Econometrica Supplementary Material

SUPPLEMENT TO "LAND-PRICE DYNAMICS AND MACROECONOMIC FLUCTUATIONS"

(*Econometrica*, Vol. 81, No. 3, May 2013, 1147–1184)

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APPENDIX D: DERIVATIONS AND ESTIMATION OF THE BENCHMARK DSGE MODEL

D.1. *The Model*

THE ECONOMY IS POPULATED BY TWO TYPES OF AGENTS—households and entrepreneurs—with a continuum and unit measure of each type. There are four types of commodities: labor, goods, land, and loanable bonds. Goods production requires labor, capital, and land as inputs. The output can be used for consumption (by both types of agents) and for capital investment (by the entrepreneurs). The representative household's utility depends on consumption goods, land services (housing), and leisure; the representative entrepreneur's utility depends on consumption goods only.

D.1.1. *The Representative Household*

Similarly to Iacoviello (2005), the household has the utility function

$$\text{(S.1)}\qquad \mathrm{E}\sum_{t=0}^{\infty}\beta^t A_t\{\log(C_{ht}-\gamma_h C_{h,t-1})+\varphi_t\log L_{ht}-\psi_t N_{ht}\},$$

where C_{ht} denotes consumption, L_{ht} denotes land holdings, and N_{ht} denotes labor hours. The parameter $\beta\in(0,1)$ is a subjective discount factor, the parameter γ_h measures the degree of habit persistence, and the term E is a mathematical expectation operator. The terms A_t, φ_t, and ψ_t are preference shocks. We assume that the intertemporal preference shock A_t follows the stochastic process

$$\text{(S.2)}\qquad A_t=A_{t-1}(1+\lambda_{at}),\qquad \ln\lambda_{at}=(1-\rho_a)\ln\bar{\lambda}_a+\rho_a\ln\lambda_{a,t-1}+\varepsilon_{at},$$

where $\bar{\lambda}_a>0$ is a constant, $\rho_a\in(-1,1)$ is the persistence parameter, and ε_{at} is an independent and identically distributed (i.i.d.) white noise process with mean zero and variance σ_a^2. The housing preference shock φ_t follows the stationary process

$$\text{(S.3)}\qquad \ln\varphi_t=(1-\rho_\varphi)\ln\bar{\varphi}+\rho_\varphi\ln\varphi_{t-1}+\varepsilon_{\varphi t},$$

[1]We are grateful to Pat Higgins, who provides invaluable research assistance. The views expressed herein are those of the authors and do not necessarily reflect the views of the Federal Reserve Banks of Atlanta and San Francisco or the Federal Reserve System.

 DOI: 10.3982/ECTA8994

where $\bar{\varphi} > 0$ is a constant, $\rho_\varphi \in (-1, 1)$ measures the persistence of the shock, and $\varepsilon_{\varphi t}$ is a white noise process with mean zero and variance σ_φ^2. The labor supply shock ψ_t follows the stationary process

$$\ln \psi_t = (1 - \rho_\psi) \ln \bar{\psi} + \rho_\psi \ln \psi_{t-1} + \varepsilon_{\psi t}, \tag{S.4}$$

where $\bar{\varphi} > 0$ is a constant, $\rho_\psi \in (-1, 1)$ measures the persistence, and $\varepsilon_{\psi t}$ is a white noise process with mean zero and variance σ_ψ^2.

Denote by q_{lt} the relative price of housing (in consumption units), R_t the gross real loan rate, and w_t the real wage; denote by S_t the household's purchase in period t of the loanable bond that pays off one unit of consumption good in all states of nature in period $t + 1$. In period 0, the household begins with $L_{h,-1} > 0$ units of housing and $S_{-1} > 0$ units of the loanable bond. The flow of funds constraint for the household is given by

$$C_{ht} + q_{lt}(L_{ht} - L_{h,t-1}) + \frac{S_t}{R_t} \leq w_t N_{ht} + S_{t-1}. \tag{S.5}$$

The household chooses C_{ht}, $L_{h,t}$, N_{ht}, and S_t to maximize (S.1) subject to (S.2)–(S.5) and the borrowing constraint $S_t \geq -\bar{S}$ for some large number $\bar{S}$.

D.1.2. *The Representative Entrepreneur*

The entrepreneur has the utility function

$$\mathrm{E} \sum_{t=0}^{\infty} \beta^t \big[\log(C_{et} - \gamma_e C_{e,t-1})\big], \tag{S.6}$$

where C_{et} denotes the entrepreneur's consumption and γ_e is the habit persistence parameter.

The entrepreneur produces goods using capital, labor, and land as inputs. The production function is given by

$$Y_t = Z_t \big[L_{e,t-1}^{\phi} K_{t-1}^{1-\phi}\big]^{\alpha} N_{et}^{1-\alpha}, \tag{S.7}$$

where Y_t denotes output, K_{t-1}, N_{et}, and $L_{e,t-1}$ denote the inputs capital, labor, and land, respectively, and the parameters $\alpha \in (0, 1)$ and $\phi \in (0, 1)$ measure the output elasticities of these production factors. We assume that the total factor productivity Z_t is composed of a permanent component Z_t^p and a transitory component ν_t such that $Z_t = Z_t^p \nu_{zt}$, where the permanent component Z_t^p follows the stochastic process

$$Z_t^p = Z_{t-1}^p \lambda_{zt}, \quad \ln \lambda_{zt} = (1 - \rho_z) \ln \bar{\lambda}_z + \rho_z \ln \lambda_{z,t-1} + \varepsilon_{zt}, \tag{S.8}$$

and the transitory component follows the stochastic process

$$\ln \nu_{zt} = \rho_{\nu_z} \ln \nu_{z,t-1} + \varepsilon_{\nu_z t}. \tag{S.9}$$

The parameter $\bar{\lambda}_z$ is the steady-state growth rate of Z_t^p; the parameters ρ_z and ρ_{ν_z} measure the degree of persistence. The innovations ε_{zt} and $\varepsilon_{\nu_z t}$ are i.i.d. white noise processes that are mutually independent with mean zero and variances given by σ_z^2 and $\sigma_{\nu_z}^2$, respectively.

The entrepreneur is endowed with K_{-1} units of initial capital stock and $L_{-1,e}$ units of initial land. Capital accumulation follows the law of motion

$$K_t = (1-\delta)K_{t-1} + \left[1 - \frac{\Omega}{2}\left(\frac{I_t}{I_{t-1}} - \bar{\lambda}_I\right)^2\right] I_t, \tag{S.10}$$

where I_t denotes investment, $\bar{\lambda}_I$ denotes the steady-state growth rate of investment, and $\Omega > 0$ is the adjustment cost parameter.

The entrepreneur faces the flow of funds constraint

$$\begin{aligned} &C_{et} + q_{lt}(L_{et} - L_{e,t-1}) + B_{t-1} \\ &\quad = Z_t\left[L_{e,t-1}^{\phi} K_{t-1}^{1-\phi}\right]^{\alpha} N_{et}^{1-\alpha} - \frac{I_t}{Q_t} - w_t N_{et} + \frac{B_t}{R_t}, \end{aligned} \tag{S.11}$$

where B_{t-1} is the amount of matured debt and B_t/R_t is the value of new debt. Following Greenwood, Hercowitz, and Krusell (1997), we interpret Q_t as the investment-specific technological change. Specifically, we assume that $Q_t = Q_t^p \nu_{qt}$, where the permanent component Q_t^p follows the stochastic process

$$Q_t^p = Q_{t-1}^p \lambda_{qt}, \quad \ln \lambda_{qt} = (1-\rho_q)\ln \bar{\lambda}_q + \rho_q \ln \lambda_{q,t-1} + \varepsilon_{qt}, \tag{S.12}$$

and the transitory component follows the stochastic process

$$\ln \nu_{qt} = \rho_{\nu_q} \ln \nu_{q,t-1} + \varepsilon_{\nu_q t}. \tag{S.13}$$

The parameter $\bar{\lambda}_q$ is the steady-state growth rate of Q_t^p; the parameters ρ_q and ρ_{ν_q} measure the degree of persistence. The innovations ε_{qt} and $\varepsilon_{\nu_q t}$ are i.i.d. white noise processes that are mutually independent with mean zero and variances given by σ_q^2 and $\sigma_{\nu_q}^2$, respectively.

The entrepreneur faces the credit constraint

$$B_t \leq \theta_t \mathrm{E}_t[q_{l,t+1}L_{et} + q_{k,t+1}K_t], \tag{S.14}$$

where $q_{k,t+1}$ is the shadow price of capital in consumption units.[2] Under this credit constraint, the amount that the entrepreneur can borrow is limited by a fraction of the value of the collateral assets—land and capital. Following

[2]Since the price of new capital is $1/Q_t$, Tobin's q in this model is given by $q_{kt}Q_t$, which is the ratio of the value of installed capital to the price of new capital.

Kiyotaki and Moore (1997), we interpret this type of credit constraints as reflecting the problem of costly contract enforcement: if the entrepreneur fails to pay the debt, the creditor can seize the land and the accumulated capital; since it is costly to liquidate the seized land and capital stock, the creditor can recoup up to a fraction θ_t of the total value of the collateral assets. We interpret θ_t as a "collateral shock" that reflects the uncertainty in the tightness of the credit market. We assume that θ_t follows the stochastic process

$$\ln \theta_t = (1-\rho_\theta)\ln \bar{\theta} + \rho_\theta \ln \theta_{t-1} + \varepsilon_{\theta t}, \tag{S.15}$$

where $\bar{\theta}$ is the steady-state value of θ_t, $\rho_\theta \in (0, 1)$ is the persistence parameter, and $\varepsilon_{\theta t}$ is an i.i.d. white noise process with mean zero and variance σ_θ^2.

The entrepreneur chooses C_{et}, N_{et}, I_t, $L_{e,t}$, K_t, and B_t to maximize (S.6) subject to (S.7) through (S.15).

D.1.3. *Market Clearing Conditions and Equilibrium*

In a competitive equilibrium, the markets for goods, labor, land, and loanable bonds all clear. The goods market clearing condition implies that

$$C_t + \frac{I_t}{Q_t} = Y_t, \tag{S.16}$$

where $C_t = C_{ht} + C_{et}$ denotes aggregate consumption. The labor market clearing condition implies that labor demand equals labor supply:

$$N_{et} = N_{ht} \equiv N_t. \tag{S.17}$$

The land market clearing condition implies that

$$L_{ht} + L_{et} = \bar{L}, \tag{S.18}$$

where $\bar{L}$ is the fixed aggregate land endowment. Finally, the bond market clearing condition implies that

$$S_t = B_t. \tag{S.19}$$

A competitive equilibrium consists of sequences of prices $\{w_t, q_{lt}, R_t\}_{t=0}^{\infty}$ and allocations $\{C_{ht}, C_{et}, I_t, N_{ht}, N_{et}, L_{ht}, L_{et}, S_t, B_t, K_t, Y_t\}_{t=0}^{\infty}$ such that (i) taking the prices as given, the allocations solve the optimizing problems for the household and the entrepreneur, and (ii) all markets clear.

D.2. *Derivations of Excess Returns and Equilibrium Conditions*

D.2.1. *The Excess Returns*

In this section, we provide an intuitive derivation of the first-order excess returns in the presence of binding credit constraints.

The representative entrepreneur has two types of assets: land and capital. Each asset can be intuitively thought of as a Lucas tree bearing fruits and growing at a gross rate of g_γ. The entrepreneur can trade a portion of the tree in the market, and the return on this tree depends on the price of a unit of the tree as well as the marginal product (fruit) of the remaining tree. In steady state, it should be g_γ/β. To see if this intuition works in the model when the entrepreneur faces the borrowing constraint, we first derive the expected return on each of these assets. We begin with the return on land.

Suppose the entrepreneur purchases one unit of land at the price q_{lt} in period t. Since she can pledge a fraction θ_t of the present value of the land as a collateral, the net out-of-pocket payment (i.e., the down payment) to purchase the land is given by

$$u_t \equiv q_{lt} - \theta_t \mathrm{E}_t \frac{q_{l,t+1}}{R_t}, \tag{S.20}$$

where R_t is the loan rate. The land is used for period $t+1$ production and yields $\phi\alpha Y_{t+1}/L_{et}$ units of extra output. In addition, the entrepreneur can keep the remaining value of the land in period $t+1$ after repaying the debt, so that the total payoff from the land is $\phi\alpha Y_{t+1}/L_{et} + q_{l,t+1} - \theta_t \mathrm{E}_t q_{l,t+1}$. The return on the land from period t to $t+1$ is thus given by

$$R_{l,t+1} = \frac{\phi\alpha Y_{t+1}/L_{et} + q_{l,t+1} - \theta_t \mathrm{E}_t q_{l,t+1}}{q_{lt} - \theta_t \mathrm{E}_t \dfrac{q_{l,t+1}}{R_t}}. \tag{S.21}$$

We can similarly derive the return on capital, which is given by

$$R_{k,t+1} = \frac{\phi\alpha Y_{t+1}/K_t + q_{k,t+1}(1-\delta) - \theta_t \mathrm{E}_t q_{k,t+1}}{q_{kt} - \theta_t \mathrm{E}_t \dfrac{q_{k,t+1}}{R_t}}. \tag{S.22}$$

To see how these returns relate to the entrepreneur's optimal decisions, we denote by μ_{et} the Lagrangian multiplier for the flow of funds constraint (S.11), μ_{kt} the multiplier for the capital accumulation equation (S.10), and μ_{bt} the multiplier for the credit constraint (S.14). With these notations, the shadow price of capital in consumption units is given by

$$q_{kt} = \frac{\mu_{kt}}{\mu_{et}},$$

and the marginal utility of income, μ_{et}, is equal to the marginal utility of consumption:

$$\mu_{et} = \frac{1}{C_{et} - \gamma_e C_{e,t-1}} - \mathrm{E}_t \frac{\beta\gamma_e}{C_{e,t+1} - \gamma_e C_{et}}.$$

The optimal decision on the entrepreneur's borrowing can be described by

$$\frac{1}{R_t} = \beta \mathrm{E}_t \frac{\mu_{e,t+1}}{\mu_{et}} + \frac{\mu_{bt}}{\mu_{et}}. \tag{S.23}$$

The above Euler equation implies that the credit constraint is binding (i.e., $\mu_{bt} > 0$) if and only if the interest rate is lower than the entrepreneur's intertemporal marginal rate of substitution. The entrepreneur's optimal decisions on land and capital can be described by the following two Euler equations:

$$q_{lt} = \beta \mathrm{E}_t \frac{\mu_{e,t+1}}{\mu_{et}} \left[\alpha \phi \frac{Y_{t+1}}{L_{et}} + q_{l,t+1} \right] + \frac{\mu_{bt}}{\mu_{et}} \theta_t \mathrm{E}_t q_{l,t+1}, \tag{S.24}$$

$$q_{kt} = \beta \mathrm{E}_t \frac{\mu_{e,t+1}}{\mu_{et}} \left[\alpha (1-\phi) \frac{Y_{t+1}}{K_t} + q_{k,t+1}(1-\delta) \right] + \frac{\mu_{bt}}{\mu_{et}} \theta_t \mathrm{E}_t q_{k,t+1}. \tag{S.25}$$

Using (S.23), we can rewrite (S.24) and (S.25) as

$$1 = \beta \mathrm{E}_t \frac{\mu_{e,t+1}}{\mu_{et}} R_{j,t+1}, \quad j \in \{l, k\}. \tag{S.26}$$

Since consumption grows at the rate g_λ in equilibrium and the utility function is of logarithmic form, (S.26) implies that $R_j = g_\lambda/\beta$.

On the other hand, the loan rate R_t is determined by the household's intertemporal Euler equation:

$$\frac{1}{R_t} = \beta \mathrm{E}_t \frac{\mu_{h,t+1}}{\mu_{ht}}, \tag{S.27}$$

where μ_{ht} is the Lagrangian multiplier for the flow of funds constraint (S.5). It represents the marginal utility of income and is equal to the marginal utility of consumption:

$$\mu_{ht} = A_t \left[\frac{1}{C_{ht} - \gamma_h C_{h,t-1}} - \mathrm{E}_t \frac{\beta \gamma_h}{C_{h,t+1} - \gamma_h C_{ht}} (1 + \lambda_{a,t+1}) \right].$$

It follows from (S.27) that, in steady state, $R = \frac{g_\gamma}{\beta(1+\bar{\lambda}_a)}$, where $\bar{\lambda}_a > 0$ measures the extent to which the household is more patient than the entrepreneur. The steady-state excess return is then given by

$$R_j^e \equiv R_j - R = \frac{g_\gamma}{\beta} \frac{\bar{\lambda}_a}{1 + \bar{\lambda}_a}, \quad j \in \{l, k\}. \tag{S.28}$$

Clearly, the steady-state excess return is positive if and only if the patience factor, $\bar{\lambda}_a$, is positive.

To see how a positive first-order excess return is related to the entrepreneur's credit constraint, one can derive from (S.23) the following steady-state relationship:

$$\frac{\beta\bar{\lambda}_a}{g_\gamma} = \frac{\tilde{\mu}_b}{\tilde{\mu}_e}.$$

Thus, the credit constraint is binding (i.e., $\tilde{\mu}_b > 0$) if and only if the household is more patient than the entrepreneur (i.e., $\bar{\lambda}_a > 0$).

This result carries over to the dynamics of excess returns. Denote by $R^e_{j,t+1} \equiv R_{j,t+1} - R_t$ the excess return for asset $j \in \{l, k\}$. By combining the bond Euler equation (S.23) and the asset-pricing equation (S.26), we obtain

$$\text{(S.29)} \qquad \beta \mathrm{E}_t \frac{\mu_{e,t+1}}{\mu_{et}} R^e_{j,t+1} = \frac{\mu_{bt}}{\mu_{et}} R_t, \quad j \in \{l, k\}.$$

As in the standard asset-pricing model, the mean excess return depends on the asset's riskiness measured by the covariance between the return and the marginal utility of consumption. Unlike the standard model, however, the excess return in our model contains a first-order term that is positive if and only if the borrowing constraint is binding (i.e., $\mu_{bt} > 0$).

D.2.2. *Euler Equations*

Denote by μ_{ht} the Lagrangian multiplier for the flow of funds constraint (S.5). The first-order conditions for the household's optimizing problem are given by

$$\text{(S.30)} \qquad \mu_{ht} = A_t \left[\frac{1}{C_{ht} - \gamma_h C_{h,t-1}} - \mathrm{E}_t \frac{\beta \gamma_h}{C_{h,t+1} - \gamma_h C_{ht}} (1 + \lambda_{a,t+1}) \right],$$

$$\text{(S.31)} \qquad w_t = \frac{A_t}{\mu_{ht}} \psi_t,$$

$$\text{(S.32)} \qquad q_{lt} = \beta \mathrm{E}_t \frac{\mu_{h,t+1}}{\mu_{ht}} q_{l,t+1} + \frac{A_t \varphi_t}{\mu_{ht} L_{ht}},$$

$$\text{(S.33)} \qquad \frac{1}{R_t} = \beta \mathrm{E}_t \frac{\mu_{h,t+1}}{\mu_{ht}}.$$

Equation (S.30) equates the marginal utility of income and of consumption; equation (S.31) equates the real wage and the marginal rate of substitution (MRS) between leisure and income; equation (S.32) equates the current relative price of land to the marginal benefit of purchasing an extra unit of land, which consists of the current utility benefits (i.e., the MRS between housing and consumption) and the land's discounted future resale value; and equation (S.33) is the standard Euler equation for the loanable bond.

