

SUPPLEMENT TO “ESTIMATING BOTH SUPPLY AND DEMAND
ELASTICITIES USING VARIATION IN A SINGLE TAX RATE”
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APPENDIX A: EXTENSIONS

IN THIS APPENDIX, we consider three extensions to our main result derived in Section 2. The extensions are (i) supply-side instead of demand-side taxation, (ii) nonlinear, instead of ad valorem taxation, and (iii) a setting with multiple goods and multiple taxes.

A.1. Supply-Side Taxation

To extend our result to supply-side taxation, we again start out with the most general formulation of the supply-demand system given by equations (1), (2). However, we now assume the tax τ_{it} is levied on the supply side. Because the tax is levied on the supply side, we need to slightly adjust the exclusion restrictions formulated in Section 2. In particular, in this setting, the logically equivalent SER is that the instrument can be excluded from the demand equation, because the tax is paid for by the supply side:

SUPPLY-SIDE STANDARD EXCLUSION RESTRICTION: *If the tax τ_{it} is levied on the supply side, then we set $\gamma = 0$.*

The logical equivalent of the RER with supply-side taxation is that supply depends only on the price net of the tax rate denoted by $P_{it}^{-\tau} = (1 - \tau_{it})P_{it}$. The motivation for the supply-side RER is similar to the motivation of the demand-side RER: it follows from rational behavior of producers, and is a standard assumption in most models of taxation. Under the supply-side RER, the supply equation is given by

$$\begin{aligned} y_{it} &= \varepsilon^S p_{it}^{-\tau} + \Gamma^S x_{it} + \nu_{it}^S \\ &= \varepsilon^S p_{it} + \varepsilon^S \log(1 - \tau_{it}) + \eta z_{it} + \Gamma^S x_{it} + \nu_{it}^S. \end{aligned}$$

Comparing this equation to (1), we see that the supply-side RER restricts the supply equation as follows:

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SUPPLY-SIDE RAMSEY EXCLUSION RESTRICTION: *If supply only depends on the price net of the tax rate, $p_{it}^{-\tau}$, it follows that $z_{it} \equiv \log(1 - \tau_{it})$, and $\eta = \varepsilon^S$.*

To see that the supply-side SER and RER jointly allow for identification of the supply and demand elasticity, consider equation (4) after imposing both restrictions:

$$\begin{bmatrix} \pi_{zy} \\ \pi_{zp} \end{bmatrix} = \begin{bmatrix} \frac{-\varepsilon^D \varepsilon^S}{\varepsilon^S - \varepsilon^D} \\ \frac{-\varepsilon^S}{\varepsilon^S - \varepsilon^D} \end{bmatrix}.$$

The equations contain two reduced-form coefficients, and two structural coefficients. The supply and demand elasticity can therefore be expressed in terms of reduced-form elasticities as follows:

$$\begin{aligned} \varepsilon^S &= \frac{\pi_{zy}}{1 + \pi_{zp}}, \\ \varepsilon^D &= \frac{\pi_{zy}}{\pi_{zp}}. \end{aligned}$$

Hence, under the supply-side SER and RER, z_{it} is a valid instrument for estimating both elasticities. Strong identification of the demand (supply) elasticity requires that the null hypothesis that $\pi_{zp} = 0$ ($\pi_{zp} = -1$) can be rejected.

A.2. Nonlinear Taxes

In this subsection, we return to the model with demand-side taxation but extend it by considering a nonlinear rather than an ad valorem tax. Specifically, let the tax rate on the good be given by the (known) time-varying tax function $\tau_{it} = T_t(y_{it}, p_{it})$. Note that this is a very general formulation of nonlinear taxes, because the tax rate may independently depend both on the quantity y_{it} and on the price p_{it} . This allows us to capture the standard format for nonlinear taxes where the tax rate depends on the value of the traded goods, $\tau_{it} = T_t(p_{it}y_{it})$, as a special case. In addition, the formulation allows us to capture per-unit taxes by letting $\tau_{it} = \frac{\theta_{it}}{p_{it}}$, where θ_{it} denotes the tax per unit of the good.

