A Comment on "Can Relaxation of Beliefs Rationalize the Winner's Curse?: An Experimental Study"

by

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Computation of Expected-Payoff Maximizing Bids in the part II auctions

In this memo, we describe how expected-payoff maximizing bids are computed. **The subject number is fixed for the procedures below. Hence, our notation omits the index for the subject number.** We use subject # 37 as an example, whose signal-bid combinations in part I are shown below (also in Table 1 of the article), to illustrate the procedure:

x_i	0	1	2	3	4	5	6	7	8	9	10
b_i	4	1	4	5	5	5	5	6	9	10	10

Sorting the Bids: We first sort the bids in ascending order, which produces the following list of signal-bid combinations. For each $l \in \{1, \ldots, 11\}$, we label each bid as $\mathbf{oppb}(l)$ and the corresponding signal as $\mathbf{oppv}(l)$.[1] Remember that the subject's opponent in part II (i.e., computer) receives the signal randomly for each auction, and mimics the subject's behavior in part I.

l	1	2	3	4	5	6	7	8	9	10	11
$\mathbf{oppv}(l)$	**1**	**0**	2	3	4	5	6	7	8	9	10
$\mathbf{oppb}(l)$	**1**	**4**	4	5	5	5	5	6	9	10	10

Bids in Consideration: Although the number of feasible bids is 100,000,001, we only look at a subset of them for each subject, denoted as $\mathbf{sb}$ whose elements are denoted by $\mathbf{sb}(k)$ for $k \in \{1, \ldots, 45\}$.

First, the vector $\mathbf{sb}$ includes the elements of the vector $\mathbf{bidsignal}$ whose elements are $\mathbf{bidsignal}(\xi)$ for $\xi \in \{1, \ldots, 22\}$. The vector $\mathbf{sb}$ consists of (i) $\mathbf{oppb}$ (11 elements) and (ii)

[1] Note that the signal-bid pairs are lexicographically ordered.

the signals (11 elements in bold in the **bidsignal** vector below). They are sorted in ascending order.[2]

`bidsignal`	**0**	**1**	1	**2**	**3**	**4**	4	4	**5**	5	5	5	5	**6**	6	**7**	**8**	**9**	9	**10**	10	10

Second, we consider a bid between every pair of two adjoining bids in **bidsignal**. Consider the current example. Given **oppb** shown above (which is known to the subject in part II), consider the set of bids $\{1.01, \ldots, 3.99\}$. Note that any bid in this set leads to the same expected payoff since the cases in which these bids win are the same (and in a second-price auction the payoff for all winning bids is the same). Hence, the examination of only one point in this set is sufficient to see whether every bid in this set is an expected-payoff maximizing bid. We simply include the average of every pair of adjoining elements in **bidsignal** (21 elements). We also add 0 (the first element) and 1000000 (the last element). Together these bids create **mid** whose elements are denoted by $\mathbf{mid}(\zeta)$ for $\zeta \in \{1, \ldots, 23\}$.[3] Some elements of **mid** may not be feasible bids. Nevertheless, our computation of expected payoffs (and hence expected-payoff maximizing bids) still works.

We sort the elements in **bidsignal** and **mid** in ascending order to create **sb** (45 elements).[4] As a consequence, each element from **bidsignal** has an even-numbered index in **sb** while every element from **mid** has an odd-numbered index in **sb**. We use the bids in **sb** as the subject's potential bid choices to evaluate their expected payoffs. We then identify the set of expected-payoff maximizing bids.

0	0	**0.5**	1	**1**	1	**1.5**	2	**2.5**
3	**3.5**	4	**4**	4	**4**	4	**4.5**	5
5	5	**5**	5	**5**	5	**5**	5	**5.5**
6	**6**	6	**6.5**	7	**7.5**	8	**8.5**	9
9	9	**9.5**	10	**10**	10	**10**	10	**1000000**

[2]The same number appears at least twice if any of the bids is an integer between 0 and 10 as the current example suggests. We are aware that the signals could be dropped from our procedure to compute expected-payoff maximizing bids.

[3]If two adjoining elements in **bidsignal** are equal, the corresponding element in **mid** is equal to both as well.

[4]Since 0 enters as a signal and as the first element of **mid**, the first two elements of **sb** are always zero.

Remark 1. For *MB*, any number strictly lower than x_i cannot be chosen.

