $C_b \to 0$, the left-hand side of (A.7) strictly decreases in x_t^{count}, which, by continuity of the left-hand side in C_b, ensures there exists a strictly positive value of C_b, $\hat{C}_b > 0$, such that for all $C_b < \hat{C}_b$, the left-hand side continues to strictly decrease. It then follows that x_t^{count} is unique for all $C_b < \hat{C}_b$. The sign of the derivative of the left-hand side of (A.7) with respect to x_t^{count}, is determined by[4]

$$-\frac{d\underline{\eta}_{1,t}}{dx_t^{\text{count}}}\phi\left(\frac{\underline{\eta}_{1,t}}{\sigma_\eta}\right)\left(1 - \Phi\left(\frac{MV_t - \hat{M}V_{R,R} - \underline{\eta}_{1,t}}{\sigma_\eta}\right)\right)$$
$$\times \frac{1}{2\sigma_\eta}\int_{\underline{\eta}_{1,t}}^{\infty}\phi\left(\frac{MV_t^{\text{count}} - \hat{M}V_{R,R} - \eta}{\sigma_\eta}\right)\phi\left(\frac{\eta}{\sigma_\eta}\right)d\eta$$
$$+ \frac{d\underline{\eta}_{1,t}}{dx_t^{\text{count}}}\frac{1}{2\sigma_\eta}\phi\left(\frac{MV_t^{\text{count}} - \hat{M}V_{R,R} - \underline{\eta}_{1,t}}{\sigma_\eta}\right)\phi\left(\frac{\underline{\eta}_{1,t}}{\sigma_\eta}\right)$$
$$\times \int_{\underline{\eta}_{1,t}}^{\infty}\left(1 - \Phi\left(\frac{MV_t^{\text{count}} - \hat{M}V_{R,R} - \eta}{\sigma_\eta}\right)\right)\phi\left(\frac{\eta}{\sigma_\eta}\right)d\eta$$
$$- \left(\frac{1}{2\sigma_\eta}\int_{\underline{\eta}_{1,t}}^{\infty}\phi\left(\frac{MV_t^{\text{count}} - \hat{M}V_{R,R} - \eta}{\sigma_\eta}\right)\phi\left(\frac{\eta}{\sigma_\eta}\right)d\eta\right)^2$$

[3]These statements require $\underline{\eta}_{1,t} < \infty$, which, by continuity, is true for C_b sufficiently small given that $\underline{\eta}_{1,t} \to -\infty$ as $C_b \to 0$.

[4]Again, the necessary conditions for applying the Leibniz integral rule with an infinite bound are satisfied.

$$- \frac{1}{4\sigma_\eta} \int_{\underline{\eta}_{1,t}}^{\infty} \phi'\left(\frac{MV_t^{\text{count}} - \hat{MV}_{R,R} - \eta}{\sigma_\eta}\right) \phi\left(\frac{\eta}{\sigma_\eta}\right) d\eta$$

$$\times \int_{\underline{\eta}_{1,t}}^{\infty} \left(1 - \Phi\left(\frac{MV_t^{\text{count}} - \hat{MV}_{R,R} - \eta}{\sigma_\eta}\right)\right) \phi\left(\frac{\eta}{\sigma_\eta}\right) d\eta. \tag{A.8}$$

By the implicit function theorem, $\frac{d\underline{\eta}_{1,t}}{dx_t}$ must satisfy (from (A.5))

$$-\phi\left(\frac{MV_t^{\text{count}} - \hat{MV}_{R,R} - \underline{\eta}_{1,t}}{\sigma_\eta}\right) \frac{1}{\sigma_\eta} \left(\frac{1}{2} - \frac{d\underline{\eta}_{1,t}}{dx_t^{\text{count}}}\right) \left(u\left(x_t^{\text{count}}, \theta_D^m\right) - u\left(q_t, \theta_D^m\right)\right)$$

$$+ \left(1 - \Phi\left(\frac{MV_t^{\text{count}} - \hat{MV}_{R,R} - \underline{\eta}_{1,t}}{\sigma_\eta}\right)\right) u_x\left(x_t^{\text{count}}, \theta_D^m\right) = 0$$

or

$$\frac{d\underline{\eta}_{1,t}}{dx_t^{\text{count}}} = \frac{1}{2} - \frac{\sigma_\eta \left(1 - \Phi\left(\frac{MV_t^{\text{count}} - \hat{MV}_{R,R} - \underline{\eta}_{1,t}}{\sigma_\eta}\right)\right) u_x\left(x_t^{\text{count}}, \theta_D^m\right)}{\phi\left(\frac{MV_t^{\text{count}} - \hat{MV}_{R,R} - \underline{\eta}_{1,t}}{\sigma_\eta}\right) \left(u\left(x_t^{\text{count}}, \theta_D^m\right) - u\left(q_t, \theta_D^m\right)\right)}. \tag{A.9}$$