With nonlinear taxes, the standard approach outlined in Section 2 fails. To see this, note that the instrument $z_{it} = f(\tau_{it}) = f(T_t(y_{it}, p_{it}))$ depends on endogenous variables. Therefore, even the reduced-form equation (3) cannot be estimated consistently using OLS.

Fortunately, there is a literature that attempts to estimate reduced-form elasticities with nonlinear taxation (Gruber and Saez (2002), Kopczuk (2005), Weber (2014)). The idea in this literature is to create a synthetic instrument by taking the new tax rules and applying them to lagged quantities and prices. That is, a synthetic instrument s_{it} is created by letting $s_{it} \equiv T_t(y_{i,t-L}, p_{i,t-L})$, where L denotes the number of lags. The synthetic instrument is plausibly exogenous provided L is large enough (see Weber (2014) for a discussion).

To combine this approach with our method, note first that we cannot simply use $z_{it} \equiv \log(1 + s_{it})$ because there is no reason to assume this instrument satisfies the RER.

Instead, we propose to first instrument the log of the gross-of-tax rate $\log(1 + \tau_{it})$ by regressing actual tax rates on synthetic tax rates:

$$\log(1 + \tau_{it}) = \beta \log(1 + s_{it}) + \lambda x_{it} + \phi_{it}, \quad (\text{A.1})$$

where β and Λ are coefficients and ϕ_{it} is the error term. Equation (A.1) is the standard first-stage regression in the empirical literature on nonlinear taxation. The instrumented log of the gross-of-tax rate $\widehat{\log(1 + \tau_{it})}$ is plausibly exogenous and plausibly satisfies the RER. Hence, one can use $z_{it} \equiv \widehat{\log(1 + \tau_{it})}$ as the instrument in the estimation method outlined in Section 3.

A potential issue is that standard errors have to be adjusted because $\widehat{\log(1 + \tau_{it})}$ is estimated with error. This can be solved using the Delta Method. However, in the nonlinear tax literature, the fit of equation (A.1) is typically close to perfect with first-stage F -values exceeding 50 (e.g., Weber (2014)). As such, it is unlikely that adjusting the standard errors will have an important effect on the estimates.

A.3. Multiple Goods

To generalize our result to a setting with multiple goods, consider a supply-demand system with J goods. Let y_{it}^j denote the logged quantity of good j . Assume that each good faces an ad valorem tax rate τ_{it}^j , and assume that variation in each of the tax rates is exogenous (after controlling for x_{it}). Again, we use as an instrument a known transformation of the tax rate such that $z_{it}^j \equiv f^j(\tau_{it}^j)$. For simplicity, we also immediately impose the SER, implying that the instruments can be excluded from the supply equations. Under the SER, the system of equations that relates the J -row vector of logged prices p_{it} to supply and demand is given by

$$y_{it}^j = p_{it} \varepsilon^{Sj} + x_{it} \Gamma^{Sj} + \nu_{it}^{Sj}, \quad \forall j = 1, \dots, J, \quad (\text{A.2})$$

$$y_{it}^j = p_{it} \varepsilon^{Dj} + z_{it} \gamma^{Dj} + x_{it} \Gamma^{Dj} + \nu_{it}^{Dj}, \quad \forall j = 1, \dots, J, \quad (\text{A.3})$$

where ε^{Sj} and ε^{Dj} are J -column vectors of supply and demand (cross) elasticities of good j with respect to each price. y_{it} is a J -row vector with all traded goods. z_{it} is a J -row vector with instruments. x_{it} is a K -row vector with linearly independent control variables. γ^{Dj} denotes a J -column vector of coefficients for the instruments. Γ^{Sj} and Γ^{Dj} denote the K -column vectors of coefficients for the control variables. ν_{it}^{Sj} (ν_{it}^{Dj}) denotes the disturbance term in supply (demand) equation j . Equations (A.2), (A.3) represent a general supply and demand system, where the demand and supply of each good can potentially depend upon the entire vector of prices p_{it} .