Denote by μ_{et} the Lagrangian multiplier for the flow of funds constraint (S.11), μ_{kt} the multiplier for the capital accumulation equation (S.10), and μ_{bt} the multiplier for the borrowing constraint (S.14). With these notations, the shadow price of capital in consumption units is given by

$$q_{kt} = \frac{\mu_{kt}}{\mu_{et}}. \tag{S.34}$$

The first-order conditions for the entrepreneur's optimizing problem are given by

$$\mu_{et} = \frac{1}{C_{et} - \gamma_e C_{e,t-1}} - \mathrm{E}_t \frac{\beta \gamma_e}{C_{e,t+1} - \gamma_e C_{et}}, \tag{S.35}$$

$$w_t = (1-\alpha) Y_t / N_{et}, \tag{S.36}$$

$$\begin{aligned} \frac{1}{Q_t} = {} & q_{kt}\left[1 - \frac{\Omega}{2}\left(\frac{I_t}{I_{t-1}} - \bar{\lambda}_I\right)^2 - \Omega\left(\frac{I_t}{I_{t-1}} - \bar{\lambda}_I\right)\frac{I_t}{I_{t-1}}\right] \\ & + \beta \Omega \mathrm{E}_t \frac{\mu_{e,t+1}}{\mu_{et}} q_{k,t+1}\left(\frac{I_{t+1}}{I_t} - \bar{\lambda}_I\right)\left(\frac{I_{t+1}}{I_t}\right)^2, \end{aligned} \tag{S.37}$$

$$q_{kt} = \beta \mathrm{E}_t \frac{\mu_{e,t+1}}{\mu_{et}}\left[\alpha(1-\phi)\frac{Y_{t+1}}{K_t} + q_{k,t+1}(1-\delta)\right] + \frac{\mu_{bt}}{\mu_{et}} \theta_t \mathrm{E}_t q_{k,t+1}, \tag{S.38}$$

$$q_{lt} = \beta \mathrm{E}_t \frac{\mu_{e,t+1}}{\mu_{et}}\left[\alpha\phi\frac{Y_{t+1}}{L_{et}} + q_{l,t+1}\right] + \frac{\mu_{bt}}{\mu_{et}} \theta_t \mathrm{E}_t q_{l,t+1}, \tag{S.39}$$

$$\frac{1}{R_t} = \beta \mathrm{E}_t \frac{\mu_{e,t+1}}{\mu_{et}} + \frac{\mu_{bt}}{\mu_{et}}. \tag{S.40}$$

Equation (S.35) equates the marginal utility of income to the marginal utility of consumption since consumption is the numé raire; equation (S.36) is the labor demand equation, which equates the real wage to the marginal product of labor; equation (S.37) is the investment Euler equation, which equates the cost of purchasing an additional unit of investment good and the benefit of having an extra unit of new capital, where the benefit includes the shadow value of the installed capital net of adjustment costs and the present value of the saved future adjustment costs; equation (S.38) is the capital Euler equation, which equates the shadow price of capital to the present value of future marginal product of capital and the resale value of the un-depreciated capital, plus the value of capital as a collateral asset for borrowing; equation (S.39) is the land Euler equation, which equates the price of the land to the present value of the future marginal product of land and the resale value, plus the value of land as a collateral asset for borrowing; equation (S.40) is the bond Euler equation for the entrepreneur, which reveals that the borrowing constraint is binding (i.e., $\mu_{bt} > 0$) if and only if the interest rate is lower than the entrepreneur's intertemporal marginal rate of substitution.

D.2.3. *Stationary Equilibrium*

We are interested in studying the fluctuations around the balanced growth path. For this purpose, we focus on a stationary equilibrium by appropriately transforming the growing variables. Specifically, we make the following transformations of the variables:

$$\text{(S.41)}\quad \begin{aligned} &\tilde{Y}_t \equiv \frac{Y_t}{\Gamma_t}, \quad \tilde{C}_{ht} \equiv \frac{C_{ht}}{\Gamma_t}, \quad \tilde{C}_{et} \equiv \frac{C_{et}}{\Gamma_t}, \quad \tilde{I}_t \equiv \frac{I_t}{Q_t \Gamma_t}, \quad \tilde{K}_t \equiv \frac{K_t}{Q_t \Gamma_t}, \\ &\tilde{B}_t \equiv \frac{B_t}{\Gamma_t}, \quad \tilde{w}_t \equiv \frac{w_t}{\Gamma_t}, \quad \tilde{\mu}_{ht} \equiv \frac{\mu_{ht}\Gamma_t}{A_t}, \quad \tilde{\mu}_{et} \equiv \mu_{et}\Gamma_t, \\ &\tilde{\mu}_{bt} \equiv \mu_{bt}\Gamma_t, \quad \tilde{q}_{lt} \equiv \frac{q_{lt}}{\Gamma_t}, \quad \tilde{q}_{kt} \equiv q_{kt} Q_t, \end{aligned}$$

where $\Gamma_t \equiv [Z_t Q_t^{(1-\phi)\alpha}]^{1/(1-(1-\phi)\alpha)}$. In Appendix F.1.2, we describe the stationary equilibrium and derive the log-linearized equilibrium conditions around the steady state for solving the model. To solve the log-linearized equilibrium system requires the input of several key steady-state values. These include the shadow value of the loanable funds $\frac{\tilde{\mu}_b}{\tilde{\mu}_e}$, the ratio of commercial real estate to aggregate output $\frac{\tilde{q}_l L_e}{\tilde{Y}}$, the ratio of residential land to commercial real estate $\frac{L_h}{L_e}$, the ratio of loanable funds to output $\frac{\tilde{B}}{\tilde{Y}}$, the capital-output ratio $\frac{\tilde{K}}{\tilde{Y}}$, and the "big ratios" $\frac{\tilde{C}_h}{\tilde{Y}}$, $\frac{\tilde{C}_e}{\tilde{Y}}$, and $\frac{\tilde{I}}{\tilde{Y}}$. The model implies a set of restrictions between these steady-state ratios and the parameters, and we will use these restrictions along with the first moments of selected time series in the data to sharpen our priors and to help identify a subset of the parameters in our estimation.

Denote by $g_{\gamma t} \equiv \frac{\Gamma_t}{\Gamma_{t-1}}$ and $g_{qt} \equiv \frac{Q_t}{Q_{t-1}}$ the growth rates for the exogenous variables Γ_t and Q_t. Denote by g_γ the steady-state value of $g_{\gamma t}$ and $\lambda_k \equiv g_\gamma \bar{\lambda}_q$ the steady-state growth rate of capital stock. On the balanced growth path, investment grows at the same rate as does capital, so we have $\bar{\lambda}_I = \lambda_k$.

The stationary equilibrium is the solution to the following system of equations:

$$\text{(S.42)}\quad \tilde{\mu}_{ht} = \frac{1}{\tilde{C}_{ht} - \gamma_h \tilde{C}_{h,t-1}\Gamma_{t-1}/\Gamma_t} - \mathrm{E}_t \frac{\beta\gamma_h}{\tilde{C}_{h,t+1}\Gamma_{t+1}/\Gamma_t - \gamma_h \tilde{C}_{ht}}(1+\lambda_{a,t+1}),$$

$$\text{(S.43)}\quad \tilde{w}_t = \frac{\psi_t}{\tilde{\mu}_{ht}},$$

$$\text{(S.44)}\quad \tilde{q}_{lt} = \beta \mathrm{E}_t \frac{\tilde{\mu}_{h,t+1}}{\tilde{\mu}_{ht}}(1+\lambda_{a,t+1})\tilde{q}_{l,t+1} + \frac{\varphi_t}{\tilde{\mu}_{ht} L_{ht}},$$

$$\frac{1}{R_t} = \beta \mathrm{E}_t \frac{\tilde{\mu}_{h,t+1}}{\tilde{\mu}_{ht}} \frac{\Gamma_t}{\Gamma_{t+1}} (1+\lambda_{a,t+1}). \tag{S.45}$$

$$\tilde{\mu}_{et} = \frac{1}{\tilde{C}_{et} - \gamma_e \tilde{C}_{e,t-1}\Gamma_{t-1}/\Gamma_t} - \mathrm{E}_t \frac{\beta\gamma_e}{\tilde{C}_{e,t+1}\Gamma_{t+1}/\Gamma_t - \gamma_e \tilde{C}_{et}}, \tag{S.46}$$

$$\tilde{w}_t = (1-\alpha)\tilde{Y}_t/N_t, \tag{S.47}$$

$$\begin{aligned} 1 = \tilde{q}_{kt}\Bigg[& 1 - \frac{\Omega}{2}\left(\frac{\tilde{I}_t}{\tilde{I}_{t-1}}\frac{Q_t\Gamma_t}{Q_{t-1}\Gamma_{t-1}} - \bar{\lambda}_I\right)^2 \\ & - \Omega\left(\frac{\tilde{I}_t}{\tilde{I}_{t-1}}\frac{Q_t\Gamma_t}{Q_{t-1}\Gamma_{t-1}} - \bar{\lambda}_I\right)\frac{\tilde{I}_t}{\tilde{I}_{t-1}}\frac{Q_t\Gamma_t}{Q_{t-1}\Gamma_{t-1}}\Bigg] \\ & + \beta\Omega \mathrm{E}_t \frac{\tilde{\mu}_{e,t+1}}{\tilde{\mu}_{et}}\frac{Q_t\Gamma_t}{Q_{t+1}\Gamma_{t+1}}\tilde{q}_{k,t+1} \\ & \times \left(\frac{\tilde{I}_{t+1}}{\tilde{I}_t}\frac{Q_{t+1}\Gamma_{t+1}}{Q_t\Gamma_t} - \bar{\lambda}_I\right)\left(\frac{\tilde{I}_{t+1}}{\tilde{I}_t}\frac{Q_{t+1}\Gamma_{t+1}}{Q_t\Gamma_t}\right)^2, \end{aligned} \tag{S.48}$$

$$\begin{aligned} \tilde{q}_{kt} = {} & \beta \mathrm{E}_t \frac{\tilde{\mu}_{e,t+1}}{\tilde{\mu}_{et}}\left[\alpha(1-\phi)\frac{\tilde{Y}_{t+1}}{\tilde{K}_t} + \tilde{q}_{k,t+1}\frac{Q_t\Gamma_t}{Q_{t+1}\Gamma_{t+1}}(1-\delta)\right] \\ & + \frac{\tilde{\mu}_{bt}}{\tilde{\mu}_{et}}\theta_t \mathrm{E}_t \tilde{q}_{k,t+1}\frac{Q_t}{Q_{t+1}}, \end{aligned} \tag{S.49}$$

$$\tilde{q}_{lt} = \beta \mathrm{E}_t \frac{\tilde{\mu}_{e,t+1}}{\tilde{\mu}_{et}}\left[\alpha\phi\frac{\tilde{Y}_{t+1}}{L_{et}} + \tilde{q}_{l,t+1}\right] + \frac{\tilde{\mu}_{bt}}{\tilde{\mu}_{et}}\theta_t \mathrm{E}_t \tilde{q}_{l,t+1}\frac{\Gamma_{t+1}}{\Gamma_t}, \tag{S.50}$$

$$\frac{1}{R_t} = \beta \mathrm{E}_t \frac{\tilde{\mu}_{e,t+1}}{\tilde{\mu}_{et}}\frac{\Gamma_t}{\Gamma_{t+1}} + \frac{\tilde{\mu}_{bt}}{\tilde{\mu}_{et}}. \tag{S.51}$$

$$\tilde{Y}_t = \left(\frac{Z_tQ_t}{Z_{t-1}Q_{t-1}}\right)^{-(1-\phi)\alpha/(1-(1-\phi)\alpha)}\left[L_{e,t-1}^{\phi}\tilde{K}_{t-1}^{1-\phi}\right]^{\alpha}N_t^{1-\alpha}, \tag{S.52}$$

$$\tilde{K}_t = (1-\delta)\tilde{K}_{t-1}\frac{Q_{t-1}\Gamma_{t-1}}{Q_t\Gamma_t} + \left[1 - \frac{\Omega}{2}\left(\frac{\tilde{I}_t}{\tilde{I}_{t-1}}\frac{Q_t\Gamma_t}{Q_{t-1}\Gamma_{t-1}} - \bar{\lambda}_I\right)^2\right]\tilde{I}_t, \tag{S.53}$$

$$\tilde{Y}_t = \tilde{C}_{ht} + \tilde{C}_{et} + \tilde{I}_t, \tag{S.54}$$

$$\bar{L} = L_{ht} + L_{et}, \tag{S.55}$$

$$\alpha\tilde{Y}_t = \tilde{C}_{et} + \tilde{I}_t + \tilde{q}_{lt}(L_{et} - L_{e,t-1}) + \tilde{B}_{t-1}\frac{\Gamma_{t-1}}{\Gamma_t} - \frac{\tilde{B}_t}{R_t}, \tag{S.56}$$

$$\tilde{B}_t = \theta_t \mathrm{E}_t\left[\tilde{q}_{l,t+1}\frac{\Gamma_{t+1}}{\Gamma_t}L_{et} + \tilde{q}_{k,t+1}\tilde{K}_t\frac{Q_t}{Q_{t+1}}\right]. \tag{S.57}$$

We solve these 16 equations for 16 variables summarized in the vector

$$[\tilde{\mu}_{ht}, \tilde{w}_t, \tilde{q}_{lt}, R_t, \tilde{\mu}_{et}, N_t, \tilde{I}_t, \tilde{Y}_t, \tilde{C}_{ht}, \tilde{C}_{et}, \tilde{q}_{kt}, L_{et}, L_{ht}, \tilde{K}_t, \tilde{B}_t, \tilde{\mu}_{bt}]'.$$

D.2.4. *Steady State*

To get the steady-state value for $\frac{\tilde{\mu}_b}{\tilde{\mu}_e}$, we use the stationary bond Euler equations (S.45) for the household and (S.51) for the entrepreneur to obtain

$$\text{(S.58)} \qquad \frac{1}{R} = \frac{\beta(1+\bar{\lambda}_a)}{g_\gamma}, \qquad \frac{\tilde{\mu}_b}{\tilde{\mu}_e} = \frac{\beta\bar{\lambda}_a}{g_\gamma}.$$

Since $\bar{\lambda}_a > 0$, we have $\tilde{\mu}_b > 0$ and the borrowing constraint is binding in the steady-state equilibrium.

To get the ratio of commercial real estate to output, we use the land Euler equation (S.50) for the entrepreneur, the definition of $\tilde{\mu}_e$ in (S.46), and the solution for $\frac{\tilde{\mu}_b}{\tilde{\mu}_e}$ in (S.58). In particular, we have

$$\text{(S.59)} \qquad \frac{\tilde{q}_l L_e}{\tilde{Y}} = \frac{\beta\alpha\phi}{1-\beta-\beta\bar{\lambda}_a\bar{\theta}}.$$

To get the investment-output ratio, we first solve for the investment-capital ratio by using the law of motion for capital stock in (S.53), and then solve for the capital-output ratio using the capital Euler equation (S.49). Specifically, we have

$$\text{(S.60)} \qquad \frac{\tilde{I}}{\tilde{K}} = 1 - \frac{1-\delta}{\lambda_k},$$

$$\text{(S.61)} \qquad \frac{\tilde{K}}{\tilde{Y}} = \left[1 - \frac{\beta}{\lambda_k}(\bar{\lambda}_a\bar{\theta} + 1 - \delta)\right]^{-1} \beta\alpha(1-\phi),$$

where we have used the steady-state condition that $\tilde{q}_k = 1$, as implied by the investment Euler equation (S.48). The investment-output ratio is then given by

$$\text{(S.62)} \qquad \frac{\tilde{I}}{\tilde{Y}} = \frac{\tilde{I}}{\tilde{K}}\frac{\tilde{K}}{\tilde{Y}} = \frac{\beta\alpha(1-\phi)[\lambda_k - (1-\delta)]}{\lambda_k - \beta(\bar{\lambda}_a\bar{\theta} + 1 - \delta)}.$$

Given the solution for the ratios $\frac{\tilde{q}_l L_e}{\tilde{Y}}$ and $\frac{\tilde{K}}{\tilde{Y}}$ in (S.59) and (S.61), the binding borrowing constraint (S.57) implies that

$$\text{(S.63)} \qquad \frac{\tilde{B}}{\tilde{Y}} = \bar{\theta} g_\gamma \frac{\tilde{q}_l L_e}{\tilde{Y}} + \frac{\bar{\theta}}{\bar{\lambda}_q}\frac{\tilde{K}}{\tilde{Y}}.$$

The entrepreneur's flow of funds constraint (S.56) implies that

$$\frac{\tilde{C}_e}{\tilde{Y}} = \alpha - \frac{\tilde{I}}{\tilde{Y}} - \frac{1-\beta(1+\bar{\lambda}_a)}{g_\gamma}\frac{\tilde{B}}{\tilde{Y}}. \tag{S.64}$$

The aggregate resource constraint (S.54) then implies that

$$\frac{\tilde{C}_h}{\tilde{Y}} = 1 - \frac{\tilde{C}_e}{\tilde{Y}} - \frac{\tilde{I}}{\tilde{Y}}. \tag{S.65}$$

To solve for $\frac{L_h}{L_e}$, we first use the household's land Euler equation (i.e., the housing demand equation) (S.44) and the definition for the marginal utility (S.42) to obtain

$$\frac{\tilde{q}_l L_h}{\tilde{C}_h} = \frac{\bar{\varphi}(g_\gamma - \gamma_h)}{g_\gamma(1-g_\gamma/R)(1-\gamma_h/R)}, \tag{S.66}$$

where the steady-state loan rate is given by (S.58).

Taking the ratio between (S.66) and (S.59) results in the solution

$$\frac{L_h}{L_e} = \frac{\bar{\varphi}(g_\gamma - \gamma_h)(1-\beta-\beta\bar{\lambda}_a\bar{\theta})}{\beta\alpha\phi g_\gamma(1-g_\gamma/R)(1-\gamma_h/R)}\frac{\tilde{C}_h}{\tilde{Y}}. \tag{S.67}$$

Finally, we can solve for the steady-state hours by combining the labor supply equation (S.43) and the labor demand equation (S.47) to get

$$N = \frac{(1-\alpha)g_\gamma(1-\gamma_h/R)}{\bar{\psi}(g_\gamma - \gamma_h)}\frac{\tilde{Y}}{\tilde{C}_h}. \tag{S.68}$$

D.2.5. *Log-Linearized Equilibrium System*

Upon obtaining the steady-state equilibrium, we log-linearize the equilibrium conditions (S.42) through (S.57) around the steady state. We define the constants $\Omega_h \equiv (g_\gamma - \beta(1+\bar{\lambda}_a)\gamma_h)(g_\gamma - \gamma_h)$ and $\Omega_e \equiv (g_\gamma - \beta\gamma_e)(g_\gamma - \gamma_e)$. The log-linearized equilibrium conditions are given by

$$\begin{aligned}\Omega_h\hat{\mu}_{ht} = &-\left[g_\gamma^2 + \gamma_h^2\beta(1+\bar{\lambda}_a)\right]\hat{C}_{ht} + g_\gamma\gamma_h(\hat{C}_{h,t-1} - \hat{g}_{\gamma t}) \\ &- \beta\bar{\lambda}_a\gamma_h(g_\gamma - \gamma_h)\mathrm{E}_t\hat{\lambda}_{a,t+1} \\ &+ \beta(1+\bar{\lambda}_a)g_\gamma\gamma_h\mathrm{E}_t(\hat{C}_{h,t+1} + \hat{g}_{\gamma,t+1}),\end{aligned} \tag{S.69}$$

$$\hat{w}_t + \hat{\mu}_{ht} = \hat{\psi}_t, \tag{S.70}$$

$$\begin{aligned}\hat{q}_{lt} + \hat{\mu}_{ht} = &\beta(1+\bar{\lambda}_a)\mathrm{E}_t[\hat{\mu}_{h,t+1} + \hat{q}_{l,t+1}] \\ &+ \left[1-\beta(1+\bar{\lambda}_a)\right](\hat{\varphi}_t - \hat{L}_{ht}) + \beta\bar{\lambda}_a\mathrm{E}_t\hat{\lambda}_{a,t+1},\end{aligned} \tag{S.71}$$

$$\hat{\mu}_{ht} - \hat{R}_t = \mathrm{E}_t\left[\hat{\mu}_{h,t+1} + \frac{\bar{\lambda}_a}{1+\bar{\lambda}_a}\hat{\lambda}_{a,t+1} - \hat{g}_{\gamma,t+1}\right], \tag{S.72}$$

$$\begin{aligned}\Omega_e\hat{\mu}_{et} = {}& -\left(g_\gamma^2 + \beta\gamma_e^2\right)\hat{C}_{e,t} + g_\gamma\gamma_e(\hat{C}_{e,t-1} - \hat{g}_{\gamma t}) \\ & + \beta g_\gamma\gamma_e\mathrm{E}_t(\hat{C}_{e,t+1} + \hat{g}_{\gamma,t+1}),\end{aligned} \tag{S.73}$$

$$\hat{w}_t = \hat{Y}_t - \hat{N}_t, \tag{S.74}$$

$$\begin{aligned}\hat{q}_{kt} = {}& (1+\beta)\Omega\lambda_k^2\hat{I}_t - \Omega\lambda_k^2\hat{I}_{t-1} + \Omega\lambda_k^2(\hat{g}_{\gamma t} + \hat{g}_{qt}) \\ & - \beta\Omega\lambda_k^2\mathrm{E}_t[\hat{I}_{t+1} + \hat{g}_{\gamma,t+1} + \hat{g}_{q,t+1}],\end{aligned} \tag{S.75}$$

$$\begin{aligned}\hat{q}_{kt} + \hat{\mu}_{et} = {}& \frac{\tilde{\mu}_b}{\tilde{\mu}_e}\frac{\bar{\theta}}{\bar{\lambda}_q}(\hat{\mu}_{bt} + \hat{\theta}_t) + \frac{\beta(1-\delta)}{\lambda_k}\mathrm{E}_t(\hat{q}_{k,t+1} - \hat{g}_{q,t+1} - \hat{g}_{\gamma,t+1}) \\ & + \left(1 - \frac{\tilde{\mu}_b}{\tilde{\mu}_e}\frac{\bar{\theta}}{\bar{\lambda}_q}\right)\mathrm{E}_t\hat{\mu}_{e,t+1} + \frac{\tilde{\mu}_b}{\tilde{\mu}_e}\frac{\bar{\theta}}{\bar{\lambda}_q}\mathrm{E}_t(\hat{q}_{k,t+1} - \hat{g}_{q,t+1}) \\ & + \beta\alpha(1-\phi)\frac{\tilde{Y}}{\tilde{K}}\mathrm{E}_t(\hat{Y}_{t+1} - \hat{K}_t),\end{aligned} \tag{S.76}$$