Remark 2. In general, we cannot exclude the possibility that there exists $\eta \in \{2, \ldots, 11\}$ such that $\mathbf{bidsignal}(k) - \mathbf{bidsignal}(k-1) = 0.01$. In this case, the corresponding element in **mid** cannot be chosen by the subject.

We explicitly take these two cases into consideration below.

Computation of Expected Payoffs: Remember that $\mathbf{oppb}(l)$ and $\mathbf{oppv}(l)$ are sorted according to $\mathbf{oppb}(l)$. For any $l \in \{1, \ldots, 11\}$, $\mathbf{oppv}(l)$ provides the signal attached to $\mathbf{oppb}(l)$ for the opponent. In addition, remember that the value of the object is $\max\{x_1, x_2\}$.

For each $j \in \{1, \ldots, 11\}$ and $l \in \{1, \ldots, 11\}$, we first identify the value of the object when the subject's signal is $x(j) = j - 1$ and the opponent's signal is $\mathbf{oppv}(l)$, which we denote by $\mathbf{m}(j, l) = \max\{x(j), \mathbf{oppv}(l)\}$. For the current example, see the sheet "m" in the excel file "example".[5]

For each $k \in \{1, \ldots, 45\}$ and $l \in \{1, \ldots, 11\}$, we construct the variable $\mathbf{a}(k, l)$ which takes three possible values;

$$\mathbf{a}(k,l) = \left\{ \begin{array}{c} 1 \\ 0.5 \\ 0 \end{array} \right\} \quad \text{if} \quad \mathbf{sb}(k) \left\{ \begin{array}{c} > \\ = \\ < \end{array} \right\} \mathbf{oppb}(l).$$

See the sheet "a" in the excel file "example".

For each $k \in \{1, \ldots, 45\}$ and $j \in \{1, \ldots, 11\}$, we compute the expected payoff with $\mathbf{sb}(k)$ and $x(j)$;

$$\mathbf{u}(k,j) = \sum_{l=1}^{11} \frac{\mathbf{a}(k,l)}{11} \left[\mathbf{m}(j,l) - \mathbf{oppb}(l)\right].$$

See the sheet "u" in the file "example".

Remark 3. To avoid rounding errors, we multiply the expected payoffs by 22 for the identification of expected-payoff maximizing bids.

[5] In this excel file, we directly computed the values to confirm that the numbers in Table 1 of the manuscript are consistent with the results obtained here.

Remark 4. For any $\mathbf{sb}(i,k)$ to which Remarks 1 or 2 apply, we let $\mathbf{u}(k,j) = -1$.[6]

We identify the highest expected payoff for each $x(j)$; for each $j \in \{1,\ldots,11\}$, let

$$\mathbf{maxu}(j) = \max_k \{u(k,j)\}.$$

We then construct an indicator variable for each $k \in \{1,\ldots,45\}$ and $j \in \{1,\ldots,11\}$:

$$\mathbf{g}(k,j) = \left\{ \begin{array}{c} 1 \\ 0 \end{array} \right\} \quad \text{if} \quad \mathbf{u}(k,j) \left\{ \begin{array}{c} = \\ < \end{array} \right\} \mathbf{maxu}(j)$$

This variable indicates that for every $x(j)$, $\mathbf{sb}(k)$ with $\mathbf{g}(k,j) = 1$ is an expected-payoff maximizing bid.

Identification of Expected-Payoff Maximizing Bids: We introduce the following variables:

- $\mathbf{noi}(j)$: the number of the sets of expected-payoff maximizing bids for $x(j)$. We use the index $\eta \in \{1,\ldots,\mathbf{noi}(j)\}$,[7]
- $\mathbf{inf}(j,\eta) \in \mathbf{bidsignal}$ and $\mathbf{sup}(j,\eta) \in \mathbf{bidsignal}$: corresponding to the inf and sup of the set of expected-payoff maximizing bids η respectively,
- $\mathbf{binf}(j,\eta) \in \{-2,-1\}$ and $\mathbf{bsup}(j,\eta) \in \{1,2\}$: $\mathbf{binf}(j,\eta)$ is -2 (-1) if the corresponding $\mathbf{inf}(j,\eta)$ is "closed" ("open"). Likewise, $\mathbf{bsup}(j,\eta)$ is 2 (1) if the corresponding $\mathbf{sup}(j,\eta)$ is "closed" ("open"), and
- `oneminusone` $\in \{-1,1\}$: 1 means "looking for inf" and -1 means "looking for sup".