In the limit as $C_b \to 0$, $\underline{\eta}_{1,t} \to -\infty$, in which case the second term of (A.9) approaches zero because x_t^{count} is bounded away from q_t and θ_D^m, and the inverse hazard rate of a standard Normal random variable approaches zero as its argument approaches infinity.[5] The limit of (A.8) as $C_b \to 0$ is then determined by the limit of its second two terms because the first two terms approach zero. Defining $z_t^{\text{count}} \equiv \frac{MV_t^{\text{count}} - \hat{MV}_{R,R}}{\sigma}$, this limit is given by

$$\lim_{\underline{\eta}_{1,t} \to -\infty} -\left(\frac{1}{2\sigma_\eta} \int_{\underline{\eta}_{1,t}}^{\infty} \phi\left(\frac{MV_t^{\text{count}} - \hat{MV}_{R,R} - \eta}{\sigma_\eta}\right) \phi\left(\frac{\eta}{\sigma_\eta}\right) d\eta\right)^2$$

$$- \frac{1}{4\sigma_\eta} \int_{\underline{\eta}^t_t}^{\infty} \phi'\left(\frac{MV_t^{\text{count}} - \hat{MV}_{R,R} - \eta}{\sigma_\eta}\right) \phi\left(\frac{\eta}{\sigma_\eta}\right) d\eta$$

$$\times \int_{\underline{\eta}_{1,t}}^{\infty} \left(1 - \Phi\left(\frac{MV_t^{\text{count}} - \hat{MV}_{R,R} - \eta}{\sigma_\eta}\right)\right) \phi\left(\frac{\eta}{\sigma_\eta}\right) d\eta$$

$$= -\left(\frac{1}{2\sigma_\eta} \int_{-\infty}^{\infty} \phi\left(\frac{MV_t^{\text{count}} - \hat{MV}_{R,R} - \eta}{\sigma_\eta}\right) \phi\left(\frac{\eta}{\sigma_\eta}\right) d\eta\right)^2$$

$$- \frac{1}{4\sigma_\eta} \int_{-\infty}^{\infty} \phi'\left(\frac{MV_t^{\text{count}} - \hat{MV}_{R,R} - \eta}{\sigma_\eta}\right) \phi\left(\frac{\eta}{\sigma_\eta}\right) d\eta$$

---

[5] $\lim_{x\to\infty} \frac{1-\Phi(x)}{\phi(x)} = \lim_{x\to\infty} \frac{-\phi(x)}{\phi'(x)} = \lim_{x\to\infty} \frac{-\phi(x)}{-x\phi(x)} = 0$ where the first equality uses L'Hôpital's rule.

$$
\begin{aligned}
&\times \int_{-\infty}^{\infty}\left(1-\Phi\left(\frac{MV_t^{\text{count}}-M\hat{V}_{R,R}-\eta}{\sigma_\eta}\right)\right)\phi\left(\frac{\eta}{\sigma_\eta}\right)d\eta \\
&= -\left(\frac{1}{2\sigma}\phi\left(\frac{MV_t^{\text{count}}-M\hat{V}_{R,R}}{\sigma}\right)\right)^2 \\
&\quad -\frac{1}{4\sigma^2}\phi'\left(\frac{MV_t^{\text{count}}-M\hat{V}_{R,R}}{\sigma}\right)\left(1-\Phi\left(\frac{MV_t^{\text{count}}-M\hat{V}_{R,R}}{\sigma}\right)\right) \\
&= -\left(\frac{1}{2\sigma}\phi\left(z_t^{\text{count}}\right)\right)^2-\frac{1}{4\sigma^2}\phi'\left(z_t^{\text{count}}\right)\left(1-\Phi\left(z_t^{\text{count}}\right)\right) \\
&= -\left(\frac{1}{2\sigma}\phi\left(z_t^{\text{count}}\right)\right)^2+\frac{1}{4\sigma^2}z_t^{\text{count}}\phi\left(z_t^{\text{count}}\right)\left(1-\Phi\left(z_t^{\text{count}}\right)\right) \\
&< -\left(\frac{1}{2\sigma}\phi\left(z_t^{\text{count}}\right)\right)^2+\frac{1}{4\sigma^2}\phi\left(z_t^{\text{count}}\right)^2 \\
&= 0,
\end{aligned}
$$

where the second equality uses properties of the convolution of Normal distributions, and the inequality follows from the fact that, for a standard Normal random variable, $x(1-\Phi(x)) < \phi(x)$.