$$\begin{aligned}\hat{q}_{lt} + \hat{\mu}_{et} = {}& \frac{\tilde{\mu}_b}{\tilde{\mu}_e}g_\gamma\bar{\theta}(\hat{\theta}_t + \hat{\mu}_{bt}) + \left(1 - \frac{\tilde{\mu}_b}{\tilde{\mu}_e}g_\gamma\theta\right)\mathrm{E}_t\hat{\mu}_{e,t+1} \\ & + \frac{\tilde{\mu}_b}{\tilde{\mu}_e}g_\gamma\bar{\theta}\mathrm{E}_t(\hat{q}_{l,t+1} + \hat{g}_{\gamma,t+1}) + \beta\mathrm{E}_t\hat{q}_{l,t+1} \\ & + (1 - \beta - \beta\bar{\lambda}_a\bar{\theta})\mathrm{E}_t[\hat{Y}_{t+1} - \hat{L}_{et}],\end{aligned} \tag{S.77}$$

$$\hat{\mu}_{et} - \hat{R}_t = \frac{1}{1+\bar{\lambda}_a}\left[\mathrm{E}_t(\hat{\mu}_{e,t+1} - \hat{g}_{\gamma,t+1}) + \bar{\lambda}_a\hat{\mu}_{bt}\right], \tag{S.78}$$

$$\begin{aligned}\hat{Y}_t = {}& \alpha\phi\hat{L}_{e,t-1} + \alpha(1-\phi)\hat{K}_{t-1} + (1-\alpha)\hat{N}_t \\ & - \frac{(1-\phi)\alpha}{1-(1-\phi)\alpha}[\hat{g}_{zt} + \hat{g}_{qt}],\end{aligned} \tag{S.79}$$

$$\hat{K}_t = \frac{1-\delta}{\lambda_k}[\hat{K}_{t-1} - \hat{g}_{\gamma t} - \hat{g}_{qt}] + \left(1 - \frac{1-\delta}{\lambda_k}\right)\hat{I}_t, \tag{S.80}$$

$$\hat{Y}_t = \frac{\tilde{C}_h}{\tilde{Y}}\hat{C}_{ht} + \frac{C_e}{\tilde{Y}}\hat{C}_{e,t} + \frac{\tilde{I}}{\tilde{Y}}\hat{I}_t, \tag{S.81}$$

$$0 = \frac{L_h}{\bar{L}}\hat{L}_{ht} + \frac{L_e}{\bar{L}}\hat{L}_{et}, \tag{S.82}$$

$$\alpha\hat{Y}_t = \frac{\tilde{C}_e}{\tilde{Y}}\hat{C}_{e,t} + \frac{\tilde{I}}{\tilde{Y}}\hat{I}_t + \frac{\tilde{q}_l L_e}{\tilde{Y}}(\hat{L}_{et} - \hat{L}_{e,t-1}) + \frac{1}{g_\gamma}\frac{\tilde{B}}{\tilde{Y}}(\hat{B}_{t-1} - \hat{g}_{\gamma t}) - \frac{1}{R}\frac{\tilde{B}}{\tilde{Y}}(\hat{B}_t - \hat{R}_t), \tag{S.83}$$

$$\hat{B}_t = \hat{\theta}_t + g_\gamma\bar{\theta}\frac{\tilde{q}_l L_e}{\tilde{B}}\mathrm{E}_t(\hat{q}_{l,t+1} + \hat{L}_{et} + \hat{g}_{\gamma,t+1}) + \left(1 - g_\gamma\bar{\theta}\frac{\tilde{q}_l L_e}{\tilde{B}}\right)\mathrm{E}_t(\hat{q}_{k,t+1} + \hat{K}_t - \hat{g}_{q,t+1}). \tag{S.84}$$

The terms $\hat{g}_{zt}$, $\hat{g}_{qt}$, and $\hat{g}_{\gamma t}$ are given by

$$\hat{g}_{zt} = \hat{\lambda}_{zt} + \hat{\nu}_{zt} - \hat{\upsilon}_{z,t-1}, \tag{S.85}$$

$$\hat{g}_{qt} = \hat{\lambda}_{qt} + \hat{\nu}_{qt} - \hat{\upsilon}_{q,t-1}, \tag{S.86}$$

$$\hat{g}_{\gamma t} = \frac{1}{(1-(1-\phi)\alpha)}\hat{g}_{zt} + \frac{(1-\phi)\alpha}{(1-(1-\phi)\alpha)}\hat{g}_{qt}. \tag{S.87}$$

The technology shocks follow the processes

$$\hat{\lambda}_{zt} = \rho_z\hat{\lambda}_{z,t-1} + \hat{\varepsilon}_{zt}, \tag{S.88}$$

$$\hat{\nu}_{zt} = \rho_{\nu_z}\hat{\nu}_{z,t-1} + \hat{\varepsilon}_{\nu_z t}, \tag{S.89}$$

$$\hat{\lambda}_{qt} = \rho_q\hat{\lambda}_{q,t-1} + \hat{\varepsilon}_{qt}, \tag{S.90}$$

$$\hat{\nu}_{qt} = \rho_{\nu_q}\hat{\nu}_{q,t-1} + \hat{\varepsilon}_{\nu_q t}. \tag{S.91}$$

The preference shocks follow the processes

$$\hat{\lambda}_{at} = \rho_a\hat{\lambda}_{a,t-1} + \hat{\varepsilon}_{at}, \tag{S.92}$$

$$\hat{\varphi}_t = \rho_\varphi\hat{\varphi}_{t-1} + \hat{\varepsilon}_{\varphi t}, \tag{S.93}$$

$$\hat{\psi}_t = \rho_\psi\hat{\psi}_{t-1} + \hat{\varepsilon}_{\psi t}. \tag{S.94}$$

The liquidity shock follows the process

$$\hat{\theta}_t = \rho_\theta\hat{\theta}_{t-1} + \hat{\varepsilon}_{\theta t}. \tag{S.95}$$

We use Sims's (2002) algorithm to solve the 19 rational expectations equations, (S.69) through (S.87), for the 19 unknowns summarized in the column vector

$$x_t = [\hat{\mu}_{ht}, \hat{w}_t, \hat{q}_{lt}, \hat{R}_t, \hat{\mu}_{et}, \hat{\mu}_{bt}, \hat{N}_t, \hat{I}_t, \hat{Y}_t, \hat{C}_{ht}, \hat{C}_{et}, \hat{q}_{kt}, \hat{L}_{ht}, \hat{L}_{et}, \hat{K}_t, \hat{B}_t, \hat{g}_{\gamma t}, \hat{g}_{zt}, \hat{g}_{qt}]',$$

where x_t is referred to as a vector of state variables. The system of solved-out equations forms a system of state equations.

D.3. *Estimation*

We log-linearized the model around the steady state in which the credit constraint is binding. We use the Bayesian method to fit the linearized model to six quarterly U.S. time series: the relative price of land (q_{lt}^{Data}), the inverse of the relative price of investment (Q_t^{Data}), real per capita consumption (C_t^{Data}), real per capita investment in consumption units (I_t^{Data}), real per capita nonfinancial business debt (B_t^{Data}), and per capita hours (L_t^{Data}). All these series are constructed to be consistent with the corresponding series in Greenwood, Hercowitz, and Krusell (1997), Cummins and Violante (2002), and Davis and Heathcote (2007). The sample period covers the first quarter of 1975 through the fourth quarter of 2010.

A system of measurement equations links the observable variables to the state variables. A standard Kalman-filter algorithm can then be applied to the system of measurement and state equations in form the likelihood function. Multiplying the likelihood by the prior distribution leads to a posterior kernel (proportional to the posterior density function).

In our model with credit constraints, we find that the posterior kernel is full of thin and winding ridges as well as local peaks. Finding the mode of the posterior distribution has proven a difficult task. Indeed, the popular Dynare software fails to find the posterior mode with its various built-in optimizing methods.

To see how such difficulty arises, we first use Dynare 4.2 to estimate our model. We choose many sets of reasonably calibrated parameters as different starting points, and the Dynare program has difficulty to converge. For quasi-Newton based optimization methods (e.g., options mode_compute=1 to 5 in Dynare), we encounter the message "POSTERIOR KERNEL OPTIMIZATION PROBLEM! (minus) the Hessian matrix at the 'mode' is not positive definite!," meaning that the results are unreliable. One method (with the option mode_compute=6 in Dynare), which triggers a Monte Carlo based optimization routine, is very inefficient and seems to be able to converge to a local peak only.

In the examples given in Liu, Wang, and Zha (2013),[3] we summarize all the output produced by different methods of Dynare:

(i) When the method "options mode_compute=1" is used, the program converges with ill-behaved Hessian matrix. According to these estimated re-

[3]The complete set of materials—source code, figures, and tables—is stored in the zip file "data_and_programs.zip." In the zip file, the estimated results under different methods can be found under the subdirectory "/Output."

sults, a housing demand shock plays almost no role in macroeconomic fluctuations. Instead, at the fourth year horizon, a permanent investment-specific technology shock contributes to 67.64% of investment fluctuations and a labor supply shock contributes to 61.68% of consumption fluctuations.

(ii) The method "options mode_compute=2" (Lester Ingber's Adaptive Simulated Annealing) is no longer available for Dynare 4.2.

(iii) The method "options mode_compute=2" cannot converge and the solver stops prematurely.

(iv) When the method "options mode_compute=4" is used, the program converges with ill-behaved Hessian matrix. According to these estimated results, a housing demand shock contributes to a majority of fluctuations in the land price (for example, 76.44% at the fourth year horizon) but little in other macroeconomic variables. Instead, at the fourth year horizon, a permanent investment-specific technology shock contributes to a majority of fluctuations in investment (81.12%) and consumption (78.9%).

(v) When the method "options mode_compute=5" is used, the program converges with ill-behaved Hessian matrix. According to these estimated results, a housing demand shock has a numerically zero impact on any variable. At the fourth year horizon, contributions to investment fluctuations are 42.17% from a preference shock, 15.21% from a labor supply shock, 18.15% from a permanent investment-specific technology shock, and 16.26% from a collateral shock.

(vi) When the method "options mode_compute=6" is used, the program converges but the converged results turn out to be at a local posterior peak. A housing demand shock plays almost no role in affecting any macroeconomic variables. A preference shock affects most of fluctuations in the land price. A permanent investment-specific technology shock explains a majority of fluctuations in macroeconomic variables (78.90% for consumption and 81.12% for investment at the fourth year horizon).

As we have discussed before, we have experimented with different sets of reasonably guessed parameter values as starting points, and none of the options in the optimization routine in Dynare can achieve decent convergence.

Our own optimization routine, based on Sims, Waggoner, and Zha (2008) and coded in C/C++, has proven to be both efficient and able to find the posterior mode.[4] Given an initial guess of the values of the parameters, our program uses a combination of a constrained optimization algorithm and a hill-climbing quasi-Newton optimization routine, with the Broyden–Fletcher–Goldfarb–Shanno (BFGS) updates of the inverse of the Hessian, to find a local

[4]The source code (with the main function file "dsgelinv1_estmcmc.c") can be downloaded from http://www.tzha.net/articles#creditconstraints. The user must be familiar with C/C++ and needs a C/C++ compiler to link the C code (in the zip file "C_Cpp_Library4LWZpaper.zip") to the C library (in the zip file "C_Cpp_Library4LWZpaper.zip"). After linking and compiling all the C functions, the user needs to generate an executable file for obtaining the estimation.

peak. We use this initial local peak to run Markov chain Monte Carlo (MCMC) simulations, and then use simulated draws as different starting points for our optimization routine to find a potentially higher peak. We iterate this process until it converges. The computation typically takes three and a half days on a single processor but less time if one avails oneself of a multiprocessor computer (a cluster of nodes, for example).

Once we complete the posterior mode estimation using our own program, we use the estimated results as a starting point for the Dynare optimization routine. The Dynare program converges instantly.[5] We are currently working with Dynare to use their preprocessor and compile part of our C/C++ code into Dynare so that the general user will be able to use our estimation procedure.

D.4. *Convergence*

In this paper, we use the Bayesian criterion to compare several models. Specifically, we compute the marginal data density (MDD) for each model and compare the MDDs. There are two related issues. One is to use the MDD to select a model. Potential problems of taking this approach blindly were addressed in Sims (2003), Geweke and Amisano (2011), and Waggoner and Zha (2012). The other pertains to the accuracy of estimating the MDD.

In this section, we focus on discussing the second issue. We adopt two techniques. First, we use an extremely long sequence, ten millions, of MCMC draws.[6] We divide this sequence into ten subsequences of one million draws and then compute the MDD from the entire sequence and from each of the subsequences. The variation among the subsequences is very small (under 1 in log value for all models studied in the paper).

Second, we use draws from the prior as starting points for multiple MCMC chains, each of which has a length of one million draws. Selecting an *appropriate* starting point is crucial for reliable MCMC draws. If the initial value is in an extremely low-probability region, an unreasonably long burn-in period would be required to obtain convergence of the MCMC chain. Most parameter values drawn from the prior have extremely low likelihood values. Thus, we draw from the prior until it reaches a reasonable likelihood value. We use ten such randomly selected starting points and record the minimum and maximum values of the MDDs calculated from these chains. The difference is under 4 in log value for all the models.

[5]See the complete set of results stored under the subdirectories /Output/Method4FromLWZmode and /Output/Method5FromLWZmode.

[6]On a standard desktop computer with dual cores, the computation would have taken more than two months. We utilize a cluster of computers to reduce an exceedingly large amount of computing time.

D.5. *Linear versus Nonlinear Models*

In addition to the benchmark model, we estimate two variants of the benchmark model in which we fix the value of $\bar{\lambda}_a$ at a relatively high value (0.012) or a low value (0.0015). We find that the parameter estimates with $\bar{\lambda}_a$ fixed a priori do not change our main results obtained from the benchmark model (where $\bar{\lambda}_a$ is estimated). Figure S.1 displays the estimated sample paths of the Lagrangian multiplier for the credit constraint for the benchmark model and the two variants. As one can see, the multipliers are above zero.

In general, the estimated parameter values for the benchmark model are almost indistinguishable from those for the model with $\bar{\lambda}_a$ fixed at 0.012. Figure S.2 shows the impulse responses to both a TFP shock and a housing demand shock for these two models. It is clear that, for the most part, the responses are hard to distinguish by eyes.

As discussed in the main text of the paper, the results reported in Figure S.1 by no means imply that the original nonlinear model has binding constraints

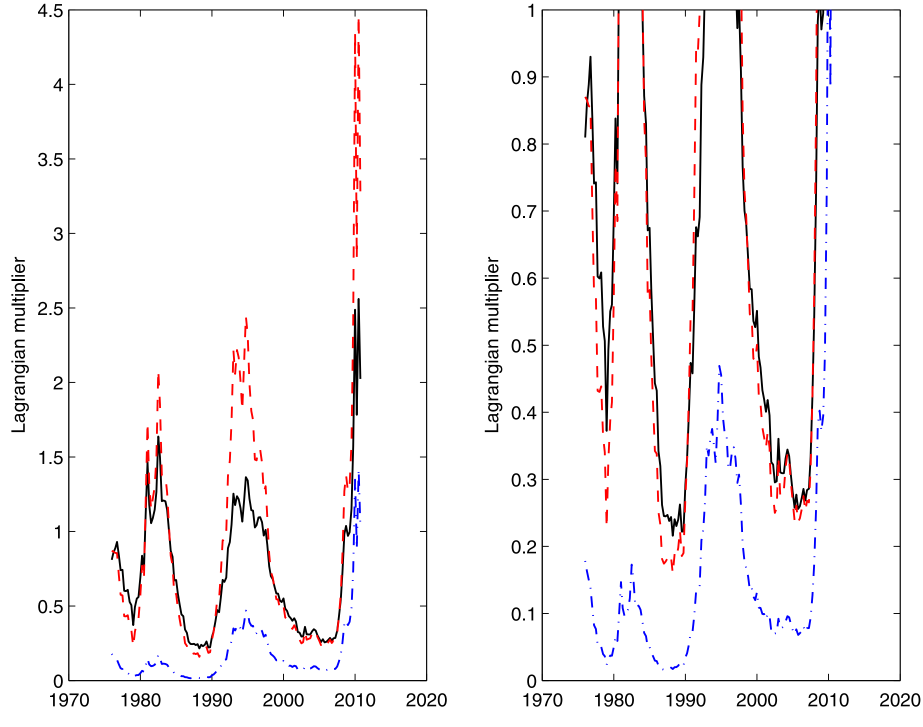

FIGURE S.1.—Lagrangian multipliers for the benchmark model (solid lines), the model with $\bar{\lambda}_a = 0.012$ (dashed lines), and the model with $\bar{\lambda}_a = 0.0015$ (dotted-dashed lines). Note that the right column is the same plot as the left column except the vertical axis is restricted to between 0 and 1 so that one can easily see how far the Lagrangian multipliers are away from zero.

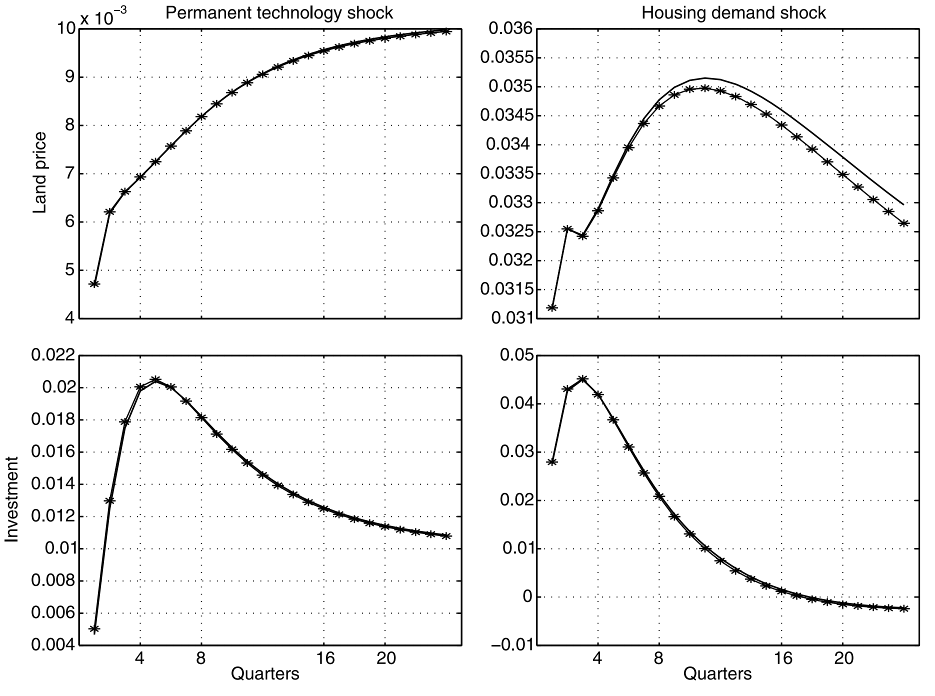

FIGURE S.2.—Impulse responses to a positive shock to neutral technology growth (left column) and to a positive shock to housing demand (right column). Lines marked by asterisks represent the responses for the benchmark model; thin solid lines represent the model with $\bar{\lambda}_a = 0.012$. Note that the results are so close that some lines are on top of one another.

always. It is possible, and even probable, that the original nonlinear model has occasionally binding constraints. In that case, one must estimate the original nonlinear model with occasionally binding constraints. Such a task is infeasible and beyond the scope of the paper.

From Figure S.3 to Figure S.6, however, we compare the impulse responses in the benchmark log-linearized model with those from two alternative nonlinear models for several key macroeconomic variables. We display the impulse responses to a positive housing demand shock and a positive collateral shock with one standard deviation as well as with three standard deviations. We solve two different nonlinear models, one in which we impose that the credit constraint is always binding (so that the multiplier for the credit constraint may be negative) and the other in which we allow the credit constraint to be occasionally binding (so that the multiplier is greater than or equal to zero). For both nonlinear models, we use a shooting algorithm to compute impulse responses, and we use the parameter estimates obtained from our log-linearized model.