We first identify any "locally" unique expected-payoff maximizing bids by looking at the elements of $\mathbf{sb}$ with even-numbered indices (the elements of $\mathbf{bidsignal}$). Given $j \in \{1,\ldots,11\}$, for every $k \in \{2,4,\ldots,44\}$ with $\mathbf{g}(k,j) = 1$ (i.e., $\mathbf{sb}(k)$ is an expected-payoff maximizing bid for $x(j)$), we check whether $\mathbf{u}(k-1,j) < \mathbf{maxu}(j)$ and $\mathbf{u}(k+1,j) < \mathbf{maxu}(j)$; i.e., whether its adjoining bids $k-1$ and $k+1$ (the elements in $\mathbf{mid}$) are expected-payoff maximizing bids for $x(j)$. If $\mathbf{sb}(k)$ is a locally unique expected-payoff maximizing bid

[6]Bidding 0 guarantees a payoff of 0 at least. Hence, the highest expected payoff cannot be negative (including -1), implying that assigning -1 is sufficient (i.e., such an **sb** cannot be a expected-payoff maximizing bid). In addition, regarding Remark 2, our computational procedure may potentially divide a set of expected-payoff maximizing bids into two sets, which did not happen with the current data.

[7]As the current example shows, it is possible to have multiple separate sets of expected-payoff maximizing bids.

for $x(j)$, we treat $\{\mathbf{sb}(k)\}$ as a set of expected-payoff maximizing bids and assign an index $\eta \in \{1, \ldots, \mathbf{noi}(j)\}$ to this set.[8]

Next, we look at the elements of $\mathbf{sb}$ with odd-numbered indices (the elements of $\mathbf{mid}$). Given $j \in \{1, \ldots, 11\}$, for every $k \in \{1, 3, \ldots, 45\}$, there are two possibilities; (a) none of its two adjoining elements $\mathbf{sb}(k-1)$ and $\mathbf{sb}(k+1)$ (which are elements of **bidsignal**) is an expected-payoff maximizing bid for $x(j)$, or (b) at least one of them is an expected-payoff maximizing bid for $x(j)$.

[INF] For $j \in \{1, \ldots, 11\}$, we first look for "inf". Starting from $\mathtt{sb}(1)$=0, we look for $\underline{k} \in \{1, 3, \ldots, 45\}$ such that $\mathbf{g}(\underline{k}, j) = 1$ (i.e., $\mathbf{u}(\underline{k}, j) = \mathbf{maxu}(j)$).

- If $\underline{k} = 1$, let $\mathbf{inf}(j, \eta) = \mathbf{sb}(1) = 0$ and $\mathbf{binf}(j, \eta) = -2$ (closed).
- For $\underline{k} > 1$, let $\mathbf{inf}(j, \eta) = \mathbf{sb}(\underline{k} - 1)$ (an element of **bidsignal**). If $\mathbf{g}(\underline{k} - 1, j) = 1$, let $\mathbf{binf}(j, \eta) = -2$ (closed). Let $\mathbf{binf}(j, \eta) = -1$ (open) otherwise.
- If $\underline{k} = 45$ (remember that $\mathbf{sb}(45) = 1000000$), let $\mathbf{sup}(j, \eta) = \mathbf{sb}(45) = 1000000$ and $\mathbf{bsup}(j, \eta) = 2$ (closed).

For the current example, see the sheet "g1-inf" of the file "example".

[SUP] We now look for "sup". Starting from $\mathbf{sb}(\underline{k} + 2)$, we check $\bar{k} \in \{\underline{k} + 2, \ldots, 45\}$:

- If $\mathbf{g}(\bar{k}, j) = 1$, check $\bar{k} + 2$.
- If instead $\mathbf{g}(\bar{k}, j) = 0$, let $\mathbf{sup}(j, \eta) = \mathbf{sb}(\bar{k} - 1)$. If $\mathbf{g}(\bar{k} - 1, j) = 1$, let $\mathbf{bsup}(j, \eta) = 2$ (closed). Let $\mathbf{bsup}(j, \eta) = 1$ (open) otherwise.
- If $\bar{k} = 45$, let $\mathbf{sup}(j, \eta) = \mathbf{sb}(45) = 1000000$ and $\mathbf{bsup}(j, \eta) = 2$ (closed).