For $q_t > \theta_D^m$ so that $x_t < q_t$, we assume party R whips against the bill (supports q_t). In case of no whip count, we can write party D's expected utility as

$$
EU_D^{\text{no count}}(q_t, x_t) = \Phi\left(\frac{MV_t - M\hat{V}_{L,R}}{\sigma}\right)\left(u\left(x_t, \theta_D^m\right) - u\left(q_t, \theta_D^m\right)\right) + u\left(q_t, \theta_D^m\right) - C_b.
$$

With a whip count, it is

$$
\begin{aligned}
&EU_D^{\text{count}}(q_t, x_t) \\
&\quad = \int_{-\infty}^{\overline{\eta}_{1,t}}\Phi\left(\frac{MV_t - M\hat{V}_{L,R}-\eta}{\sigma_\eta}\right)\frac{1}{\sigma_\eta}\phi\left(\frac{\eta}{\sigma_\eta}\right)d\eta\left(u\left(x_t, \theta_D^m\right) - u\left(q_t, \theta_D^m\right)\right) \\
&\qquad - \Phi\left(\frac{\overline{\eta}_{1,t}}{\sigma_\eta}\right)C_b + u\left(q_t, \theta_D^m\right).
\end{aligned}
$$

Using these expressions, the optimal policy candidates, x_t^{count} and $x_t^{\text{no count}}$, can be shown to be unique (provided C_b is not too large) as in the previous case. *Q.E.D.*

To prove Lemma 4, we first define and prove Lemma A.1.

LEMMA A.1: *Fix $C_b < \hat{C}_b$ such that the optimal alternative policies, x_t^{count} and $x_t^{\text{no count}}$, are unique. Then the alternative policies that satisfy the first-order conditions with and without a whip count ((A.7) and (A.3) are such that):*

(1) For $q_t \neq \theta_D^m$, the optimal alternative policy with a whip count, x_t^{count}, lies strictly closer to party D's ideal point, θ_D^m, than that without, $x_t^{\text{no count}}$.

(2) $MV_t^{count}(q_t)$ and $MV_t^{no\ count}(q_t)$ strictly increase for $q_t < \theta_D^m$ and strictly increase for $q_t > \theta_D^m$.

PROOF OF LEMMA A.1: Part 1. Consider the case of $q_t < \theta_D^m$. We can write the first-order condition in the case of no whip count as an integration over the second aggregate shock (as in the case of the whip count):

$$\int_{-\infty}^{\infty}\left[1-\Phi\left(\frac{MV_t^{\text{no count}}-\hat{M}V_{R,R}-\eta}{\sigma_\eta}\right) - \frac{1}{2\sigma_\eta}\phi\left(\frac{MV_t^{\text{no count}}-\hat{M}V_{R,R}-\eta}{\sigma_\eta}\right)\left(\frac{u(x_t^{\text{no count}},\theta_D^m)-u(q_t,\theta_D^m)}{u'(x_t^{\text{no count}},\theta_D^m)}\right)\right] \times \phi\left(\frac{\eta}{\sigma_\eta}\right)d\eta = 0.$$

Consider the left-hand side of this expression, evaluated instead at x_t^{count}:

$$\begin{aligned}
&\int_{-\infty}^{\infty}\left[1-\Phi\left(\frac{MV_t^{\text{count}}-\hat{M}V_{R,R}-\eta}{\sigma_\eta}\right) - \frac{1}{2\sigma_\eta}\phi\left(\frac{MV_t^{\text{count}}-\hat{M}V_{R,R}-\eta}{\sigma_\eta}\right)\left(\frac{u(x_t^{\text{count}},\theta_D^m)-u(q_t,\theta_D^m)}{u'(x_t^{\text{count}},\theta_D^m)}\right)\right]\phi\left(\frac{\eta}{\sigma_\eta}\right)d\eta \\
&= \int_{\underline{\eta}_{1,t}}^{\infty}\left[1-\Phi\left(\frac{MV_t^{\text{count}}-\hat{M}V_{R,R}-\eta}{\sigma_\eta}\right) - \frac{1}{2\sigma_\eta}\phi\left(\frac{MV_t^{\text{count}}-\hat{M}V_{R,R}-\eta}{\sigma_\eta}\right)\left(\frac{u(x_t^{\text{count}},\theta_D^m)-u(q_t,\theta_D^m)}{u'(x_t^{\text{count}},\theta_D^m)}\right)\right]\phi\left(\frac{\eta}{\sigma_\eta}\right)d\eta \\
&\quad + \int_{-\infty}^{\underline{\eta}_{1,t}}\left[1-\Phi\left(\frac{MV_t^{\text{count}}-\hat{M}V_{R,R}-\eta}{\sigma_\eta}\right) - \frac{1}{2\sigma_\eta}\phi\left(\frac{MV_t^{\text{count}}-\hat{M}V_{R,R}-\eta}{\sigma_\eta}\right)\left(\frac{u(x_t^{\text{count}},\theta_D^m)-u(q_t,\theta_D^m)}{u'(x_t^{\text{count}},\theta_D^m)}\right)\right]\phi\left(\frac{\eta}{\sigma_\eta}\right)d\eta \\
&= +\int_{-\infty}^{\underline{\eta}_{1,t}}\left[1-\Phi\left(\frac{MV_t^{\text{count}}-\hat{M}V_{R,R}-\eta}{\sigma_\eta}\right) - \frac{1}{2\sigma_\eta}\phi\left(\frac{MV_t^{\text{count}}-\hat{M}V_{R,R}-\eta}{\sigma_\eta}\right)\left(\frac{u(x_t^{\text{count}},\theta_D^m)-u(q_t,\theta_D^m)}{u'(x_t^{\text{count}},\theta_D^m)}\right)\right]\phi\left(\frac{\eta}{\sigma_\eta}\right)d\eta,
\end{aligned} \tag{A.10}$$

where the last equality follows from the fact that x_t^{count} satisfies the first-order condition for the case of a whip count. Consider the sign of the integrand in (A.10):

$$\left[1-\Phi\left(\frac{MV_t^{\text{count}}-\hat{M}V_{R,R}-\eta}{\sigma_\eta}\right) - \frac{1}{2\sigma_\eta}\phi\left(\frac{MV_t^{\text{count}}-\hat{M}V_{R,R}-\eta}{\sigma_\eta}\right)\left(\frac{u(x_t^{\text{count}},\theta_D^m)-u(q_t,\theta_D^m)}{u'(x_t^{\text{no count}},\theta_D^m)}\right)\right]\phi\left(\frac{\eta}{\sigma_\eta}\right) \gtrless 0$$

$$\iff \quad \frac{1-\Phi\left(\dfrac{MV_t^{\text{count}}-\hat{M}V_{R,R}-\eta}{\sigma_\eta}\right)}{\dfrac{1}{2\sigma_\eta}\phi\left(\dfrac{MV_t^{\text{count}}-\hat{M}V_{R,R}-\eta}{\sigma_\eta}\right)}-\left(\frac{u(x_t^{\text{count}},\theta_D^m)-u(q_t,\theta_D^m)}{u_x(x_t^{\text{no count}},\theta_D^m)}\right)\gtrless 0.$$

The left-hand side of this inequality is a strictly increasing function of η, so that there is at most one value of η at which the integrand is zero. As $\eta\to\infty$, the integrand approaches 1. Thus, to satisfy the first-order condition for the case of a whip count at x_t^{count}, the integrand evaluated at $\underline{\eta}_{1,t}$ must be strictly negative so that the single zero-crossing is contained in $[\underline{\eta}_{1,t},\infty)$ (otherwise the integrand is positive over the whole range and cannot integrate to zero). Thus, the integrand in (A.10) must be strictly negative over $[-\infty,\underline{\eta}_{1,t}]$ so that the integral is strictly negative: the marginal expected utility for the case of no whip count must be negative when evaluated at the optimal alternative policy for the case of a whip count. But, then we must have $x_t^{\text{no count}}<x_t^{\text{count}}$ to ensure that the first-order condition for the case of no whip count is satisfied (given that $x_t^{\text{no count}}$ is the unique optimum, for every $x_t<x_t^{\text{no count}}$, the marginal expected utility is positive). The case of $q_t>\theta_D^m$ can be shown similarly.