Figures S.3 and S.5 show that, when the shock is moderate, the difference between all these models is negligible. Figures S.4 and S.6 show that, when the

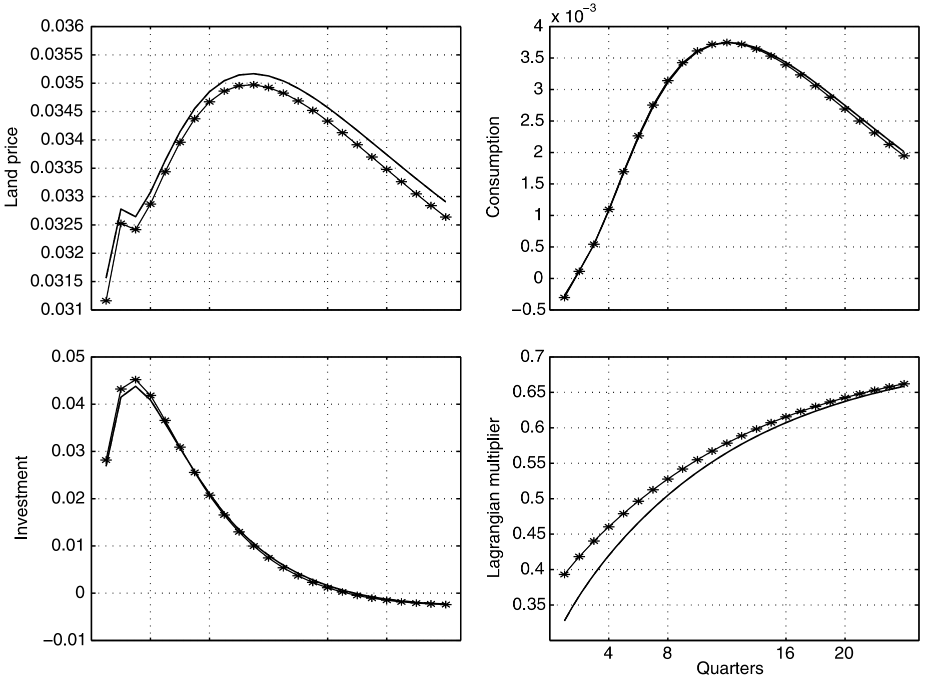

FIGURE S.3.—Impulse responses to a moderate positive housing demand shock (one standard deviation) in the benchmark model. Lines marked by asterisks represent the impulse responses for the log-linearized model; solid lines represent the original nonlinear model but with the credit constraint imposed to be always binding; dashed lines represent the original nonlinear model with the credit constraint allowed to be occasionally binding. Note that solid and dashed lines are on top of each other so that one cannot distinguish by eyes.

shock is large, the difference remains small. Even when the constraint is occasionally binding, as shown in the initial responses of the multiplier in Figures S.4 and S.6 in response to a large shock, much of the difference is driven by other parts of nonlinearity in the model rather than the occasionally binding constraint: although the responses of the Lagrangian multiplier following large shocks to housing demand (or credit limit) are very different between the linearized model and the nonlinear model, the responses of the land price and macroeconomic variables are very similar. For the impulse responses to other structural shocks, we obtain similar results.

While the preceding exercise is reassuring, it would be misleading to infer that one can simply calibrate the original nonlinear model with the estimates obtained from the log-linearized version. As shown in Section D.3, the estimation of the log-linearized version has already posed a challenging task, as many estimation procedures have been inadequate and, consequently, misleading conclusions may be drawn if the model parameters are not properly estimated.

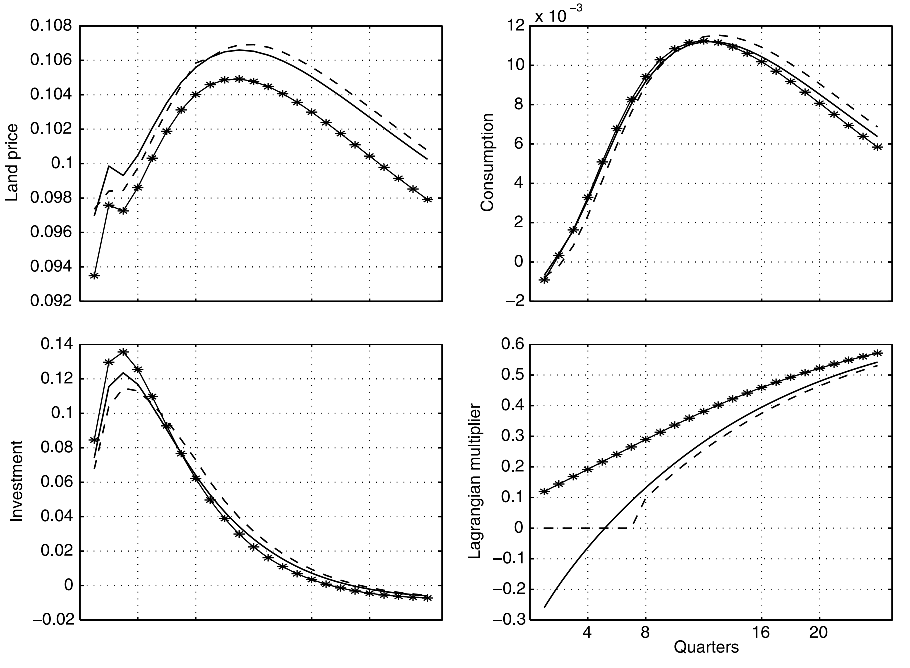

FIGURE S.4.—Impulse responses to a large positive housing demand shock (three standard deviations) in the benchmark model. Lines marked by asterisks represent the impulse responses for the log-linearized model; solid lines represent the original nonlinear model but with the credit constraint imposed to be always binding; dashed lines represent the original nonlinear model with the credit constraint allowed to be occasionally binding.

This lesson is particularly true for the nonlinear model with occasionally binding constraints. We are in the process of developing a robust empirical method that can tackle the estimation of such a model.

D.6. *Estimation Issues*

In this section, we discuss several estimation challenges we have faced during this project.

We use our own algorithm to estimate the log-linearized model and the corresponding model with regime-switching volatilities. One natural question is why we do not use Dynare to estimate these models. Dynare does not yet have capability to estimate the DSGE model with Markov-switching features. For the benchmark model, one could use Dynare. But because the posterior distribution is full of thin winding ridges as well as local peaks, finding its mode has proven to be a difficult task. To see exactly how such difficulty arises, we first use Dynare 4.2 to estimate our model. We choose many sets of reasonably calibrated parameters as different starting points, but the Dynare program has

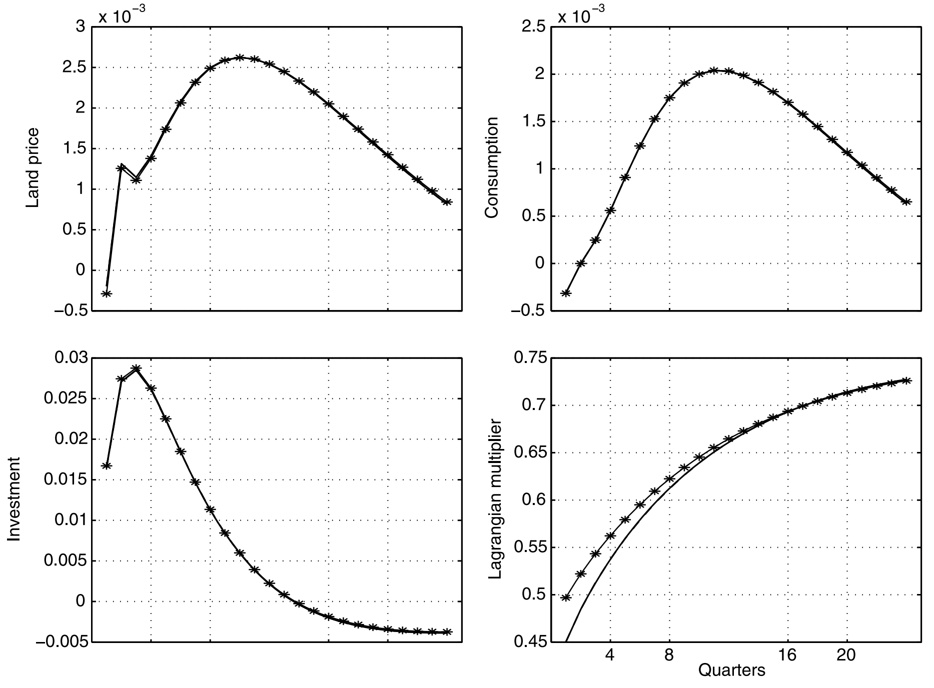

FIGURE S.5.—Impulse responses to a moderate positive collateral shock (one standard deviation) in the benchmark model. Lines marked by asterisks represent the impulse responses for the log-linearized model; solid lines represent the original nonlinear model but with the credit constraint imposed to be always binding; dashed lines represent the original nonlinear model with the credit constraint allowed to be occasionally binding. Note that solid and dashed lines are on top of each other so that one cannot distinguish by eyes.

difficulty converging. Most options in Dynare lead to an ill-behaved Hessian matrix due to thin winding ridges in the posterior distribution. One option, similar to a simulated annealing algorithm, converges but to a local posterior peak (see details in Section D.3 of this Supplemental Material).

Our own optimization routine, based on Sims, Waggoner, and Zha (2008) and coded in C/C++, has proven to be both efficient and able to find the posterior mode. The routine relies, in part, on the Broyden–Fletcher–Goldfarb–Shanno (BFGS) updates of the inverse of the Hessian matrix. When the inverse Hessian matrix is close to being numerically ill-conditioned, our program resets it to a diagonal matrix. Given an initial guess of the values of the parameters, our program uses a combination of a constrained optimization algorithm and an unconstrained BFGS optimization routine to find a local peak. We then use the local peak to generate a long sequence of Markov chain Monte Carlo (MCMC) posterior draws. These simulated draws are randomly selected as different starting points for our optimization routine to find a potentially higher

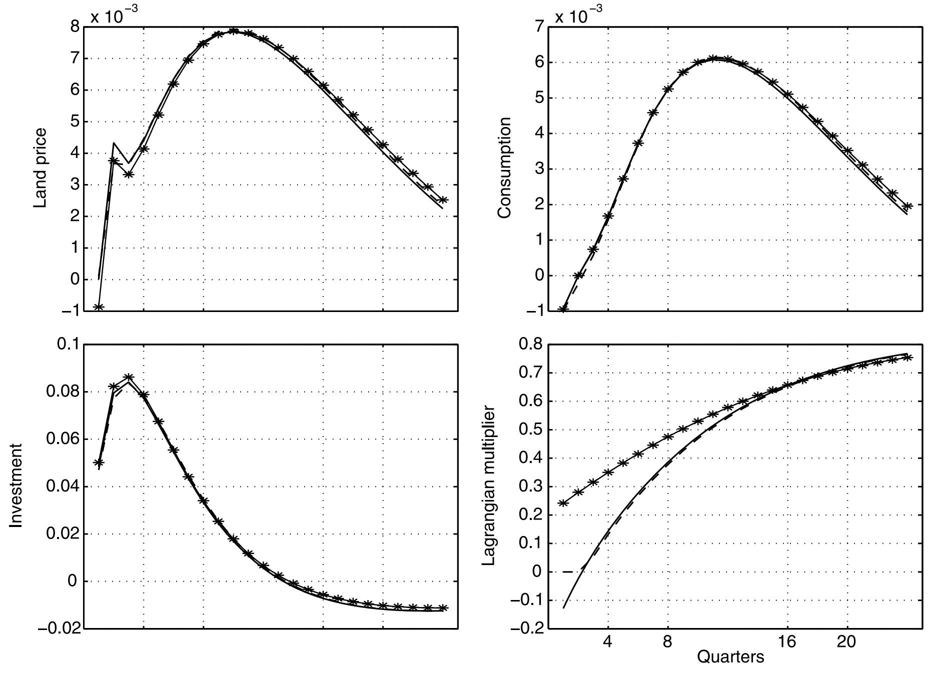

FIGURE S.6.—Impulse responses to a large positive collateral shock (three standard deviations) in the benchmark model. Lines marked by asterisks represent the impulse responses for the log-linearized model; solid lines represent the original nonlinear model but with the credit constraint imposed to be always binding; dashed lines represent the original nonlinear model with the credit constraint allowed to be occasionally binding.

peak. We iterate this process until the highest peak is found. The computation typically takes four and a half days on a cluster of five dual-core processors. We are in the process of collaborating with the Dynare team to incorporate our estimation software into the Dynare package.

APPENDIX E: LAND PRICES AND QUANTITY

In this appendix, we discuss some issues related to the measurement of land prices and quantities.

E.1. *The Price of Land*

The house value is composed of two diametrically different components: (1) the cost of structures that is specific to the cost of basic materials and the productivity of the construction industry relative to other sectors of the economy and (2) the price of land. As documented in Davis and Heathcote (2007), it is changes in the price of land, not those in the cost of structures, that constitute a driving force behind large house price fluctuations at both low and business-cycle frequencies.

The land price in our benchmark model is based on the Federal Housing Finance Agency (FHFA) house price index. The FHFA series is used in the literature (Chaney, Sraer, and Thesmar (2012)) because it has a comprehensive geographic coverage. The FHFA publishes the house price index for each of all 50 states based on all transactions. The disadvantage of the FHFA series is that it covers only conforming (conventional) mortgages. On the other hand, the CoreLogic house price index series, provided by CoreLogic Databases, has the same time series pattern as the Case–Shiller–Weiss (CSW) house price index but covers far more counties than does the CSW house price index. Indeed, the CoreLogic data cover all 50 states and, unlike the FHFA data, include both conforming and nonconforming mortgages.

The purchase-only FHFA house price index (Haver Data key: USPHPI@USECON) is available only from 1991Q1 to present. For 1975Q1 to 1990Q4, the FHFA house price index is spliced to be consistent with the purchase-only series. We then follow the methodology of Davis and Heathcote (2007) and compute the FHFA land-price index. The series is seasonally adjusted.

Both FHFA and CoreLogic data are all transactions, but the CoreLogic data include nonconforming mortgages. Why do we not use the CoreLogic data in place of the FHFA data? The reason is that the CoreLogic house price data have serious problems in the early part of the sample. First of all, the number of repeat sales in the early part of the sample is much less than in the later part. For example, the total number of repeat sales per year as a percentage of the total number of existing single-family home sales from the National Association of Realtors does not exceed 15% until 1980.

Second, the geographic coverage of the CoreLogic index is not as broad in the early part of the sample. For example, the CoreLogic index did not include all states until 2000. By contrast, the FHFA publishes an all-transactions state index for each of the U.S. states all the way back until 1975. Thus, the FHFA had comprehensive geographic coverage even in the early part of the sample.[7]

Third, CoreLogic overweighs certain states, especially California and Florida, in the early part of the sample. We compute the share of single-family homes in the United States that are in California and Florida using the 10-year Census[8] and linearly interpolate them. Then we compute the share of repeat sales in the CoreLogic data by year that are in California and Florida. From 1976 to 1981, for example, roughly 40% of the sales in the CoreLogic sample are in California or Florida.

[7]Given the very large swings in FHFA home prices for some states in the early part of the sample, there probably exist small sample issues for some states early on.

[8]The data are available at http://www.census.gov/hhes/www/housing/census/historic/units.html.

To overcome these problems in the early part of the sample, we seasonally adjust FHFA home price index for 1975Q1–1980Q4 and splice this index together with Haver Data's seasonally adjusted CoreLogic home price index for the third month of a quarter (Haver Data key: USLPHPIS@USECON) for 1981Q1 to present. We then follow the methodology of Davis and Heathcote (2007) and compute the CoreLogic land price index.

E.2. *The Quantity of Land: Model Implications and Some Evidence*

As we discuss in the paper (Section 4.3), our model implies a land-reallocation effect when the land price rises. The mechanism works in the following way. Following a positive housing demand shock, the land price rises and the entrepreneur's net worth increases. The entrepreneur is able to borrow more to finance investment and production. As production expands, the entrepreneur needs to acquire more land and labor (as well as capital). The expansion in production raises the household's wealth and triggers competing demand for land between the household and the entrepreneur. Such competing demand for land further pushes up the land price. The extent to which land is reallocated depends on parameter values, although the competition for land between the two sectors raises the land price unambiguously.

In our estimated model, the entrepreneur ends up with owning moderately more land in equilibrium. Figure S.7 shows the impulse responses land holdings by the household and by the entrepreneur following a positive housing demand shock. The figure shows that the quantity of land reallocated between the two sectors is small. With estimated parameters, the entrepreneur's land holdings increase by a bit less than 3% of total land (and symmetrically, the household's land holdings decrease by a bit less than 3% of total land).

To examine whether the model's land reallocation mechanism is empirically plausible, we need data on land quantities. Unfortunately, land quantity, especially commercial land quantity, is poorly measured and extremely unreliable. The main measures of land quantity that we can find were constructed by Davis and Heathcote (2007) based on data from the Bureau of Economic Analysis (BEA) and Bureau of Labor Statistics (BLS).

The BEA–BLS measure shows that total land quantity has grown slightly over time. If some residential land is converted into commercial land in periods when land prices boom, then we should expect to see residential land growth slow down when land prices are rising. Figure S.8 displays the real land price (left scale) and the growth rate of residential land (right scale). The figure is based on the CoreLogic data, whose broad coverage of mortgage types is likely to improve the quality of the measurement of the land quantity, especially for the period after 1990. The figure shows that residential land growth slowed down substantially during the land-price booms in the first half of the 2000s. Since aggregate land supply grows slowly, we take this observation as suggestive evidence that land flows from the household sector to the business sector when land prices rise.

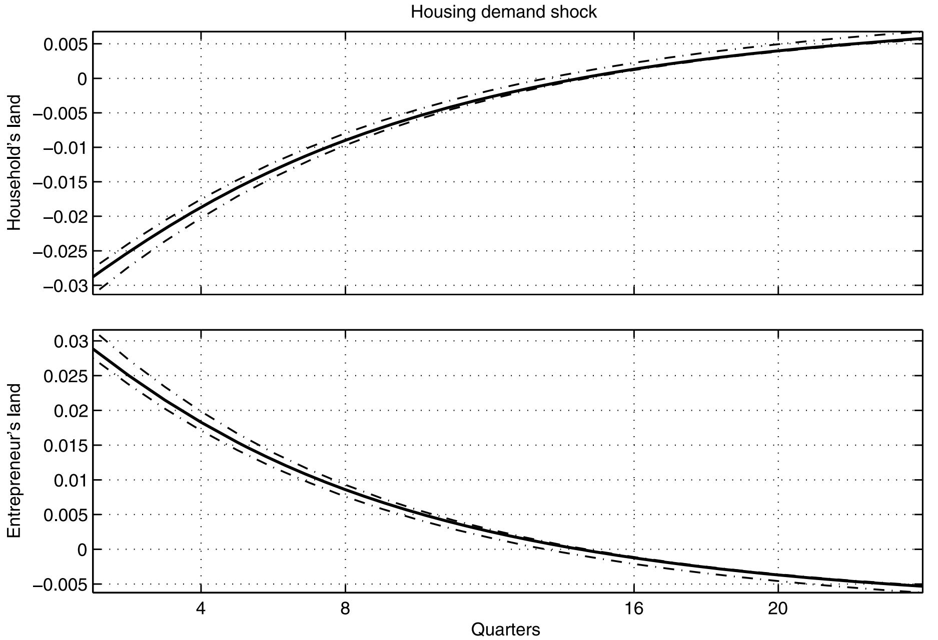

FIGURE S.7.—Impulse responses of land in each sector following a positive housing demand shock in the benchmark model.

To obtain the quantity of commercial land directly, the best matched series is probably measured by the land in nonfarm business sector, which is available only on an annual basis. As the growth rate of commercial land before 2001 is extrapolated by the BLS relying on the strong assumption that land-structure ratios are based on data from 2001 for *all counties in Ohio*, the quality of the series before 2001 is extremely poor because of this highly unreliable extrapolation. Even for residential land, Davis and Heathcote (2007) are most confident in their land estimate only from 2000 on. The BLS measure suggests that commercial land growth accelerated from a little under 1% in 2001 to about 2% in 2006 during the booming years of land prices. Thus, the available data do not seem to contradict our model's implications.

While the data do not seem to contradict our theoretical predictions about reallocation between residential land and commercial land, we caution against overinterpretation. The quality of data on land quantities is so poor and their measurement is so fragmentary that future studies into this issue are warranted.

E.3. *Commercial and Residential Real Estate Prices*

In our paper, we use prices of residential real estate as a proxy for those of commercial real estate for three main reasons. First, prices of commercial

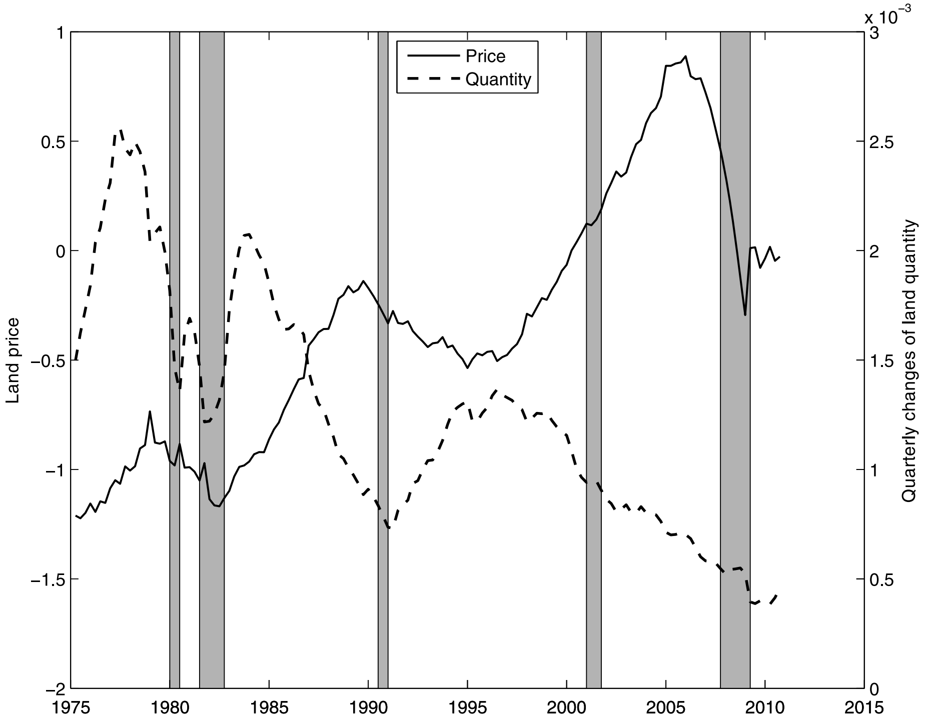

FIGURE S.8.—Log real land prices (on the left scale) and quarterly changes of land quantity (on the right scale). The shaded area marks NBER recession dates.

real estate are not as well measured as those of residential real estate. Second, the data history is much shorter for commercial real estate than for residential real estate. Third, the two series are highly correlated. Figure S.9 displays the CoreLogic national house price index and the RCA-based national commercial real estate price index (both series come from the HAVER data analytics). Despite the short sample for commercial real estate prices, one can see clearly that the two series, residential and commercial real estate prices, are strongly correlated.