For the current example, see "g1-sup" of the file "example".

Repeat the same procedure for higher η's and then for higher j's. The highest $\mathbf{noi}(j)$ we found is 2. See "g2" of the file "example".

Remark 5. Note that for our search of "sup", we only look at $k \in \{1, 3, \ldots, 45\}$ (the elements of $\mathbf{mid}$). One may wonder we may be missing the case where for some $k' \in \{2, 4, \ldots, 44\}$ (the elements of **bidsignal**), we have $\mathbf{g}(k' - 1, j) = \mathbf{g}(k' + 1, j) = 1$ and

[8] In the current example, we do not have such a case.

$\mathbf{g}(k', j) = 0$.[9] Three possible cases listed below show that we do not have to explicitly take this into account:

- $\mathbf{sb}(k') = \mathbf{sb}(k'+1) = \mathbf{sb}(k'+2)$: This implies $\mathbf{g}(k', j) = \mathbf{g}(k'+1, j) = \mathbf{g}(k'+2, j)$.

- $\mathbf{sb}(k'+2) - \mathbf{sb}(k') = 0.01$: $\mathbf{u}(k'+1, j) = -1$.[10] Although this produces the correct sets of expected-payoff maximizing bids, it unnecessarily divides a set into two (which did not occur with the current data).

- $\mathbf{sb}(k'+2) - \mathbf{sb}(k') > 0.01$:[11] That $\mathbf{g}(k'-1, j) = \mathbf{g}(k'+1, j)$ implies that the tie at $\mathbf{sb}(k')$ (if there is one – if this is a signal, the argument does not apply) contributes zero to $\mathbf{u}(k', j)$ – otherwise, $\mathbf{u}(k'-1, j) \neq \mathbf{u}(k'+1, j)$. This implies $\mathbf{g}(k', j) = 1$ as well.

We now identify the sets of expected-payoff maximizing bids. For each set of expected-payoff maximizing bids, we use two variables: $\mathbf{dinf}(j, \eta)$ and $\mathbf{dsup}(j, \eta)$. For each $j \in \{1, \ldots, 11\}$ and $\eta \in \{1, \ldots, \mathbf{noi}(j)\}$, there are five possibilities:

- $\mathbf{inf}(j, \eta) = \mathbf{sup}(j, sup)$ (i.e., "locally unique" expected-payoff maximizing bid): $\mathbf{dinf}(j, \eta) = \mathbf{inf}(j, \eta)$ and $\mathbf{dsup}(j, \eta) = \mathbf{sup}(j, \eta)$.

- $\mathbf{binf}(j, \eta) = -2$ (closed) and $\mathbf{bsup}(j, \eta) = 2$ (closed): $\mathbf{dinf}(j, \eta) = \mathbf{inf}(j, \eta)$ and $\mathbf{dsup}(j, \eta) = \mathbf{sup}(j, \eta)$.

- $\mathbf{binf}(j, \eta) = -1$ (open) and $\mathbf{bsup}(j, \eta) = 2$ (closed): $\mathbf{dinf}(j, \eta) = \mathbf{inf}(j, \eta) + 0.01$ and $\mathbf{dsup}(j, \eta) = \mathbf{sup}(j, \eta)$.

- $\mathbf{binf}(j, \eta) = -2$ (closed) and $\mathbf{bsup}(j, \eta) = 1$ (open): $\mathbf{dinf}(j, \eta) = \mathbf{inf}(j, \eta)$ and $\mathbf{dsup}(j, \eta) = \mathbf{sup}(j, \eta) - 0.01$.

- $\mathbf{binf}(j, \eta) = -1$ (open) and $\mathbf{bsup}(j, \eta) = 1$ (open): $\mathbf{dinf}(j, \eta) = \mathbf{inf}(j, \eta) + 0.01$ and $\mathbf{dsup}(j, \eta) = \mathbf{sup}(j, \eta) - 0.01$.

Please compare the sheet "g2" in the file "example" and Table 1 in the manuscript.

[9] This implies that $\mathbf{sb}(k'-2) < \mathbf{sb}(k')$ and hence the opponent does not choose a bid between these two numbers, including $\mathbf{sb}(k'-1)$.

[10] See Remarks 2 and 3.

[11] This means that the opponent does not choose a bid between these two numbers, including $\mathbf{sb}(k'+1)$.