Part 2. Consider the case of $q_t<\theta_D^m$ when a whip count is conducted. MV_t^{count} is determined implicitly by the first-order condition, (A.7). Taking its derivative with respect to q_t, we have

$$\frac{\partial}{\partial q_t}\left[\frac{\displaystyle\int_{\underline{\eta}_{1,t}}^{\infty}\left(1-\Phi\left(\frac{MV_t^{\text{count}}-\hat{M}V_{R,R}-\eta}{\sigma_\eta}\right)\right)\phi\left(\frac{\eta}{\sigma_\eta}\right)d\eta}{\displaystyle\frac{1}{2\sigma_\eta}\int_{\underline{\eta}_{1,t}}^{\infty}\phi\left(\frac{MV_t^{\text{count}}-\hat{M}V_{R,R}-\eta}{\sigma_\eta}\right)\phi\left(\frac{\eta}{\sigma_\eta}\right)d\eta}-\frac{\left(u(x_t^{\text{count}},\theta_D^m)-u(q_t,\theta_D^m)\right)}{u_x(x_t^{\text{count}},\theta_D^m)}\right]$$
$$=0$$

$$\iff \quad \frac{\partial}{\partial MV_t^{\text{count}}}\left(\frac{\displaystyle\int_{\underline{\eta}_{1,t}}^{\infty}\left(1-\Phi\left(\frac{MV_t^{\text{count}}-\hat{M}V_{R,R}-\eta}{\sigma_\eta}\right)\right)\phi\left(\frac{\eta}{\sigma_\eta}\right)d\eta}{\displaystyle\frac{1}{2\sigma_\eta}\int_{\underline{\eta}_{1,t}}^{\infty}\phi\left(\frac{MV_t^{\text{count}}-\hat{M}V_{R,R}-\eta}{\sigma_\eta}\right)\phi\left(\frac{\eta}{\sigma_\eta}\right)d\eta}\right)\frac{\partial MV_t^{\text{count}}}{\partial q_t}$$
$$-\frac{\partial}{\partial x_t^{\text{count}}}\left(\frac{u(x_t^{\text{count}},\theta_D^m)-u(q_t,\theta_D^m)}{u_x(x_t^{\text{count}},\theta_D^m)}\right)\frac{\partial x_t^{\text{count}}}{\partial q_t}=0$$

$$\iff \quad \frac{\partial}{\partial MV_t^{\text{count}}}\left(\frac{\displaystyle\int_{\underline{\eta}_{1,t}}^{\infty}\left(1-\Phi\left(\frac{MV_t^{\text{count}}-\hat{M}V_{R,R}-\eta}{\sigma_\eta}\right)\right)\phi\left(\frac{\eta}{\sigma_\eta}\right)d\eta}{\displaystyle\frac{1}{2\sigma_\eta}\int_{\underline{\eta}_{1,t}}^{\infty}\phi\left(\frac{MV_t^{\text{count}}-\hat{M}V_{R,R}-\eta}{\sigma_\eta}\right)\phi\left(\frac{\eta}{\sigma_\eta}\right)d\eta}\right)\frac{\partial MV_t^{\text{count}}}{\partial q_t}$$
$$-\frac{\partial}{\partial x_t^{\text{count}}}\left(\frac{u(x_t^{\text{count}},\theta_D^m)-u(q_t,\theta_D^m)}{u_x(x_t^{\text{count}},\theta_D^m)}\right)\left(2\frac{\partial MV_t^{\text{count}}}{\partial q_t}-1\right)=0$$

$$\iff \frac{\partial MV_t^{\text{count}}}{\partial q_t}\left[\frac{\partial}{\partial MV_t^{\text{count}}}\left(\frac{\int_{\underline{\eta}_{1,t}}^{\infty}\left(1-\Phi\left(\frac{MV_t^{\text{count}}-\hat{M}V_{R,R}-\eta}{\sigma_\eta}\right)\right)\phi\left(\frac{\eta}{\sigma_\eta}\right)d\eta}{\frac{1}{2\sigma_\eta}\int_{\underline{\eta}_{1,t}}^{\infty}\phi\left(\frac{MV_t^{\text{count}}-\hat{M}V_{R,R}-\eta}{\sigma_\eta}\right)\phi\left(\frac{\eta}{\sigma_\eta}\right)d\eta}\right)\right.$$

$$\left.-2\frac{\partial}{\partial x_t^{\text{count}}}\left(\frac{u\left(x_t^{\text{count}},\theta_D^m\right)-u\left(q_t,\theta_D^m\right)}{u_x\left(x_t^{\text{count}},\theta_D^m\right)}\right)\right]$$

$$-\frac{\partial}{\partial x_t^{\text{count}}}\left(\frac{u\left(x_t^{\text{count}},\theta_D^m\right)-u\left(q_t,\theta_D^m\right)}{u_x\left(x_t^{\text{count}},\theta_D^m\right)}\right)=0.$$

As shown in the proof of Proposition 1, the term in brackets on the left-hand side is strictly negative for $C_b < \hat{C}_b$, and the last term on the left-hand side is also strictly positive so that we must have $\frac{\partial MV_t^{\text{count}}}{\partial q_t} > 0$. Similarly, $\frac{\partial MV_t^{\text{no count}}}{\partial q_t} > 0$. For $q_t > \theta_D^m$, we can similarly establish $\frac{\partial MV_t^{\text{count}}}{\partial q_t} < 0$ and $\frac{\partial MV_t^{\text{no count}}}{\partial q_t} < 0$. *Q.E.D.*