APPENDIX F: SOME VARIATIONS IN THE MODEL, THE DATA, AND THE ESTIMATION APPROACH

In this appendix, we discuss a few variations in our model setup, the data that we use, and our estimation approach.

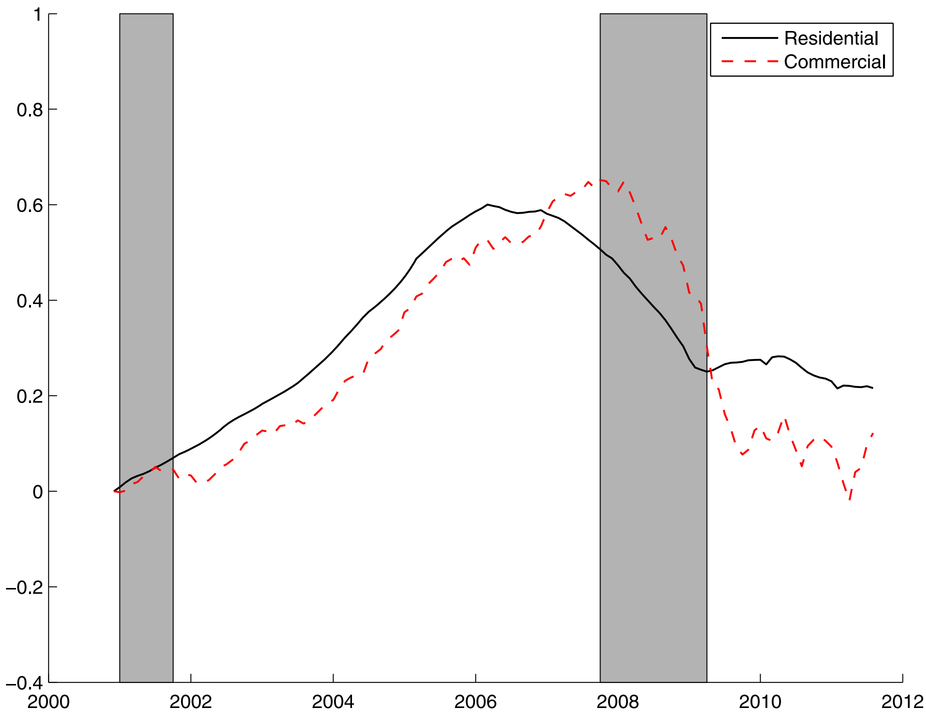

FIGURE S.9.—Log values of CoreLogic national house price index and the RCA-based national commercial real estate price index. The shaded bars mark the NBER-dated recessions.

F.1. *Model With Working Capital and Growth of Land Supply*

This section presents a variation of our benchmark model by incorporating working capital and growth in land supply.

F.1.1. *The Model*

As in the benchmark model, the economy consists of two types of agents—a representative household and a representative entrepreneur. There are four types of commodities: labor, goods, land, and loanable bonds. The representative household's utility depends on consumption goods, land services (housing), and leisure; the representative entrepreneur's utility depends on consumption goods only. Goods production requires labor, capital, and land as inputs. The entrepreneur needs external financing for investment spending. Imperfect contract enforcement implies that the entrepreneur's borrowing capacity is constrained by the value of collateral assets, consisting of land and capital stocks.

We add two variations here. First, we assume that a fraction ϕ_w of firms' wage payments needs to be financed by working capital, which is repaid within the period and carries no interest. This modification implies that the total amount of debt, including the intertemporal loans and the working capital, is limited by the value of the firms' collateral assets (land and capital). Second, we assume that aggregate land endowment grows at a constant rate of $\bar{\lambda}_l$. To maintain balanced growth, we assume that existing land holdings in each sector also grow at the same rate absent any shocks. Shocks that drive land reallocation would lead to different growth rates of land held in the two sectors.

We obtain two main results. First, absent working capital (i.e., $\phi_w = 0$), the model with exogenous land growth has an identical steady-state equilibrium and fluctuations around the steady state as in the benchmark model. Second, the parameter estimates in the model with working capital are very similar to those in the benchmark model.

F.1.1.1. *The Representative Household.* Similarly to Iacoviello (2005), the household has the utility function

$$\text{(S.96)} \qquad \mathrm{E}\sum_{t=0}^{\infty}\beta^t A_t\big\{\log(C_{ht}-\gamma_h C_{h,t-1})+\varphi_t\log L_{ht}-\psi_t N_{ht}\big\},$$

where C_{ht} denotes consumption, L_{ht} denotes land holdings, and N_{ht} denotes labor hours. The parameter $\beta \in (0, 1)$ is a subjective discount factor, the parameter γ_h measures the degree of habit persistence, and the term E is a mathematical expectation operator. The terms A_t, φ_t, and ψ_t are preference shocks. We assume that the intertemporal preference shock A_t follows the stochastic process

$$\text{(S.97)} \qquad A_t = A_{t-1}(1+\lambda_{at}), \quad \ln\lambda_{at} = (1-\rho_a)\ln\bar{\lambda}_a + \rho_a\ln\lambda_{a,t-1} + \varepsilon_{at},$$

where $\bar{\lambda}_a > 0$ is a constant, $\rho_a \in (-1, 1)$ is the persistence parameter, and ε_{at} is an independent and identically distributed (i.i.d.) white noise process with mean zero and variance σ_a^2. The housing preference shock φ_t follows the stationary process

$$\text{(S.98)} \qquad \ln\varphi_t = (1-\rho_\varphi)\ln\bar{\varphi} + \rho_\varphi\ln\varphi_{t-1} + \varepsilon_{\varphi t},$$

where $\bar{\varphi} > 0$ is a constant, $\rho_\varphi \in (-1, 1)$ measures the persistence of the shock, and $\varepsilon_{\varphi t}$ is a white noise process with mean zero and variance σ_φ^2. The labor supply shock ψ_t follows the stationary process

$$\text{(S.99)} \qquad \ln\psi_t = (1-\rho_\psi)\ln\bar{\psi} + \rho_\psi\ln\psi_{t-1} + \varepsilon_{\psi t},$$

where $\bar{\varphi} > 0$ is a constant, $\rho_\psi \in (-1, 1)$ measures the persistence, and $\varepsilon_{\psi t}$ is a white noise process with mean zero and variance σ_ψ^2.

Denote by q_{lt} the relative price of housing (in consumption units), R_t the gross real loan rate, and w_t the real wage; denote by S_t the household's purchase in period t of the loanable bond that pays off one unit of consumption good in all states of nature in period $t+1$. In period 0, the household begins with $L_{h,-1} > 0$ units of housing and $S_{-1} > 0$ units of the loanable bond. The flow of funds constraint for the household is given by

$$C_{ht} + q_{lt}(L_{ht} - \bar{\lambda}_l L_{h,t-1}) + \frac{S_t}{R_t} \le w_t N_{ht} + S_{t-1}, \tag{S.100}$$

where we have imposed the implicit assumption that the household's land holding grows "naturally" at the constant rate $\bar{\lambda}_l$.

The household chooses C_{ht}, $L_{h,t}$, N_{ht}, and S_t to maximize (S.96) subject to (S.97)–(S.100) and the borrowing constraint $S_t \ge -\bar{S}$ for some large number $\bar{S}$.

F.1.1.2. *The Representative Entrepreneur.* The entrepreneur has the utility function

$$\mathrm{E}\sum_{t=0}^{\infty} \beta^t\big[\log(C_{et} - \gamma_e C_{e,t-1})\big], \tag{S.101}$$

where C_{et} denotes the entrepreneur's consumption and γ_e is the habit persistence parameter.

The entrepreneur produces goods using capital, labor, and land as inputs. The production function is given by

$$Y_t = Z_t\big[L_{e,t-1}^{\phi} K_{t-1}^{1-\phi}\big]^{\alpha} N_{et}^{1-\alpha}, \tag{S.102}$$

where Y_t denotes output, K_{t-1}, N_{et}, and $L_{e,t-1}$ denote the inputs capital, labor, and land, respectively, and the parameters $\alpha \in (0,1)$ and $\phi \in (0,1)$ measure the output elasticities of these production factors. We assume that the total factor productivity Z_t is composed of a permanent component Z_t^p and a transitory component ν_t such that $Z_t = Z_t^p \nu_{zt}$, where the permanent component Z_t^p follows the stochastic process

$$Z_t^p = Z_{t-1}^p \lambda_{zt}, \quad \ln \lambda_{zt} = (1-\rho_z)\ln \bar{\lambda}_z + \rho_z \ln \lambda_{z,t-1} + \varepsilon_{zt}, \tag{S.103}$$

and the transitory component follows the stochastic process

$$\ln \nu_{zt} = \rho_{\nu_z} \ln \nu_{z,t-1} + \varepsilon_{\nu_z t}. \tag{S.104}$$

The parameter $\bar{\lambda}_z$ is the steady-state growth rate of Z_t^p; the parameters ρ_z and ρ_{ν_z} measure the degree of persistence. The innovations ε_{zt} and $\varepsilon_{\nu_z t}$ are i.i.d. white noise processes that are mutually independent with mean zero and variances given by σ_z^2 and $\sigma_{\nu_z}^2$, respectively.

The entrepreneur is endowed with K_{-1} units of initial capital stock and $L_{-1,e}$ units of initial land. Capital accumulation follows the law of motion

$$K_t = (1-\delta)K_{t-1} + \left[1 - \frac{\Omega}{2}\left(\frac{I_t}{I_{t-1}} - \bar{\lambda}_I\right)^2\right] I_t, \tag{S.105}$$

where I_t denotes investment, $\bar{\lambda}_I$ denotes the steady-state growth rate of investment, and $\Omega > 0$ is the adjustment cost parameter.

The entrepreneur faces the flow of funds constraint

$$\text{(S.106)}\quad C_{et} + q_{lt}(L_{et} - \bar{\lambda}_l L_{e,t-1}) + B_{t-1} = Z_t\big[L_{e,t-1}^{\phi} K_{t-1}^{1-\phi}\big]^{\alpha} N_{et}^{1-\alpha} - \frac{I_t}{Q_t} - w_t N_{et} + \frac{B_t}{R_t},$$

where B_{t-1} is the amount of matured debt and B_t/R_t is the value of new debt. Following Greenwood, Hercowitz, and Krusell (1997), we interpret Q_t as the investment-specific technological change. Specifically, we assume that $Q_t = Q_t^p \nu_{qt}$, where the permanent component Q_t^p follows the stochastic process

$$\text{(S.107)}\quad Q_t^p = Q_{t-1}^p \lambda_{qt}, \quad \ln \lambda_{qt} = (1-\rho_q)\ln \bar{\lambda}_q + \rho_q \ln \lambda_{q,t-1} + \varepsilon_{qt},$$

and the transitory component μ_t follows the stochastic process

$$\text{(S.108)}\quad \ln \nu_{qt} = \rho_{\nu_q} \ln \nu_{q,t-1} + \varepsilon_{\nu_q t}.$$

The parameter $\bar{\lambda}_q$ is the steady-state growth rate of Q_t^p; the parameters ρ_q and ρ_{ν_q} measure the degree of persistence. The innovations ε_{qt} and $\varepsilon_{\nu_q t}$ are i.i.d. white noise processes that are mutually independent with mean zero and variances given by σ_q^2 and $\sigma_{\nu_q}^2$, respectively.

The entrepreneur's consumption, investment, and production can be partly financed externally. In addition, a fraction ϕ_w of the wage payments need to be financed externally. In particular, the entrepreneur faces the borrowing constraint

$$\text{(S.109)}\quad B_t \leq \theta_t \mathrm{E}_t[\phi_k q_{k,t+1} K_t + \phi_l \bar{\lambda}_l q_{l,t+1} L_{et}] - \phi_w w_t N_{et} R_t,$$

where $q_{k,t+1}$ is the shadow price of capital in consumption units.[9] Under this credit constraint, the amount that the entrepreneur can borrow is limited by a fraction of the value of the collateral assets—land and capital—net of the required repayment of working capital. The constants ϕ_k and ϕ_l represent the fractions of capital and land that can be pledged as collateral. Note that the collateral value of land grows at the rate $\bar{\lambda}_l$, as does the entrepreneur's land holdings.

Following Kiyotaki and Moore (1997), we interpret this type of credit constraints as reflecting the problem of costly contract enforcement: if the entrepreneur fails to pay the debt, the creditor can seize the land and the accumulated capital; since it is costly to liquidate the seized land and capital stock,

[9]Since the price of new capital is $1/Q_t$, Tobin's q in this model is given by $q_{kt}Q_t$, which is the ratio of the value of installed capital to the price of new capital.

the creditor can recoup up to a fraction θ_t of the total value of the collateral assets. We interpret θ_t as a "collateral shock" that reflects the tightness of the credit market. We assume that θ_t follows the stochastic process

$$\ln \theta_t = (1-\rho_\theta)\ln\bar{\theta} + \rho_\theta \ln \theta_{t-1} + \varepsilon_{\theta t}, \tag{S.110}$$

where $\bar{\theta}$ is the steady-state value of θ_t, $\rho_\theta \in (0,1)$ is the persistence parameter, and $\varepsilon_{\theta t}$ is an i.i.d. white noise process with mean zero and variance σ_θ^2.

The entrepreneur chooses C_{et}, N_{et}, I_t, $L_{e,t}$, K_t, and B_t to maximize (S.101) subject to (S.102) through (S.110).

F.1.1.3. *Market Clearing Conditions and Equilibrium.* In a competitive equilibrium, the markets for goods, labor, land, and loanable bonds all clear. The goods market clearing condition implies that

$$C_t + \frac{I_t}{Q_t} = Y_t, \tag{S.111}$$

where $C_t = C_{ht} + C_{et}$ denotes aggregate consumption. The labor market clearing condition implies that labor demand equals labor supply:

$$N_{et} = N_{ht} \equiv N_t. \tag{S.112}$$

The land market clearing condition implies that

$$L_{ht} + L_{et} = \bar{\lambda}_l^t \bar{L}, \tag{S.113}$$

where $\bar{L}$ is the fixed aggregate land endowment. Finally, the bond market clearing condition implies that

$$S_t = B_t. \tag{S.114}$$

A competitive equilibrium consists of sequences of prices $\{w_t, q_{lt}, R_t\}_{t=0}^\infty$ and allocations $\{C_{ht}, C_{et}, I_t, N_{ht}, N_{et}, L_{ht}, L_{et}, S_t, B_t, K_t, Y_t\}_{t=0}^\infty$ such that (i) taking the prices as given, the allocations solve the optimizing problems for the household and the entrepreneur, and (ii) all markets clear.

F.1.2. *Derivations*

F.1.2.1. *Euler Equations.* Denote by μ_{ht} the Lagrangian multiplier for the flow of funds constraint (S.100). The first-order conditions for the household's optimizing problem are given by

$$\mu_{ht} = A_t\left[\frac{1}{C_{ht} - \gamma_h C_{h,t-1}} - \mathrm{E}_t \frac{\beta\gamma_h}{C_{h,t+1} - \gamma_h C_{ht}}(1 + \lambda_{a,t+1})\right], \tag{S.115}$$

$$w_t = \frac{A_t}{\mu_{ht}}\psi_t, \tag{S.116}$$

$$q_{lt} = \beta \mathrm{E}_t \frac{\mu_{h,t+1}}{\mu_{ht}} \bar{\lambda}_l q_{l,t+1} + \frac{A_t \varphi_t}{\mu_{ht} L_{ht}}, \tag{S.117}$$

$$\frac{1}{R_t} = \beta \mathrm{E}_t \frac{\mu_{h,t+1}}{\mu_{ht}}. \tag{S.118}$$

Equation (S.115) equates the marginal utility of income and of consumption; equation (S.116) equates the real wage and the marginal rate of substitution (MRS) between leisure and income; equation (S.117) equates the current relative price of land to the marginal benefit of purchasing an extra unit of land, which consists of the current utility benefits (i.e., the MRS between housing and consumption) and the land's discounted future resale value; and equation (S.118) is the standard Euler equation for the loanable bond.

Denote by μ_{et} the Lagrangian multiplier for the flow of funds constraint (S.106), μ_{kt} the multiplier for the capital accumulation equation (S.105), and μ_{bt} the multiplier for the borrowing constraint (S.109). With these notations, the shadow price of capital in consumption units is given by

$$q_{kt} = \frac{\mu_{kt}}{\mu_{et}}. \tag{S.119}$$

The first-order conditions for the entrepreneur's optimizing problem are given by

$$\mu_{et} = \frac{1}{C_{et} - \gamma_e C_{e,t-1}} - \mathrm{E}_t \frac{\beta \gamma_e}{C_{e,t+1} - \gamma_e C_{et}}, \tag{S.120}$$

$$(1-\alpha)\frac{Y_t}{N_{et}} = \left[1 + \phi_w \frac{\mu_{bt}}{\mu_{et}} R_t\right] w_t, \tag{S.121}$$

$$\begin{aligned} \frac{1}{Q_t} = {} & q_{kt}\left[1 - \frac{\Omega}{2}\left(\frac{I_t}{I_{t-1}} - \bar{\lambda}_I\right)^2 - \Omega\left(\frac{I_t}{I_{t-1}} - \bar{\lambda}_I\right)\frac{I_t}{I_{t-1}}\right] \\ & + \beta \Omega \mathrm{E}_t \frac{\mu_{e,t+1}}{\mu_{et}} q_{k,t+1}\left(\frac{I_{t+1}}{I_t} - \bar{\lambda}_I\right)\left(\frac{I_{t+1}}{I_t}\right)^2, \end{aligned} \tag{S.122}$$

$$q_{kt} = \beta \mathrm{E}_t \frac{\mu_{e,t+1}}{\mu_{et}}\left[\alpha \frac{Y_{t+1}}{K_t} + q_{k,t+1}(1-\delta)\right] + \frac{\mu_{bt}}{\mu_{et}} \theta_t \phi_k \mathrm{E}_t q_{k,t+1}, \tag{S.123}$$

$$q_{lt} = \beta \mathrm{E}_t \frac{\mu_{e,t+1}}{\mu_{et}}\left[\alpha \phi \frac{Y_{t+1}}{L_{et}} + \bar{\lambda}_l q_{l,t+1}\right] + \frac{\mu_{bt}}{\mu_{et}} \theta_t \phi_l \bar{\lambda}_l \mathrm{E}_t q_{l,t+1}, \tag{S.124}$$

$$\frac{1}{R_t} = \beta \mathrm{E}_t \frac{\mu_{e,t+1}}{\mu_{et}} + \frac{\mu_{bt}}{\mu_{et}}. \tag{S.125}$$

Equation (S.120) equates the marginal utility of income to the marginal utility of consumption since consumption is the numéraire. Equation (S.121) is the labor demand equation. Since part of the wage payment needs to be externally financed by working capital ($\phi_w > 0$), the effective cost of labor exceeds the market wage rate if the borrowing constraint is binding. Equation (S.122) is the investment Euler equation, which equates the cost of purchasing an additional unit of investment good and the benefit of having an extra unit of new capital, where the benefit includes the shadow value of the installed capital net of adjustment costs and the present value of the saved future adjustment costs. Equation (S.123) is the capital Euler equation, which equates the shadow price of capital to the present value of future marginal product of capital and the resale value of the un-depreciated capital, plus the value of capital as a collateral asset for borrowing. Equation (S.124) is the land Euler equation, which equates the price of the land to the present value of the future marginal product of land and the resale value, plus the value of land as a collateral asset for borrowing. Equation (S.125) is the bond Euler equation for the entrepreneur, which reveals that the borrowing constraint is binding (i.e., $\mu_{bt} > 0$) if and only if the interest rate is lower than the entrepreneur's intertemporal marginal rate of substitution.

F.1.2.2. *Stationary Equilibrium.* We are interested in studying the fluctuations around the balanced growth path. For this purpose, we focus on a stationary equilibrium by appropriately transforming the growing variables. Specifically, we make the following transformations of the variables:

$$\text{(S.126)} \quad \tilde{Y}_t \equiv \frac{Y_t}{\Gamma_t}, \quad \tilde{C}_{ht} \equiv \frac{C_{ht}}{\Gamma_t}, \quad \tilde{C}_{et} \equiv \frac{C_{et}}{\Gamma_t}, \quad \tilde{I}_t \equiv \frac{I_t}{Q_t \Gamma_t}, \quad \tilde{K}_t \equiv \frac{K_t}{Q_t \Gamma_t},$$

$$\tilde{B}_t \equiv \frac{B_t}{\Gamma_t}, \quad \tilde{w}_t \equiv \frac{w_t}{\Gamma_t}, \quad \tilde{\mu}_{ht} \equiv \frac{\mu_{ht}\Gamma_t}{A_t}, \quad \tilde{\mu}_{et} \equiv \mu_{et}\Gamma_t, \quad \tilde{\mu}_{bt} \equiv \mu_{bt}\Gamma_t,$$

$$\tilde{q}_{lt} \equiv \frac{q_{lt}\bar{\lambda}_l^t}{\Gamma_t}, \quad \tilde{q}_{kt} \equiv q_{kt} Q_t, \quad \tilde{L}_{ht} \equiv \frac{L_{ht}}{\bar{\lambda}_l^t}, \quad \tilde{L}_{et} \equiv \frac{L_{et}}{\bar{\lambda}_l^t},$$

where $\Gamma_t \equiv [Z_t Q_t^{(1-\phi)\alpha} (\bar{\lambda}_l^t)^{\alpha\phi}]^{1/(1-(1-\phi)\alpha)}$.

Denote by $g_{\gamma t} \equiv \frac{\Gamma_t}{\Gamma_{t-1}}$ and $g_{qt} \equiv \frac{Q_t}{Q_{t-1}}$ the growth rates for the exogenous variables Γ_t and Q_t. Denote by g_γ the steady-state value of $g_{\gamma t}$ and by $\lambda_k \equiv g_\gamma \bar{\lambda}_q$ the steady-state growth rate of capital stock. On the balanced growth path, investment grows at the same rate as does capital, so we have $\bar{\lambda}_I = \lambda_k$.

The stationary equilibrium is the solution to the following system of equations:

$$\text{(S.127)} \quad \tilde{\mu}_{ht} = \frac{1}{\tilde{C}_{ht} - \gamma_h \tilde{C}_{h,t-1}\Gamma_{t-1}/\Gamma_t} - \mathrm{E}_t \frac{\beta\gamma_h}{\tilde{C}_{h,t+1}\Gamma_{t+1}/\Gamma_t - \gamma_h \tilde{C}_{ht}} (1 + \lambda_{a,t+1}),$$

$$\text{(S.128)} \quad \tilde{w}_t = \frac{\psi_t}{\tilde{\mu}_{ht}},$$

$$\tilde{q}_{lt} = \beta \mathrm{E}_t \frac{\tilde{\mu}_{h,t+1}}{\tilde{\mu}_{ht}}(1+\lambda_{a,t+1})\tilde{q}_{l,t+1} + \frac{\varphi_t}{\tilde{\mu}_{ht} L_{ht}}, \tag{S.129}$$

$$\frac{1}{R_t} = \beta \mathrm{E}_t \frac{\tilde{\mu}_{h,t+1}}{\tilde{\mu}_{ht}} \frac{\Gamma_t}{\Gamma_{t+1}}(1+\lambda_{a,t+1}). \tag{S.130}$$

$$\tilde{\mu}_{et} = \frac{1}{\tilde{C}_{et} - \gamma_e \tilde{C}_{e,t-1}\Gamma_{t-1}/\Gamma_t} - \mathrm{E}_t \frac{\beta\gamma_e}{\tilde{C}_{e,t+1}\Gamma_{t+1}/\Gamma_t - \gamma_e \tilde{C}_{et}}, \tag{S.131}$$

$$(1-\alpha)\frac{\tilde{Y}_t}{N_t} = \left[1 + \phi_w \frac{\tilde{\mu}_{bt}}{\tilde{\mu}_{et}} R_t\right]\tilde{w}_t, \tag{S.132}$$

$$\begin{aligned} 1 = \tilde{q}_{kt}\Bigg[1 &- \frac{\Omega}{2}\left(\frac{\tilde{I}_t}{\tilde{I}_{t-1}}\frac{Q_t\Gamma_t}{Q_{t-1}\Gamma_{t-1}} - \bar{\lambda}_I\right)^2 \\ &- \Omega\left(\frac{\tilde{I}_t}{\tilde{I}_{t-1}}\frac{Q_t\Gamma_t}{Q_{t-1}\Gamma_{t-1}} - \bar{\lambda}_I\right)\frac{\tilde{I}_t}{\tilde{I}_{t-1}}\frac{Q_t\Gamma_t}{Q_{t-1}\Gamma_{t-1}}\Bigg] \\ &+ \beta\Omega \mathrm{E}_t \frac{\tilde{\mu}_{e,t+1}}{\tilde{\mu}_{et}}\frac{Q_t\Gamma_t}{Q_{t+1}\Gamma_{t+1}}\tilde{q}_{k,t+1} \\ &\times \left(\frac{\tilde{I}_{t+1}}{\tilde{I}_t}\frac{Q_{t+1}\Gamma_{t+1}}{Q_t\Gamma_t} - \bar{\lambda}_I\right)\left(\frac{\tilde{I}_{t+1}}{\tilde{I}_t}\frac{Q_{t+1}\Gamma_{t+1}}{Q_t\Gamma_t}\right)^2, \end{aligned} \tag{S.133}$$

$$\begin{aligned} \tilde{q}_{kt} = \beta \mathrm{E}_t \frac{\tilde{\mu}_{e,t+1}}{\tilde{\mu}_{et}}\left[\alpha(1-\phi)\frac{\tilde{Y}_{t+1}}{\tilde{K}_t} + \tilde{q}_{k,t+1}\frac{Q_t\Gamma_t}{Q_{t+1}\Gamma_{t+1}}(1-\delta)\right] \\ + \frac{\tilde{\mu}_{bt}}{\tilde{\mu}_{et}}\theta_t\phi_k \mathrm{E}_t \tilde{q}_{k,t+1}\frac{Q_t}{Q_{t+1}}, \end{aligned} \tag{S.134}$$

$$\tilde{q}_{lt} = \beta \mathrm{E}_t \frac{\tilde{\mu}_{e,t+1}}{\tilde{\mu}_{et}}\left[\alpha\phi\frac{\tilde{Y}_{t+1}}{L_{et}} + \tilde{q}_{l,t+1}\right] + \frac{\tilde{\mu}_{bt}}{\tilde{\mu}_{et}}\theta_t\phi_l \mathrm{E}_t \tilde{q}_{l,t+1}\frac{\Gamma_{t+1}}{\Gamma_t}, \tag{S.135}$$

$$\frac{1}{R_t} = \beta \mathrm{E}_t \frac{\tilde{\mu}_{e,t+1}}{\tilde{\mu}_{et}}\frac{\Gamma_t}{\Gamma_{t+1}} + \frac{\tilde{\mu}_{bt}}{\tilde{\mu}_{et}}, \tag{S.136}$$

$$\tilde{Y}_t = \left(\frac{Z_t Q_t}{Z_{t-1}Q_{t-1}}\right)^{-(1-\phi)\alpha/(1-(1-\phi)\alpha)}(\bar{\lambda}_l)^{-\alpha\phi/(1-(1-\phi)\alpha)}\left[\tilde{L}_{e,t-1}^{\phi}\tilde{K}_{t-1}^{1-\phi}\right]^{\alpha}N_t^{1-\alpha}, \tag{S.137}$$

$$\tilde{K}_t = (1-\delta)\tilde{K}_{t-1}\frac{Q_{t-1}\Gamma_{t-1}}{Q_t\Gamma_t} + \left[1 - \frac{\Omega}{2}\left(\frac{\tilde{I}_t}{\tilde{I}_{t-1}}\frac{Q_t\Gamma_t}{Q_{t-1}\Gamma_{t-1}} - \bar{\lambda}_I\right)^2\right]\tilde{I}_t, \tag{S.138}$$

$$\tilde{Y}_t = \tilde{C}_{ht} + \tilde{C}_{et} + \tilde{I}_t, \tag{S.139}$$

$$\bar{L} = \tilde{L}_{ht} + \tilde{L}_{et}, \tag{S.140}$$

$$\tilde{Y}_t = \tilde{C}_{et} + \tilde{I}_t + \tilde{q}_{lt}(\tilde{L}_{et} - \tilde{L}_{e,t-1}) + \tilde{B}_{t-1}\frac{\Gamma_{t-1}}{\Gamma_t} + \tilde{w}_t N_t - \frac{\tilde{B}_t}{R_t}, \tag{S.141}$$

$$\tilde{B}_t = \theta_t \mathrm{E}_t\left[\phi_k \tilde{q}_{k,t+1}\tilde{K}_t \frac{Q_t}{Q_{t+1}} + \phi_l \tilde{q}_{l,t+1}\frac{\Gamma_{t+1}}{\Gamma_t}\tilde{L}_{et}\right] - \phi_w R_t \tilde{w}_t N_t. \tag{S.142}$$

We solve these 16 equations for 16 variables summarized in the vector

$$[\tilde{\mu}_{ht}, \tilde{w}_t, \tilde{q}_{lt}, R_t, \tilde{\mu}_{et}, N_t, \tilde{I}_t, \tilde{Y}_t, \tilde{C}_{ht}, \tilde{C}_{et}, \tilde{q}_{kt}, L_{et}, L_{ht}, \tilde{K}_t, \tilde{B}_t, \tilde{\mu}_{bt}]'.$$

F.1.2.3. *Steady State.* To solve the model's equilibrium dynamics, we log-linearize the stationary equilibrium conditions summarized in (S.127)–(S.142) around a deterministic steady state. The set of steady-state values required for solving the model include the shadow value of the loanable funds $\frac{\tilde{\mu}_b}{\tilde{\mu}_e}$, the labor income share $\frac{\tilde{w}N}{\tilde{Y}}$, the ratio of commercial real estate to aggregate output $\frac{\tilde{q}_l\tilde{L}_e}{\tilde{Y}}$, the ratio of residential land to commercial real estate $\frac{\tilde{L}_h}{\tilde{L}_e}$, the ratio of loanable funds to output $\frac{\tilde{B}}{\tilde{Y}}$, the capital-output ratio $\frac{\tilde{K}}{\tilde{Y}}$, and the "big ratios" $\frac{\tilde{C}_h}{\tilde{Y}}$, $\frac{\tilde{C}_e}{\tilde{Y}}$, and $\frac{\tilde{I}}{\tilde{Y}}$.

To get the steady-state value for $\frac{\tilde{\mu}_b}{\tilde{\mu}_e}$, we use the stationary bond Euler equations (S.130) for the household and (S.136) to obtain

$$\frac{1}{R} = \frac{\beta(1+\bar{\lambda}_a)}{g_\gamma}, \qquad \frac{\tilde{\mu}_b}{\tilde{\mu}_e} = \frac{\beta\bar{\lambda}_a}{g_\gamma}. \tag{S.143}$$

Since $\bar{\lambda}_a > 0$, we have $\tilde{\mu}_b > 0$ and the borrowing constraint is binding in the steady-state equilibrium.

Given $\frac{\tilde{\mu}_b}{\tilde{\mu}_e}$, (S.132) implies that the labor share (denoted by s_n) is given by

$$s_n \equiv \frac{\tilde{w}N}{\tilde{Y}} = \frac{1-\alpha}{1+\dfrac{\phi_w\bar{\lambda}_a}{1+\bar{\lambda}_a}}. \tag{S.144}$$

To get the ratio of commercial real estate to output, we use the land Euler equation (S.135) for the entrepreneur, the definition of $\tilde{\mu}_e$ in (S.131), and the solution for $\frac{\tilde{\mu}_b}{\tilde{\mu}_e}$ in (S.143). In particular, we have

$$\frac{\tilde{q}_l L_e}{\tilde{Y}} = \frac{\beta\alpha\phi}{1-\beta-\beta\bar{\lambda}_a\bar{\theta}\phi_l}. \tag{S.145}$$

To get the investment-output ratio, we first solve for the investment-capital ratio by using the law of motion for capital stock in (S.138) and then solve for the capital-output ratio using the capital Euler equation (S.134). Specifically, we have

$$\text{(S.146)}\quad \frac{\tilde{I}}{\tilde{K}} = 1 - \frac{1-\delta}{\lambda_k},$$

$$\text{(S.147)}\quad \frac{\tilde{K}}{\tilde{Y}} = \left[1 - \frac{\beta}{\lambda_k}(\bar{\lambda}_a \bar{\theta}\phi_k + 1 - \delta)\right]^{-1} \beta\alpha(1-\phi),$$

where we have used the steady-state condition that $\tilde{q}_k = 1$, as implied by the investment Euler equation (S.133). The investment-output ratio is then given by

$$\text{(S.148)}\quad \frac{\tilde{I}}{\tilde{Y}} = \frac{\tilde{I}}{\tilde{K}}\frac{\tilde{K}}{\tilde{Y}} = \frac{\beta\alpha(1-\phi)[\lambda_k - (1-\delta)]}{\lambda_k - \beta(\bar{\lambda}_a \bar{\theta}\phi_k + 1 - \delta)}.$$

Given the solution for the ratios $\frac{\tilde{q}_l L_e}{\tilde{Y}}$ and $\frac{\tilde{K}}{\tilde{Y}}$ in (S.145) and (S.147), the binding borrowing constraint (S.142) implies that

$$\text{(S.149)}\quad \frac{\tilde{B}}{\tilde{Y}} = \bar{\theta} g_\gamma \phi_l \frac{\tilde{q}_l L_e}{\tilde{Y}} + \frac{\bar{\theta}\phi_k}{\bar{\lambda}_q}\frac{\tilde{K}}{\tilde{Y}} - \phi_w R s_n.$$

The entrepreneur's flow of funds constraint (S.141) implies that

$$\text{(S.150)}\quad 1 = \frac{\tilde{C}_e}{\tilde{Y}} + \frac{\tilde{I}}{\tilde{Y}} + \frac{\tilde{B}}{\tilde{Y}}\left(\frac{1}{g_\gamma} - \frac{1}{R}\right) + s_n.$$

The aggregate resource constraint (S.139) then implies that

$$\text{(S.151)}\quad \frac{\tilde{C}_h}{\tilde{Y}} = 1 - \frac{\tilde{C}_e}{\tilde{Y}} - \frac{\tilde{I}}{\tilde{Y}}.$$

To solve for $\frac{L_h}{L_e}$, we first use the household's land Euler equation (i.e., the housing demand equation) (S.129) and the definition for the marginal utility (S.127) to obtain

$$\text{(S.152)}\quad \frac{\tilde{q}_l L_h}{\tilde{C}_h} = \frac{\bar{\varphi}(g_\gamma - \gamma_h)}{g_\gamma(1 - g_\gamma/R)(1-\gamma_h/R)},$$

where the steady-state loan rate is given by (S.143).

Taking the ratio between (S.152) and (S.145) results in the solution

$$\text{(S.153)}\quad \frac{L_h}{L_e} = \frac{\bar{\varphi}(g_\gamma - \gamma_h)(1-\beta-\beta\bar{\lambda}_a\bar{\theta}\phi_l)}{\beta\alpha\phi g_\gamma(1-g_\gamma/R)(1-\gamma_h/R)}\frac{\tilde{C}_h}{\tilde{Y}}.$$

Finally, we can solve for the steady-state hours by combining the labor supply equation (S.128) and the labor demand equation (S.132) to get

$$N = \frac{g_\gamma(1-\gamma_h/R)}{g_\gamma - \gamma_h}\frac{s_n}{\bar{\psi}}\frac{\tilde{Y}}{\tilde{C}_h}. \tag{S.154}$$

F.1.2.4. *Log-Linearized Equilibrium System.* Upon obtaining the steady-state equilibrium, we log-linearize the equilibrium conditions (S.127) through (S.142) around the steady state. We define the constants $\Omega_h \equiv (g_\gamma - \beta(1+\bar{\lambda}_a)\gamma_h)(g_\gamma - \gamma_h)$ and $\Omega_e \equiv (g_\gamma - \beta\gamma_e)(g_\gamma - \gamma_e)$. The log-linearized equilibrium conditions are given by

$$\begin{aligned}\Omega_h\hat{\mu}_{ht} = {} & -\big[g_\gamma^2 + \gamma_h^2\beta(1+\bar{\lambda}_a)\big]\hat{C}_{ht} + g_\gamma\gamma_h(\hat{C}_{h,t-1} - \hat{g}_{\gamma t}) \\ & - \beta\bar{\lambda}_a\gamma_h(g_\gamma - \gamma_h)\mathrm{E}_t\hat{\lambda}_{a,t+1} \\ & + \beta(1+\bar{\lambda}_a)g_\gamma\gamma_h\mathrm{E}_t(\hat{C}_{h,t+1} + \hat{g}_{\gamma,t+1}),\end{aligned} \tag{S.155}$$

$$\hat{w}_t + \hat{\mu}_{ht} = \hat{\psi}_t, \tag{S.156}$$

$$\begin{aligned}\hat{q}_{lt} + \hat{\mu}_{ht} = {} & \beta(1+\bar{\lambda}_a)\mathrm{E}_t[\hat{\mu}_{h,t+1} + \hat{q}_{l,t+1}] \\ & + \big[1-\beta(1+\bar{\lambda}_a)\big](\hat{\varphi}_t - \hat{L}_{ht}) + \beta\bar{\lambda}_a\mathrm{E}_t\hat{\lambda}_{a,t+1},\end{aligned} \tag{S.157}$$

$$\hat{\mu}_{ht} - \hat{R}_t = \mathrm{E}_t\left[\hat{\mu}_{h,t+1} + \frac{\bar{\lambda}_a}{1+\bar{\lambda}_a}\hat{\lambda}_{a,t+1} - \hat{g}_{\gamma,t+1}\right], \tag{S.158}$$

$$\begin{aligned}\Omega_e\hat{\mu}_{et} = {} & -\big(g_\gamma^2 + \beta\gamma_e^2\big)\hat{C}_{e,t} + g_\gamma\gamma_e(\hat{C}_{e,t-1} - \hat{g}_{\gamma t}) \\ & + \beta g_\gamma\gamma_e\mathrm{E}_t(\hat{C}_{e,t+1} + \hat{g}_{\gamma,t+1}),\end{aligned} \tag{S.159}$$

$$\hat{Y}_t - \hat{N}_t = \frac{\phi_w\bar{\lambda}_a}{1+\bar{\lambda}_a + \phi_w\bar{\lambda}_a}(\hat{R}_t + \hat{\mu}_{bt} - \hat{\mu}_{et}) + \hat{w}_t, \tag{S.160}$$

$$\begin{aligned}\hat{q}_{kt} = {} & (1+\beta)\Omega\lambda_k^2\hat{I}_t - \Omega\lambda_k^2\hat{I}_{t-1} + \Omega\lambda_k^2(\hat{g}_{\gamma t} + \hat{g}_{qt}) \\ & - \beta\Omega\lambda_k^2\mathrm{E}_t[\hat{I}_{t+1} + \hat{g}_{\gamma,t+1} + \hat{g}_{q,t+1}],\end{aligned} \tag{S.161}$$

$$\begin{aligned}\hat{q}_{kt} + \hat{\mu}_{et} = {} & \frac{\tilde{\mu}_b}{\tilde{\mu}_e}\frac{\bar{\theta}\phi_k}{\bar{\lambda}_q}(\hat{\mu}_{bt} + \hat{\theta}_t) + \frac{\beta(1-\delta)}{\lambda_k}\mathrm{E}_t(\hat{q}_{k,t+1} - \hat{g}_{q,t+1} - \hat{g}_{\gamma,t+1}) \\ & + \left(1 - \frac{\tilde{\mu}_b}{\tilde{\mu}_e}\frac{\bar{\theta}\phi_k}{\bar{\lambda}_q}\right)\mathrm{E}_t\hat{\mu}_{e,t+1} + \frac{\tilde{\mu}_b}{\tilde{\mu}_e}\frac{\bar{\theta}\phi_k}{\bar{\lambda}_q}\mathrm{E}_t(\hat{q}_{k,t+1} - \hat{g}_{q,t+1}) \\ & + \beta\alpha(1-\phi)\frac{\tilde{Y}}{\tilde{K}}\mathrm{E}_t(\hat{Y}_{t+1} - \hat{K}_t),\end{aligned} \tag{S.162}$$

$$\hat{q}_{lt} + \hat{\mu}_{et} = \frac{\tilde{\mu}_b}{\tilde{\mu}_e} g_\gamma \bar{\theta} \phi_l (\hat{\theta}_t + \hat{\mu}_{bt}) + \left(1 - \frac{\tilde{\mu}_b}{\tilde{\mu}_e} g_\gamma \bar{\theta} \phi_l\right) \mathrm{E}_t \hat{\mu}_{e,t+1} + \frac{\tilde{\mu}_b}{\tilde{\mu}_e} g_\gamma \bar{\theta} \phi_l \mathrm{E}_t (\hat{q}_{l,t+1} + \hat{g}_{\gamma,t+1}) + \beta \mathrm{E}_t \hat{q}_{l,t+1} + (1 - \beta - \beta \bar{\lambda}_a \bar{\theta} \phi_l) \mathrm{E}_t [\hat{Y}_{t+1} - \hat{L}_{et}], \tag{S.163}$$

$$\hat{\mu}_{et} - \hat{R}_t = \frac{1}{1 + \bar{\lambda}_a} \big[\mathrm{E}_t (\hat{\mu}_{e,t+1} - \hat{g}_{\gamma,t+1}) + \bar{\lambda}_a \hat{\mu}_{bt}\big], \tag{S.164}$$

$$\hat{Y}_t = \alpha \phi \hat{L}_{e,t-1} + \alpha (1 - \phi) \hat{K}_{t-1} + (1 - \alpha) \hat{N}_t - \frac{(1 - \phi)\alpha}{1 - (1 - \phi)\alpha} [\hat{g}_{zt} + \hat{g}_{qt}], \tag{S.165}$$

$$\hat{K}_t = \frac{1 - \delta}{\lambda_k} [\hat{K}_{t-1} - \hat{g}_{\gamma t} - \hat{g}_{qt}] + \left(1 - \frac{1 - \delta}{\lambda_k}\right) \hat{I}_t, \tag{S.166}$$

$$\hat{Y}_t = \frac{\tilde{C}_h}{\tilde{Y}} \hat{C}_{ht} + \frac{C_e}{\tilde{Y}} \hat{C}_{e,t} + \frac{\tilde{I}}{\tilde{Y}} \hat{I}_t, \tag{S.167}$$

$$0 = \frac{\tilde{L}_h}{\bar{L}} \hat{L}_{ht} + \frac{\tilde{L}_e}{\bar{L}} \hat{L}_{et}, \tag{S.168}$$

$$\hat{Y}_t = \frac{\tilde{C}_e}{\tilde{Y}} \hat{C}_{e,t} + \frac{\tilde{I}}{\tilde{Y}} \hat{I}_t + \frac{\tilde{q}_l \tilde{L}_e}{\tilde{Y}} (\hat{L}_{et} - \hat{L}_{e,t-1}) + s_n (\hat{w}_t + \hat{N}_t) + \frac{1}{g_\gamma} \frac{\tilde{B}}{\tilde{Y}} (\hat{B}_{t-1} - \hat{g}_{\gamma t}) - \frac{1}{R} \frac{\tilde{B}}{\tilde{Y}} (\hat{B}_t - \hat{R}_t), \tag{S.169}$$

$$\hat{B}_t = \hat{\theta}_t + g_\gamma \bar{\theta} \phi_l \frac{\tilde{q}_l L_e}{\tilde{Y}} \frac{\tilde{Y}}{\tilde{B}} \mathrm{E}_t (\hat{q}_{l,t+1} + \hat{L}_{et} + \hat{g}_{\gamma,t+1}) + \bar{\theta} \frac{\phi_k}{\bar{\lambda}_q} \frac{\tilde{K}}{\tilde{Y}} \frac{\tilde{Y}}{\tilde{B}} \mathrm{E}_t (\hat{q}_{k,t+1} + \hat{K}_t - \hat{g}_{q,t+1}) - \phi_w s_n R \frac{\tilde{Y}}{\tilde{B}} (\hat{w}_t + \hat{N}_t + \hat{R}_t). \tag{S.170}$$