PROOF OF LEMMA 4: $V_D^{\text{count}}(q_t) > V_D^{\text{no count}}(q_t)$ because, for C_b sufficiently small, $\underline{\eta}_{1,t} < \infty$ and $\overline{\eta}_{1,t} > -\infty$ (see footnote 3) so that an alternative policy is pursued for a nonzero measure of the support of $\eta_{1,t}$. Therefore, for the same alternative policy, party D's expected utility with a whip count must strictly exceed that without because over this support of $\eta_{1,t}$, the cost, C_b, is avoided and the probability of the alternative passing is the same. If party D pursues a different alternative policy with a whip count (which it generally does), then it must because it does even better.

Consider the case of $q_t < \theta_D^m$. We claim both value functions decrease with q_t, but the difference $V_D^{\text{count}}(q_t) - V_D^{\text{no count}}(q_t)$ increases. By the envelope theorem, the derivative of the value function for the case of no whip count with respect to q_t is given by

$$\begin{aligned}
&\frac{\partial V_D^{\text{no count}}(q_t)}{\partial q_t}\\
&= -\left(1-\Phi\left(\frac{MV_t^{\text{no count}}-\hat{M}V_{R,R}}{\sigma}\right)\right)u_q\left(q_t,\theta_D^m\right)\\
&\quad -\frac{1}{2\sigma}\phi\left(\frac{MV_t^{\text{no count}}-\hat{M}V_{R,R}}{\sigma}\right)\left(u\left(x_t^{no\ \text{count}},\theta_D^m\right)-u\left(q_t,\theta_D^m\right)\right)\\
&= -\left(1-\Phi\left(\frac{MV_t^{\text{no count}}-\hat{M}V_{R,R}}{\sigma}\right)\right)u_q\left(q_t,\theta_D^m\right)\\
&\quad -\left(1-\Phi\left(\frac{MV_t^{\text{no count}}-\hat{M}V_{R,R}}{\sigma}\right)\right)u_x\left(x_t^{\text{no count}},\theta_D^m\right)\\
&= -\left(1-\Phi\left(\frac{MV_t^{\text{no count}}-\hat{M}V_{R,R}}{\sigma}\right)\right)\left(u_q\left(q_t,\theta_D^m\right)+u_x\left(x_t^{\text{no count}},\theta_D^m\right)\right),
\end{aligned}$$

where the first equality follows from applying the first-order condition. With unbounded aggregate shocks and $q_t, x_t^{\text{no count}} < \theta_D^m$, the marginal utilities are strictly positive so that the overall derivative is negative.

In a similar manner, for the case of a whip count, we have

$$\begin{aligned}
&\frac{\partial V_D^{\text{count}}(q_t)}{\partial q_t} \\
&= -\frac{1}{2\sigma_\eta^2}\int_{\underline{\eta}_{1,t}}^{\infty}\phi\left(\frac{MV_t^{\text{count}} - \hat{MV}_{R,R} - \eta}{\sigma_\eta}\right)\phi\left(\frac{\eta}{\sigma_\eta}\right)d\eta\big(u(x_t, \theta_D^m) - u(q_t, \theta_D^m)\big) \\
&\quad - \frac{1}{\sigma_\eta}u_q(q_t, \theta_D^m)\int_{\underline{\eta}_{1,t}}^{\infty}\left(1 - \Phi\left(\frac{MV_t^{\text{count}} - \hat{MV}_{R,R} - \eta}{\sigma_\eta}\right)\right)\phi\left(\frac{\eta}{\sigma_\eta}\right)d\eta \\
&= -\frac{1}{\sigma_\eta}\big(u_q(q_t, \theta_D^m) + u_x(x_t^{\text{count}}, \theta_D^m)\big) \\
&\quad \times \int_{\underline{\eta}_{1,t}}^{\infty}\left(1 - \Phi\left(\frac{MV_t^{\text{count}} - \hat{MV}_{R,R} - \eta}{\sigma_\eta}\right)\right)\phi\left(\frac{\eta}{\sigma_\eta}\right)d\eta
\end{aligned}$$

which is also strictly negative, given $\underline{\eta}_{1,t} < \infty$.