The terms $\hat{g}_{zt}$, $\hat{g}_{qt}$, and $\hat{g}_{\gamma t}$ are given by

$$\hat{g}_{zt} = \hat{\lambda}_{zt} + \hat{\nu}_{zt} - \hat{\upsilon}_{z,t-1}, \tag{S.171}$$

$$\hat{g}_{qt} = \hat{\lambda}_{qt} + \hat{\nu}_{qt} - \hat{\upsilon}_{q,t-1}, \tag{S.172}$$

$$\hat{g}_{\gamma t} = \frac{1}{(1 - (1 - \phi)\alpha)} \hat{g}_{zt} + \frac{(1 - \phi)\alpha}{(1 - (1 - \phi)\alpha)} \hat{g}_{qt}. \tag{S.173}$$

The technology shocks follow the processes

$$\hat{\lambda}_{zt} = \rho_z \hat{\lambda}_{z,t-1} + \hat{\varepsilon}_{zt}, \tag{S.174}$$

$$\hat{\nu}_{zt} = \rho_{\nu_z} \hat{\nu}_{z,t-1} + \hat{\varepsilon}_{\nu_z t}, \tag{S.175}$$

$$\hat{\lambda}_{qt} = \rho_q \hat{\lambda}_{q,t-1} + \hat{\varepsilon}_{qt}, \tag{S.176}$$

$$\hat{\nu}_{qt} = \rho_{\nu_q} \hat{\nu}_{q,t-1} + \hat{\varepsilon}_{\nu_q t}. \tag{S.177}$$

There preference shocks follow the processes

$$\hat{\lambda}_{at} = \rho_a \hat{\lambda}_{a,t-1} + \hat{\varepsilon}_{at}, \tag{S.178}$$

$$\hat{\varphi}_t = \rho_\varphi \hat{\varphi}_{t-1} + \hat{\varepsilon}_{\varphi t}, \tag{S.179}$$

$$\hat{\psi}_t = \rho_\psi \hat{\psi}_{t-1} + \hat{\varepsilon}_{\psi t}. \tag{S.180}$$

The liquidity shock follows the process

$$\hat{\theta}_t = \rho_\theta \hat{\theta}_{t-1} + \hat{\varepsilon}_{\theta t}. \tag{S.181}$$

We solve the 19 equations (S.155) through (S.173) for the 19 unknowns in the vector

$$x_t = [\hat{\mu}_{ht}, \hat{w}_t, \hat{q}_{lt}, \hat{R}_t, \hat{\mu}_{et}, \hat{\mu}_{bt}, \hat{N}_t, \hat{I}_t, \hat{Y}_t, \hat{C}_{ht}, \hat{C}_{et}, \hat{q}_{kt}, \hat{L}_{ht}, \hat{L}_{et}, \hat{K}_t, \hat{B}_t, \hat{g}_{\gamma t}, \hat{g}_{zt}, \hat{g}_{qt}]'.$$

The state variables consist of the predetermined variables and the exogenous forcing processes summarized in the vector

$$s_t = [\hat{C}_{h,t-1}, \hat{C}_{e,t-1}, \hat{I}_{t-1}, \hat{L}_{e,t-1}, \hat{K}_{t-1}, \hat{B}_{t-1}, \hat{\lambda}_{zt}, \hat{\nu}_t, \hat{\lambda}_{qt}, \hat{\mu}_t, \hat{\lambda}_{at}, \hat{\varphi}_t, \hat{\psi}_t, \hat{\theta}_t]'.$$

We use Chris Sims's gensys algorithm to solve the model.

F.1.2.5. *Estimation Results.* To estimate the model, we follow the literature by calibrating the value of ϕ_w at 0.75 (Christiano, Motto, and Rostagno (2010)). Since growth of land supply does not affect equilibrium dynamics, we set $\bar{\lambda}_l = 1$, corresponding to no growth in aggregate land supply, consistent with the evidence on land quantity discussed in Appendix E of this Supplemental Material.

As shown in Tables S.I and S.II, the estimated parameters in the model with working capital (in the columns under the heading "WK") are similar to those in the benchmark model (in the column under "Bench"). The estimation results are even closer to those in the benchmark model when the value of ϕ_w is calibrated to be less than 0.75.

TABLE S.I

POSTERIOR MODE ESTIMATES OF STRUCTURAL PARAMETERS[a]

Parameter	Bench	Alt	Regime	WK	CoreLogic	No Patience	Latent IST
γ_h	0.4976	0.5660	0.4922	0.4988	0.5323	0.5392	0.5819
γ_e	0.6584	0.8398	0.6592	0.6674	0.5444	0.7375	0.6335
Ω	0.1753	6.3513	0.1642	0.1603	0.2923	0.1997	0.1831
$100(g_\gamma - 1)$	0.4221	0.3736	0.4493	0.4072	0.4352	0.3200	0.4335
$100(\bar{\lambda}_q - 1)$	1.2126	1.2484	1.2206	1.2071	1.1889	1.2110	1.3750
β	0.9855	0.9706	0.9848	0.9860	0.9852	0.9884	0.9853
$\bar{\lambda}_a$	0.0089	0.0239	0.0099	0.0083	0.0094	0.0050	0.0093
$\bar{\varphi}$	0.0457	0.0495	0.0436	0.0468	0.0447	0.0535	0.0449
ϕ	0.0695		0.0694	0.0696	0.0695	0.0698	0.0694
δ	0.0368	0.0369	0.0364	0.0370	0.0369	0.0378	0.0351

[a]The columns of numbers are the posterior mode estimates in various models. “Bench” denotes the benchmark model; “Alt” denotes the alternative model in which land is not used as collateral; “Regime” denotes the Markov regime-switching model; “WK” denotes the model with working capital; “CoreLogic” denotes the benchmark model estimated using land prices constructed based on the CoreLogic home price index; “No patience” denotes the benchmark model with no patience shocks; and “Latent IST” denotes the benchmark model estimated by treating the investment-specific technology shocks as a latent variable.

F.2. *Other Variations in the Model and the Data*

This section presents a few other variations of the benchmark model and some alternative data for estimation.

F.2.1. *DSGE Model Estimated With CoreLogic Data*

The land-price series we use for the benchmark model is based on the FHFA home price index. In Appendix E of this Supplemental Material, we discuss advantages and disadvantages of using this price index series relative to using other land-price indices. To examine whether our main findings are robust to different land-price series, we fit our model to the data in which the FHFA land-price series is replaced by the CoreLogic land-price series.

Tables S.I and S.II show that the estimated parameters using CoreLogic data are similar to those using the FHFA data, except that the volatility of housing demand shocks is substantially higher. The CoreLogic data imply a more volatile land-price series because CoreLogic has a broader coverage of home prices than does FHFA: it includes both conforming loans and jumbo loans and it covers distressed home sales including short sales and foreclosures.

With the CoreLogic land-price data, housing demand shocks account for over 50% of investment fluctuations (see Table S.III, the column under “CoreLogic”). Thus, our results are robust with this different measure of land prices.

F.2.2. *Regime-Switching Models: VAR and DSGE*

Our land-price series spans the sample from 1975 to 2010, covering several recession periods with changes in macroeconomic volatility (Stock and Watson

TABLE S.II

POSTERIOR MODE ESTIMATES OF SHOCK PARAMETERS[a]

Parameter	Bench	Alt	RS	WK	CL	No Patience	Latent IST
ρ_a	0.9055	0.0883	0.9034	0.9479	0.9040		0.9928
ρ_z	0.4263	0.8798	0.4256	0.4040	0.5333	0.6826	0.4671
ρ_{ν_z}	0.0095	0.8770	0.0106	0.0501	0.3804	0.8782	0.0000
ρ_q	0.5620	0.4172	0.5494	0.5671	0.6649	0.5719	0.5856
ρ_{ν_q}	0.2949	0.4471	0.2836	0.2838	0.3947	0.3246	0.2914
ρ_φ	0.9997	0.9690	0.9995	0.9998	0.9999	0.9998	0.9979
ρ_ψ	0.9829	0.9749	0.9816	0.9959	0.9810	0.9886	0.9997
ρ_θ	0.9804	0.9623	0.9793	0.9894	0.9884	0.9852	0.9663
σ_a	0.1013	3.8118	0.0887	0.0941	0.1463		0.0001
σ_z	0.0042	0.0012	0.0042	0.0043	0.0033	0.0033	0.0057
σ_{ν_z}	0.0037	0.0063	0.0037	0.0036	0.0044	0.0071	0.0037
σ_q	0.0042	0.0057	0.0042	0.0041	0.0041	0.0041	0.0062
σ_{ν_q}	0.0029	0.0001	0.0029	0.0030	0.0029	0.0029	0.0001
$\sigma_\varphi(1)$	0.0462	0.1985	0.0316	0.0485	0.1080	0.0436	0.0566
$\sigma_\varphi(2)$			0.0785				
σ_ψ	0.0073	0.0106	0.0073	0.0076	0.0081	0.0079	0.0087
σ_θ	0.0112	0.0171	0.0112	0.0114	0.0237	0.0116	0.0116
p_{11}			0.9792				
p_{22}			0.9664				

[a]The columns of numbers are the posterior mode estimates in various models. "Bench" denotes the benchmark model; "Alt" denotes the alternative model in which land is not used as collateral; "Regime" denotes the Markov regime-switching model; "WK" denotes the model with working capital; "CoreLogic" denotes the benchmark model estimated using land prices constructed based on the CoreLogic home price index; "No patience" denotes the benchmark model with no patience shocks; and "Latent IST" denotes the benchmark model estimated by treating the investment-specific technology shocks as a latent variable.

TABLE S.III

CONTRIBUTIONS (IN PERCENT) TO INVESTMENT FLUCTUATIONS FROM A HOUSING DEMAND SHOCK[a]

Horizon	Bench	WK	No Patience	Latent IST	CoreLogic	High Vol	Low Vol
1Q	35.46	37.26	34.10	41.10	55.74	60.49	19.19
4Q	41.19	40.17	39.31	46.35	58.68	66.31	23.39
8Q	38.71	38.18	37.27	39.02	57.90	63.96	21.59
16Q	33.70	34.99	31.74	28.48	54.60	58.85	18.16
24Q	30.67	33.51	28.66	23.48	52.18	55.46	16.19

[a]The column labeled by "Bench" reports the contributions in the benchmark model; the column "WK" reports those in the model with working capital; the column "No patience" reports the results in the benchmark model with no patience shocks; the column "Latent IST" reports those in the benchmark model estimated by treating the investment-specific technology shocks as a latent variable; the column "CoreLogic" reports the results from the benchmark model with the CoreLogic data on the land price; and the columns "High vol" and "Low vol" report the contributions under the high- and low-volatility regimes from the regime-switching benchmark model.

(2003), Sims and Zha (2006), Taylor (2007)). It is therefore important to investigate how our results are affected when volatility changes are explicitly taken into account.

We first fit our bivariate BVAR with a regime-switching process on the volatility of a shock to land prices, following the approach of Sims, Waggoner, and Zha (2008). We find that the best-fit model is a Markov-switching BVAR with two volatility regimes. Figure S.10 shows that the high-volatility regime is associated with the periods in the late 1970s, the early 1980s, and the recent deep recession; and that the low-volatility regime corresponds to the Great Moderation period. In Figure S.11, we display the joint dynamics of land prices and business investment for both volatility regimes. Comparing to the impulse responses estimated from the constant-parameter BVAR model (reported in Figure 2 of the paper), the qualitative patterns of the impulse responses do not change. The main difference lies in the magnitude of responses to the land-price shock under the two volatility regimes.

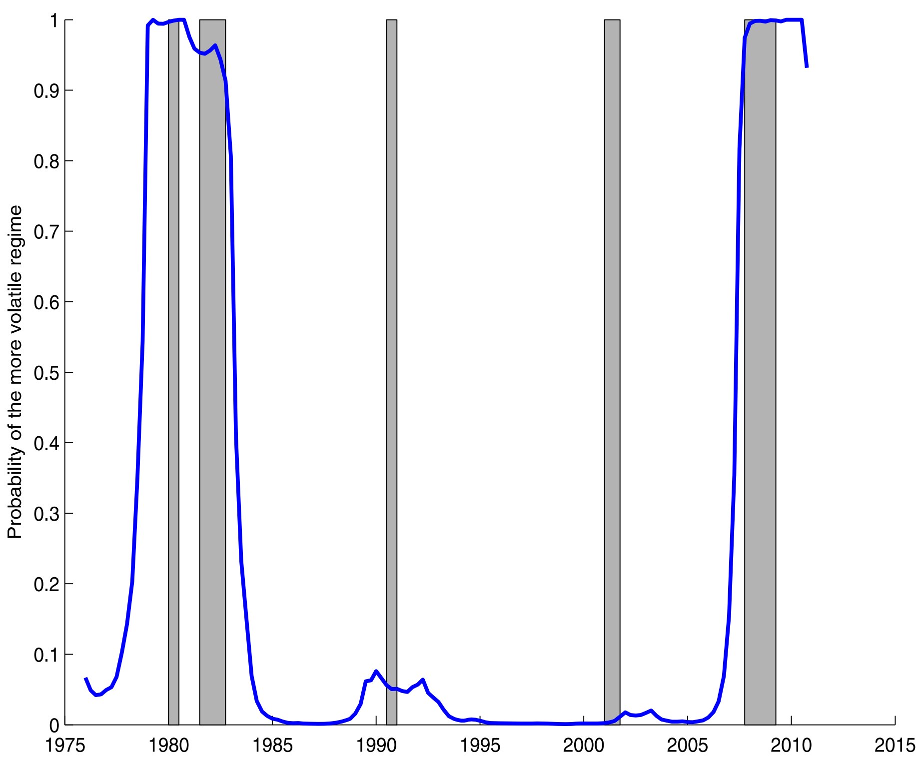

FIGURE S.10.—The posterior probability of the regime with a high volatility from the regime-switching BVAR model. The shaded area marks NBER recession dates.

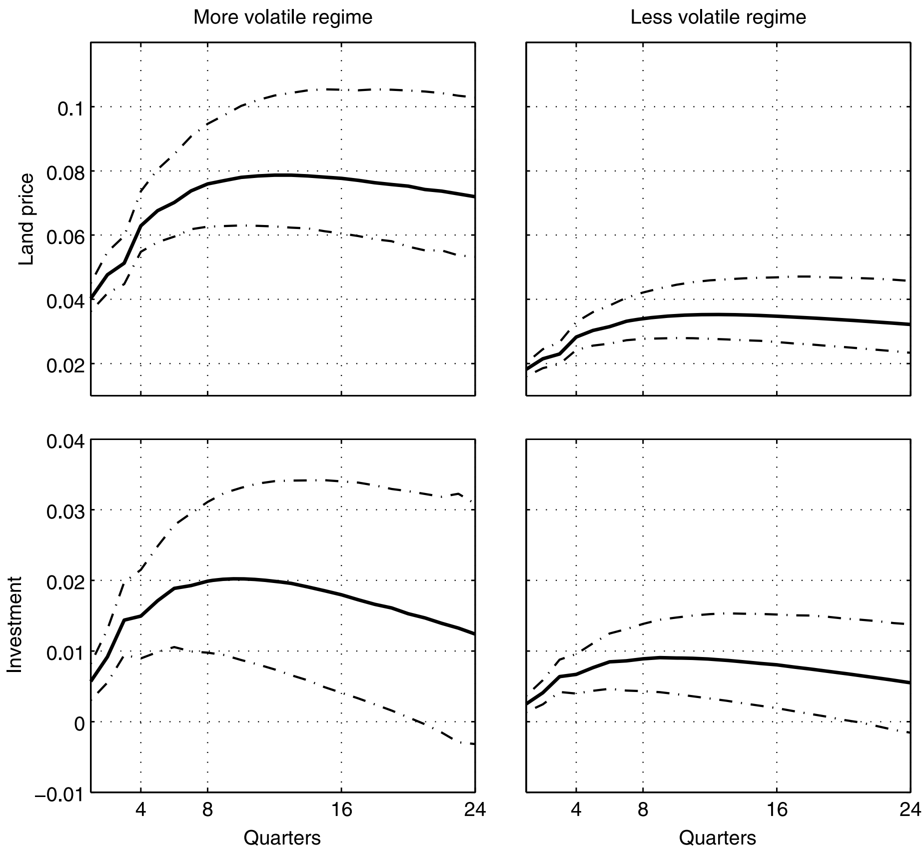

FIGURE S.11.—Impulse responses to a shock to the land price from a Markov-switching BVAR model.

To examine implications of regime shifts in shock volatility in our DSGE model, we generalize the benchmark model to allow for regime shifts in the volatility of a housing demand shock, with the following heteroskedastic process:

$$\ln \varphi_t = (1 - \rho_\varphi) \ln \bar{\varphi} + \rho_\varphi \ln \varphi_{t-1} + \sigma_\varphi(s_t)\varepsilon_{\varphi t}, \tag{S.182}$$

where the shock volatility $\sigma_\varphi(s_t)$ varies with the regime s_t. We assume that the shock volatility switches between two regimes ($s_t = 1$ or $s_t = 2$), with the Markov transition probabilities summarized by the matrix $P = [p_{ij}]$, where $p_{ij} = \text{Prob}(s_{t+1} = i | s_t = j)$ for $i, j \in \{1, 2\}$, $p_{12} = 1 - p_{22}$, and $p_{21} = 1 - p_{11}$.

We estimate this regime-switching DSGE model using the approach described in Liu, Waggoner, and Zha (2011). In the estimation, we adopt the same prior distributions for the parameters and use the same data set as in our benchmark model. The posterior mode estimates of the structural parameters and the shock parameters are very similar to those in the benchmark model, as shown in Tables S.I and S.II (in the columns under the heading "RS").

The estimated volatility of a housing demand shock has two distinct regimes: a low-volatility regime (regime 1 with $\sigma_\varphi = 0.03$) and a high-volatility regime (regime 2 with $\sigma_\varphi = 0.08$). The posterior mode estimates of the Markov-switching probabilities ($p_{11} = 0.9794$ and $p_{22} = 0.9662$) indicate that both regimes are highly persistent, although the low-volatility regime is more persistent than the high-volatility regime.

Figure S.12 shows the probability of the high-volatility regime throughout the sample periods. It indicates that the high-volatility regime is associated with periods of large declines in land prices (covering the two recessions between 1978 and 1983 and the recent deep recession).

According to the estimated variance decompositions, a housing demand shock accounts for about 20% of investment fluctuations in the low-volatility regime and 55–65% in the high-volatility regime (see the last two columns in Table S.III). Since the high-volatility regime captures periods with both large recessions and large declines in the land price, a housing demand shock plays a more important role for explaining the dynamics in land prices and business

FIGURE S.12.—Log real land prices (left scale) and the posterior probability of the regime with larger volatility from the regime-switching model (right scale). The shaded area marks NBER recession dates.

investment during recessions. This finding is consistent with Claessens, Kose, and Terrones (2011), who found that a recession is typically deeper than other recessions if there is a sharp fall in housing prices.

In addition, we estimate our DSGE model by increasing the number of volatility regimes to three, and compute the marginal data density for each of the three models: the benchmark DSGE model (2344.0), the DSGE with two volatility regimes (2354.1), and the DSGE model with three volatility regimes (2353.2). According to these marginal data density results, the data favor the DSGE with two volatility regimes. We did not estimate a DSGE model with possible shifts in coefficients partly because there is no consensus on which parameters should be allowed to switch regime and partly because the task of solving and estimating a DSGE model with coefficients switching regime continues to be daunting. (Farmer, Waggoner, and Zha (2009) and Famer, Waggoner, and Zha (2011) discussed conceptual issues regarding a regime-switching rational expectations model.) Computing time for obtaining the accurate marginal data density for the DSGE model with two or three volatility regimes is two and a half weeks on a cluster of dual-core computers.

Although it is infeasible to estimate DSGE models with coefficients switching regimes, we explore estimation of various bivariate (land-price and investment) BVAR models with both volatility regimes and coefficient regimes. We compute the marginal data density for each model. The BVAR model with three volatility regimes has the highest marginal data density (632.3). The marginal data density is 613.2 for the BVAR model with no regime switching, 623.6 for the BVAR model with two volatility regimes, and 632.0 for the BVAR model with four volatility regimes. When we allow the coefficients for the land-price equation in the BVAR model to switch between two regimes, where the Markov process controlling coefficient changes is independent of the three-regime volatility Markov process, the marginal data density (627.2) becomes considerably lower. These results confirm the finding by Sims and Zha (2006) that once volatility changes are allowed, it is difficult to find changes in coefficients favored by the data.

F.2.3. *No Patience Shocks*

An intertemporal preference (patience) shock λ_{at} has been used in the DSGE literature as one of important shocks in driving business cycles (Smets and Wouters (2007)). In our estimated model, the patience shock accounts for a nontrivial fraction of investment fluctuations (about 15–20%). Therefore, it is important to examine whether abstracting from this shock would change the model's quantitative implications in a significant way. We reestimate the model without patience shocks. The estimation results are shown in Tables S.I and S.II (under the heading "No patience"). The estimates are broadly similar to those in the benchmark model. Without patience shocks, we find that a housing demand shock remains the most important driving force for investment dynamics, accounting for about 30–40% of investment fluctuations (see Table S.III, the column under "No patience").

F.2.4. *Latent IST Shocks*

Justiniano, Primiceri, and Tambalotti (2011) argued that if the price of investment goods is not used in fitting the model, investment-specific shocks can be interpreted as "financial" shocks and may have a large impact on macroeconomic fluctuations. When we reestimate the model by treating the IST shocks as a latent variable (i.e., without fitting to the time series data of the relative price of investment), we find that the estimation results are similar to the benchmark model and a housing demand shock still accounts for 23–46% of investment fluctuations (see Table S.III, the column under "Latent IST").

F.2.5. *How Important is Land as a Collateral Asset?*

In the data, real estate represents a large fraction of firms' tangible assets and, as discussed in the Introduction, changes in the real estate value have a significant impact on firms' investment spending. In our benchmark model, we assume that land is a collateral asset for firms. A positive housing demand shock raises the land price and thereby expands the firm's borrowing capacity, enabling the firm to finance expansions of investment and production.

How important is land as a collateral asset in our macroeconomic model? To answer this question, we study an alternative model specification with the general setup of a collateral constraint as

$$B_t \leq \theta_t \mathrm{E}_t[\omega_l q_{l,t+1} L_{et} + \omega_k q_{k,t+1} K_t,], \tag{S.183}$$

where ω_l is the weight put on land value and ω_k is the weight put on capital value. The weight parameters ω_l and ω_k cannot be identified separately, but one can identify the relative weight ω_k/ω_l. This model nests our benchmark model as a special case when $\omega_k/\omega_l = 1$.

We estimate this alternative model using the same set of time series data. The estimation implies that the relative weight for capital value ω_k/ω_l is 1.2. As shown in Figure S.13, the impulse responses to both a TFP shock and a housing demand shock are very close to those from the benchmark model (compare the dotted-dashed lines and the asterisk lines).

F.2.6. *Frisch Labor Supply Elasticity*

In the benchmark model, we assume that labor is indivisible so that the utility of leisure is linear and aggregate labor supply elasticity is infinity (Hansen (1985), Rogerson (1988)). We now consider the alternative specification of the utility function

$$\mathrm{E}\sum_{t=0}^{\infty} \beta^t A_t \left\{ \log(C_{ht} - \gamma_h C_{h,t-1}) + \varphi_t \log L_{ht} - \psi_t \frac{N_{ht}^{1+\eta}}{1+\eta} \right\}, \tag{S.184}$$

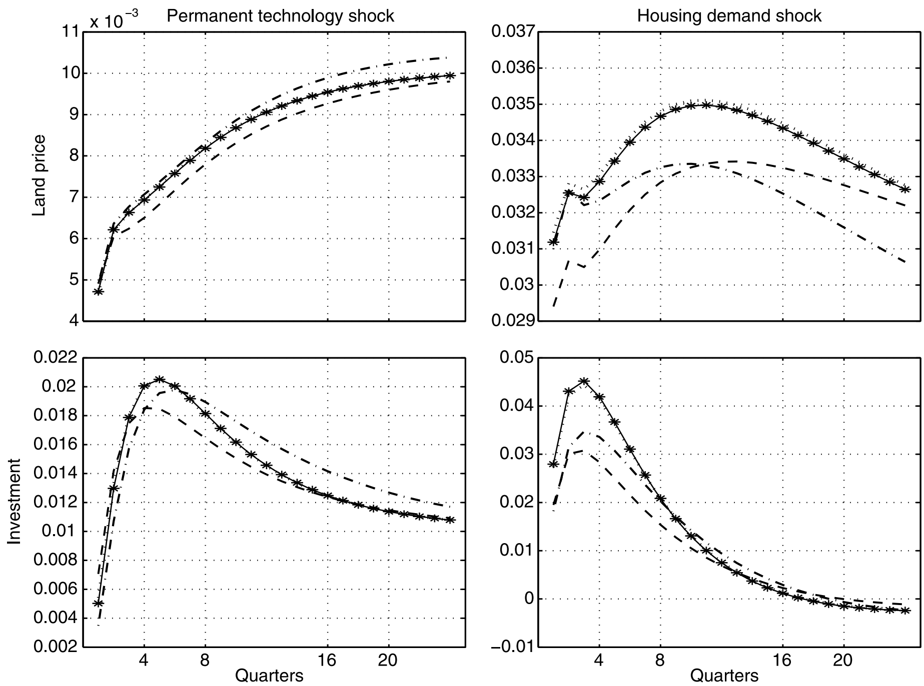

FIGURE S.13.—Impulse responses to a positive shock to neutral technology growth (left column) and to a positive shock to housing demand (right column). Lines marked by asterisks represent the responses for the benchmark model; dashed lines represent the model with a general form of the disutility function of labor; dotted-dashed lines represent the model with flexible relative weight on capital value in the credit constraint; dotted lines represent the model with external adjustments costs to capital. Note that the results are so close that some lines are on top of one another.

where $\eta > 0$ is the inverse Frisch elasticity of labor supply.

With this utility function, all equilibrium conditions are identical except that the labor supply equation (S.116) is changed into

$$w_t = \frac{A_t}{\mu_{ht}} \psi_t N_{ht}^{\eta}. \tag{S.185}$$

This change affects the steady-state labor as well as the log-linearized version of the labor supply decision. In particular, the steady-state solution for labor (S.154) becomes

$$N = \left\{ \frac{(1-\alpha) g_\gamma (1-\gamma_h/R)}{\bar{\psi}(g_\gamma - \gamma_h)} \frac{\tilde{Y}}{\tilde{C}_h} \right\}^{1/(1+\eta)}. \tag{S.186}$$

The linearized labor supply equation (S.156) is replaced by

$$\text{(S.187)}\quad \hat{w}_t + \hat{\mu}_{ht} = \hat{\psi}_t + \eta \hat{N}_t.$$

We reestimate the benchmark model with the disutility function of labor replaced by this more flexible form. The posterior mode estimate for η is about 2.1. The estimates for the other parameters are similar to those in the benchmark model. As shown in Figure S.13, the impulse responses of the land price and business investment to a TFP shock and to a housing demand shock do not differ much from those in the benchmark model (compare the dashed lines and the asterisk lines).

F.2.7. *External Capital Producers*

In the benchmark model, capital adjustment is internal to the entrepreneurs, so that the price of capital reflects the shadow value of capital that is specific to the entrepreneurs and does not reflect the market value of capital. In this sense, capital is less pledgable than land for external financing. We now consider an alternative specification such that capital adjustment is done by an external capital producer (Carlstrom and Fuerst (1997), Bernanke, Gertler, and Gilchrist (1999)). Entrepreneurs still accumulate capital, but they need to purchase new capital at a market price. We assume that the capital producers are owned by the household sector, so that capital is as pledgable as land as a collateral for borrowing.

There is a continuum of identical capital producers. The representative capital producer purchases one unit of investment goods at the price $1/Q_t$ and transforms the goods into usable capital. The transformation from investment goods to capital incurs an adjustment cost. Capital goods are then sold to the entrepreneurs at the market price q_{kt}.

The capital producer chooses investment I_t to maximize the cum-dividend (including dividend) value

$$\text{(S.188)}\quad V_t = \sum_{j=0}^{\infty} \mathrm{E}_t \beta^j \frac{\mu_{h,t+j}}{\mu_{ht}} \left\{ q_{kt+j} \left[1 - \frac{\Omega}{2} \left(\frac{I_{t+j}}{I_{t+j-1}} - \bar{\lambda}_I \right)^2 \right] I_{t+j} - \frac{I_{t+j}}{Q_{t+j}} \right\}.$$

The first-order condition is given by

$$\text{(S.189)}\quad \frac{1}{Q_t} = q_{kt} \left[1 - \frac{\Omega}{2} \left(\frac{I_t}{I_{t-1}} - \bar{\lambda}_I \right)^2 - \Omega \left(\frac{I_t}{I_{t-1}} - \bar{\lambda}_I \right) \frac{I_t}{I_{t-1}} \right] + \beta \Omega \mathrm{E}_t \frac{\mu_{h,t+1}}{\mu_{ht}} q_{k,t+1} \left(\frac{I_{t+1}}{I_t} - \bar{\lambda}_I \right) \left(\frac{I_{t+1}}{I_t} \right)^2.$$

This equation replaces the investment Euler equation (S.122) in the benchmark model.

The representative entrepreneur purchases capital goods from the capital producers at the market price q_{kt}. The entrepreneur's problem is given by

$$\mathrm{E}\sum_{t=0}^{\infty}\beta^t\big[\log(C_{et}-\gamma_e C_{e,t-1})\big], \tag{S.190}$$

subject to the flow-of-funds constraint

$$\begin{aligned} C_{et}+q_{lt}(L_{et}-L_{e,t-1})+B_{t-1}+q_{kt}\big(K_{t+1}-(1-\delta)K_t\big) \\ = Z_t\big[L_{e,t-1}^{\phi}K_{t-1}^{1-\phi}\big]^{\alpha}N_{et}^{1-\alpha}-w_tN_{et}+\frac{B_t}{R_t}, \end{aligned} \tag{S.191}$$

and the borrowing constraint

$$B_t \le \theta_t \mathrm{E}_t[q_{l,t+1}L_{et}+q_{k,t+1}K_t]. \tag{S.192}$$

The flow-of-funds constraint (S.191) here differs from (S.106) in the benchmark model in that the entrepreneur does not pay any internal investment adjustment costs, but instead, the entrepreneur acquires capital at the market price.

This change does not affect the capital Euler equation (S.123) in the benchmark model, which is rewritten here for convenience of references:

$$q_{kt}=\beta\mathrm{E}_t\frac{\mu_{e,t+1}}{\mu_{et}}\left[\alpha(1-\phi)\frac{Y_{t+1}}{K_t}+q_{k,t+1}(1-\delta)\right]+\frac{\mu_{bt}}{\mu_{et}}\theta_t\mathrm{E}_t q_{k,t+1}. \tag{S.193}$$

In equilibrium, capital goods market clearing implies that gross investment equals new capital goods produced (net of adjustment costs) so that

$$K_{t+1}-(1-\delta)K_t=\left[1-\frac{\Omega}{2}\left(\frac{I_t}{I_{t-1}}-\bar{\lambda}_I\right)^2\right]I_t. \tag{S.194}$$

This equation corresponds to the capital law of motion (S.105) in the benchmark model.

To summarize, if capital goods are produced by an external sector, then we need to change two equilibrium conditions in the benchmark model. First, the investment Euler equation should be replaced by (S.189). Second, the flow-of-funds constraint for the entrepreneur should be replaced by (S.191).

These changes do not affect the steady-state conditions (since the steady-state adjustment cost is zero). They do affect the log-linearized equilibrium dynamics. In particular, the log-linearized investment Euler equation (S.161) is replaced by

$$\begin{aligned} \hat{q}_{kt} &= \big(1+\beta(1+\bar{\lambda}_a)\big)\Omega\lambda_k^2\hat{I}_t-\Omega\lambda_k^2\hat{I}_{t-1}+\Omega\lambda_k^2(\hat{g}_{\gamma t}+\hat{g}_{qt}) \\ &\quad -\beta(1+\bar{\lambda}_a)\Omega\lambda_k^2\mathrm{E}_t[\hat{I}_{t+1}+\hat{g}_{\gamma,t+1}+\hat{g}_{q,t+1}], \end{aligned} \tag{S.195}$$

and the linearized flow-of-funds equation for the entrepreneur (S.169) is replaced by

$$\alpha \hat{Y}_t = \frac{\tilde{C}_e}{\tilde{Y}} \hat{C}_{e,t} + \frac{\tilde{I}}{\tilde{Y}} (\hat{q}_{kt} + \hat{I}_t) + \frac{\tilde{q}_l L_e}{\tilde{Y}} (\hat{L}_{et} - \hat{L}_{e,t-1}) + \frac{1}{g_\gamma} \frac{\tilde{B}}{\tilde{Y}} (\hat{B}_{t-1} - \hat{g}_{\gamma t}) - \frac{1}{R} \frac{\tilde{B}}{\tilde{Y}} (\hat{B}_t - \hat{R}_t). \tag{S.196}$$

The estimated results for this alternative model are similar to those for the benchmark. Figure S.13 displays the impulse responses of the land price and business investment to a TFP shock and to a housing demand shock for both models. As one can see, the results do not differ much (compare the dotted lines and the asterisk lines).

F.3. *Stock Prices, Land Prices, and Investment*

In our model, there are two types of collateral assets: land and capital. We choose to fit our model to land prices but not to stock prices. We find that shocks to land prices can explain a substantial fraction of investment fluctuations. We choose not to fit the model to stock prices because our model, as most of the DSGE models in the literature, is not equipped with the necessary frictions and shocks to explain the joint dynamics between stock prices and macroeconomic variables.

To examine the joint dynamics between land prices, business investment, and stock prices in the U.S. data, we estimate a recursive Bayesian VAR with these three variables, where the land price is ordered first, investment second, and the stock price third. This identification implies that stocks prices respond to all variables instantly, consistent with the argument put forth in the literature (for example, Leeper, Sims, and Zha (1996) and Bloom (2009)).

Figure S.14 displays the impulse responses estimated from the BVAR model. The first column reports the responses following a shock to the land price. The shock leads to persistent increases in the land price, investment spending, and the stock price. The second column shows the responses following a shock to the stock price. This stock-price shock leads to a large increase in the stock price as well as a persistent increase in investment spending. The land price, however, does not seem to respond to the stock-price shock and, if anything, the point estimates show that the land price actually declines slightly. This BVAR evidence suggests that the positive co-movements between land prices and investment spending are driven by land-price shocks rather than stock-price shocks.

Further, Figure S.14 shows that, following a land-price shock, the stock price rises but the magnitude of its increase is much smaller than that for the land price. This evidence supports our model's implication that, following a housing

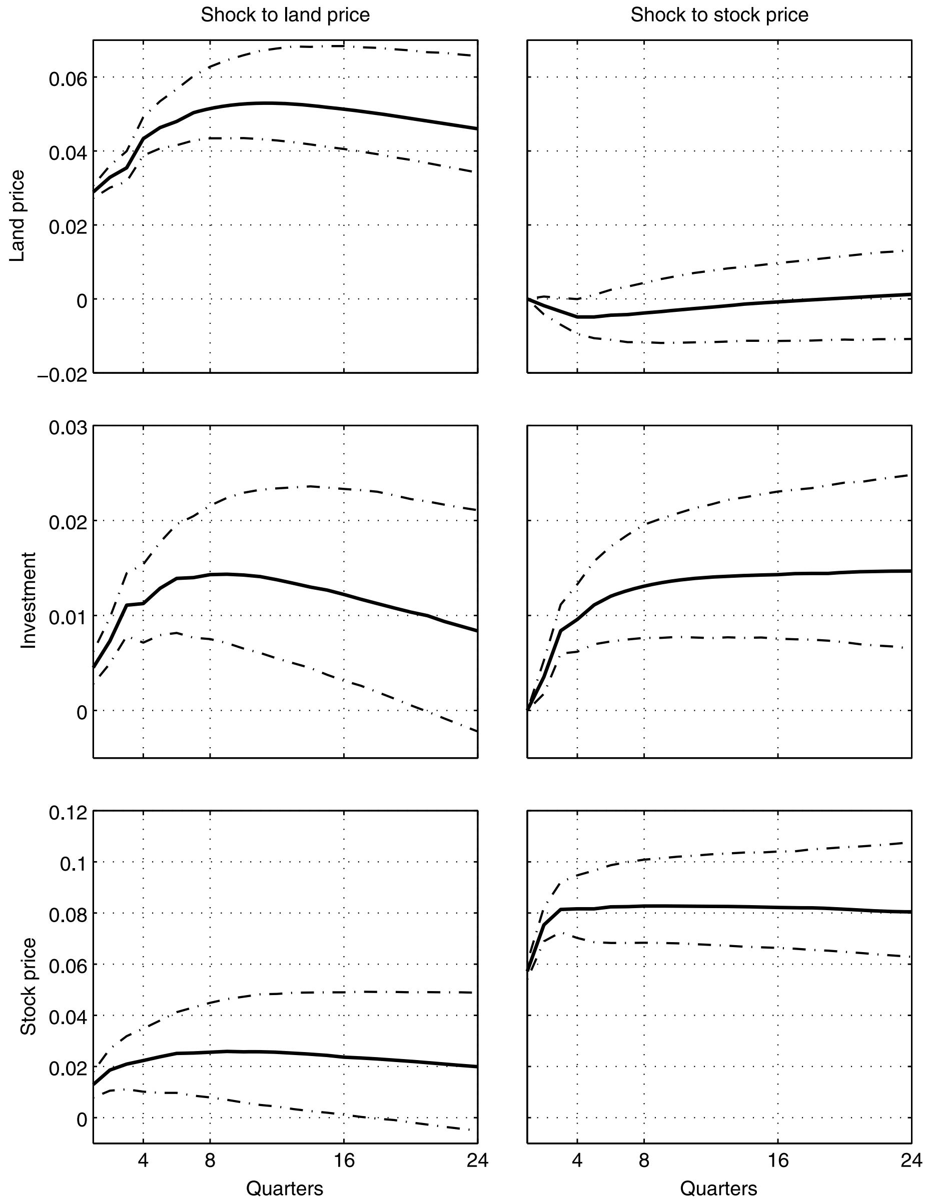

FIGURE S.14.—Impulse responses estimated from a BVAR model. Solid lines represent the estimated responses and dotted-dashed lines represent the 68% probability bands.

demand shock, the land price rises much faster than does the value of capital. Thus, the land-price dynamics play a key role in the amplification mechanism in our model.

Stock-price dynamics are likely to be driven by other economic shocks that differ from a housing demand shock. As we discuss in the Conclusion section, a challenging task for future research is to build a model to explain the empirical facts revealed by the BVAR impulse responses. Our current model is not designed to meet this challenge, partly because our focus is on the link between land-price dynamics and macroeconomic fluctuations, which, in our view, is of substantive interest by itself; and partly because such a task is beyond the scope of this paper. In a related but very different setup, however, Christiano, Motto, and Rostagno (2008) fitted a DSGE model to stock prices along with other macroeconomic variables. A more ambitious project in future research should fit a DSGE model to both land prices and stock prices, and we hope that our model is a step toward that direction.

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Manuscript received December, 2009; final revision received April, 2012.