Finally, consider the marginal difference of the value functions:

$$\begin{aligned}
&\frac{\partial\big(V_D^{\text{count}}(q_t) - V_D^{\text{no count}}(q_t)\big)}{\partial q_t} \\
&= -\frac{1}{\sigma_\eta}\big(u_q(q_t, \theta_D^m) + u_x(x_t^{\text{count}}, \theta_D^m)\big) \\
&\quad \times \int_{\underline{\eta}_{1,t}}^{\infty}\left(1 - \Phi\left(\frac{MV_t^{\text{count}} - \hat{MV}_{R,R} - \eta}{\sigma_\eta}\right)\right)\phi\left(\frac{\eta}{\sigma_\eta}\right)d\eta \\
&\quad + \big(u_q(q_t, \theta_D^m) + u_x(x_t^{\text{no count}}, \theta_D^m)\big)\left(1 - \Phi\left(\frac{MV_t^{\text{no count}} - \hat{MV}_{R,R}}{\sigma}\right)\right).
\end{aligned}$$

From the first part of Lemma A.1, $x_t^{\text{no count}} < x_t^{\text{count}}$, which ensures $u_x(x_t^{\text{no count}}, \theta_D^m) > u_x(x_t^{\text{count}}, \theta_D^m)$. Furthermore,

$$\begin{aligned}
&1 - \Phi\left(\frac{MV_t^{\text{no count}} - \hat{MV}_{R,R}}{\sigma}\right) \\
&\quad > 1 - \Phi\left(\frac{MV_t^{\text{count}} - \hat{MV}_{R,R}}{\sigma}\right) \\
&\quad = \frac{1}{\sigma_\eta}\int_{-\infty}^{\infty}\left(1 - \Phi\left(\frac{MV_t^{\text{count}} - \hat{MV}_{R,R} - \eta}{\sigma_\eta}\right)\right)\phi\left(\frac{\eta}{\sigma_\eta}\right)d\eta
\end{aligned}$$

$$> \frac{1}{\sigma_\eta}\int_{\underline{\eta}_{1,t}}^{\infty}\left(1-\Phi\left(\frac{MV_t^{\text{count}}-\hat{MV}_{R,R}-\eta}{\sigma_\eta}\right)\right)\phi\left(\frac{\eta}{\sigma_\eta}\right)d\eta$$

$$> 0$$

given $\underline{\eta}_{1,t} < \infty$. Therefore, the difference in expected utility strictly increases with q_t.

For $q_t > \theta_D^m$, we can establish that both value functions increase in q_t, but their difference decreases, in an identical manner. *Q.E.D.*

PROOF OF PROPOSITION 2: Assume $C_b < \hat{C}_b$ so that, from Proposition 1, x_t^{count} is unique. Consider $q_t < \theta_D^m$. We first show that as $q_t \to \theta_D^m$, $V_D^{\text{no count}}(q_t) \to -C_b$ and $V_D^{\text{count}}(q_t) \to 0$. The first follows from simple inspection of $EU_D^{\text{no count}}(q_t, x_t)$, noting that $x_t^{\text{no count}}$ must approach θ_D^m as $q_t \to \theta_D^m$ because it is contained in the interval, (q_t, θ_D^m), by Proposition 1. Similarly, inspecting $EU_D^{\text{count}}(q_t, x_t)$, we see that $V_D^{\text{count}}(q_t) \to -(1 - \Phi(\frac{\underline{\eta}_{1,t}}{\sigma_\eta}))C_b$. But, as $q_t \to \theta_D^m$, we can see from (A.5) that $\underline{\eta}_{1,t}$ must approach infinity such that $\Phi(\frac{\underline{\eta}_{1,t}}{\sigma_\eta}) \to 1$.

Given these facts, strictly positive costs, and the result of Lemma 4 that both value functions strictly decrease with $|q_t - \theta_D^m|$, there exists a status quo cutoff, $\overline{q}_l < \theta_D^m$, such that for all $q_t \in (\overline{q}_l, \theta_D^m)$, no alternative policy is pursued. Specifically, $\overline{q}_l$ is given by the larger of the two policies, q_1 and q_2 which satisfy $V_D^{\text{no count}}(q_1) = 0$ and $V_D^{\text{count}}(q_2) = C_w$, respectively.

For $q_t < \overline{q}_l$, there are two possibilities. If $q_1 > q_2$, then set $\underline{q}_l = \overline{q}_l = q_1$ so that $V_D^{\text{count}}(q_1) < C_w$ and $V_D^{\text{no count}}(q_1) = 0$. In this case, for any $q_t < q_1$, an alternative policy is pursued without a whip count: by Lemma 4, over this range, $V_D^{\text{no count}}(q_1) > 0$ so that an alternative policy without a whip count is preferred over not pursuing an alternative policy and, as q_t decreases from q_1, $V_D^{\text{count}}(q_t) - V_D^{\text{no count}}(q_t)$ decreases so that not conducting a whip count remains more valuable than conducting one.

If $q_1 < q_2$, then set $\overline{q}_l = q_2$ and define $\underline{q}_l < \overline{q}_l$ to be the policy for which $V_D^{\text{count}}(\underline{q}_l) - C_w = V_D^{\text{no count}}(\underline{q}_l)$. Such a point must exist because, by Lemma 4, as q_t decreases from $\overline{q}_l$, $V_D^{\text{count}}(q_t) - V_D^{\text{no count}}(q_t)$ decreases and so must eventually approach zero. Thus, for q_t sufficiently small, $V_D^{\text{count}}(q_t) - C_w < V_D^{\text{no count}}(q_t)$. With these cutoffs, for $q_t \in (-\infty, \underline{q}_l]$, an alternative policy is pursued without a whip count because $V_D^{\text{no count}}(q_t) > V_D^{\text{count}}(q_t) - C_w > 0$ for all $q_t < \underline{q}_l$. For $q_t \in (\underline{q}_l, \overline{q}_l]$, an alternative policy is pursued with a whip count because $V_D^{\text{count}}(q_t) - C_w > 0$ and, by Lemma 4, $V_D^{\text{count}}(q_t) - V_D^{\text{no count}}(q_t)$ increases with q_t over this range so that $V_D^{\text{count}}(q_t) - C_w > V_D^{\text{no count}}(q_t)$.

Symmetric arguments establish cutoffs, $\underline{q}_r$ and $\overline{q}_r$, for the bill pursuit decisions over the range $q_t > \theta_D^m$. *Q.E.D.*

A.2. *Additional Figures for the Model*

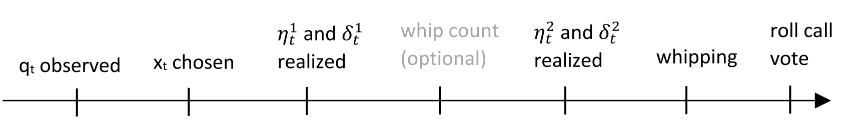

FIGURE 9.—Timeline.

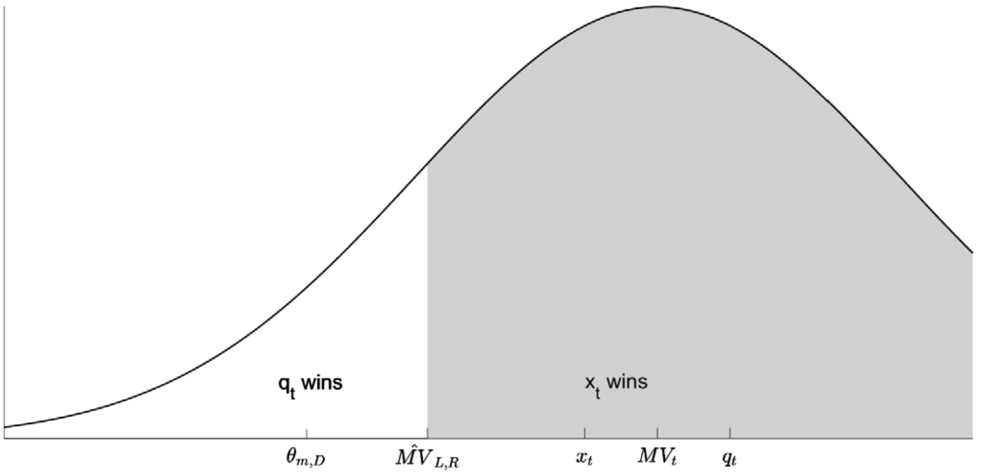

FIGURE 10.—Optimal Policy Alternative. Notes: Optimal policy selection by the Democratic party for a status quo, q_t, right of their ideal point, $\theta_{m,D}$, for a bill that goes directly to roll call. The shaded area is the probability that the policy alternative, x_t, wins. x_t wins if the sum of the aggregate shocks is such that the realized marginal voter lies to the right of $\hat{MV}_{L,R}$, the position of the marginal voter for which votes are equally split between q_t and x_t. A policy alternative chosen closer to the Democratic ideal point is preferred, but is less likely to pass because as it shifts left, the marginal voter, MV_t, also shifts left, reducing the size of the shaded area.

Co-editor Fabrizio Zilibotti handled this manuscript.

Manuscript received 17 October, 2018; final version accepted 30 December, 2019; available online 22 January, 2020.