The reduced-form representation of (A.2), (A.3) is given by

$$[y_{it} \quad p_{it}] = z_{it} [\Pi_{zy} \quad \Pi_{zp}] + x_{it} [\Pi_{xy} \quad \Pi_{xp}] + \xi_{it},$$

where Π_{zy} and Π_{zp} are $J \times J$ -matrices of reduced-form coefficients between the instruments and the traded quantities and prices, respectively. Π_{xy} and Π_{xp} are $K \times J$ -matrices of coefficients between the control variables and quantities and prices. Finally, ξ_{it} presents the $2J$ -row vector of disturbance terms.

As in the single-good case, the instruments z_{it} are valid for estimating the structural coefficients in the supply equations (A.2), because we assume the SER. Strong identification additionally requires that the null hypothesis that matrix Π_{zp} has insufficient rank can be rejected (see, e.g., Hausman (1983)). However, without imposing further restrictions it is not possible to estimate the structural coefficients in the demand equations (A.3). The restriction we impose is the multiple-good equivalent of the RER (MRER).

MULTIPLE-GOOD RAMSEY EXCLUSION RESTRICTION: *If demand for each good only depends on prices after taxation, $p_{it}^j + \log(1 + \tau_{it}^j)$, it follows that $z_{it}^j \equiv \log(1 + \tau_{it}^j)$ and $\gamma^{D_j} = \varepsilon^{D_j}$ for all demand equations $j \in 1, \dots, J$.*

As in the single-good case, the MRER is consistent with rational behavior by consumers. The MRER allows us to write the demand equations as follows

$$y_{it}^j = p_{it} \varepsilon^{D_j} + \log(1 + \tau_{it}^j) \varepsilon^{D_j} + x_{it} \Gamma^{D_j} + \nu_{it}^{D_j}, \quad \forall j = 1, \dots, J.$$

Proposition 1 gives a formal proof showing that MRER allows for identification of all demand and supply (cross) elasticities in equations (A.2), (A.3). It also provides the condition necessary for strong identification.

PROPOSITION 1: *The instruments z_{it} are valid for estimating all structural coefficients in the system of supply-demand equations (A.2), (A.3) if the MRER holds. The coefficients in (A.2) are strongly identified if the null that Π_{zp} has insufficient rank can be rejected. The coefficients in (A.3) are strongly identified if the null hypothesis that $\Pi_{zp}^\tau \equiv \Pi_{zp} + I_J$ has insufficient rank can be rejected.*

PROOF: To link the system of equations (A.2), (A.3) to the literature on identification in simultaneous equation models, it is useful to stack the equations. Let N denote the total number of observations. y^j is the $N \times 1$ vector of observations of the log quantity of good j , and p^j is the corresponding vector of prices. $Y = [y^1, \dots, y^J, p^1, \dots, p^J]$ denotes the $N \times 2J$ -matrix of endogenous variables. Similarly, let z denote the $N \times J$ -matrix of instruments and x the $N \times K$ -matrix of control variables. $Z = [z, x]$ denotes the $N \times (J + K)$ -matrix of exogenous variables. Finally, let $\nu = [\nu^{D_1}, \dots, \nu^{D_J}, \nu^{S_1}, \dots, \nu^{S_J}]$ denote the $N \times 2J$ -matrix of disturbance terms. The ordering implies that the demand equations are stacked on the left side, while supply equations appear on the right side. We can now present the system of equations, (A.2), (A.3), as follows:

$$YB + Z\Gamma = -\nu, \tag{A.4}$$

where B is a $2J \times 2J$ -matrix of coefficients for the endogenous variables in Y , and Γ is the $(J + K) \times 2J$ -matrix of coefficients for the exogenous variables.

We prove that all coefficients in the system of equations (A.2), (A.3) are identified by showing that the structural coefficients in each of the $j = 1, \dots, 2J$ individual equations are identified. Denote by $\{B_j, \Gamma_j\}$ the j th column of B and Γ . The two column vectors contain the full set of structural coefficients in equation j . Denote the restrictions on the coefficients in matrix form as follows:

$$[\Phi^{B_j} \Phi^{\Gamma_j}] \begin{bmatrix} B_j \\ \Gamma_j \end{bmatrix} = \phi_j,$$

where Φ^{B_j} (Φ^{Γ_j}) is a $g \times 2J$ ($g \times (J + K)$)-matrix with restrictions on the coefficients in B_j (Γ_j), ϕ_j is a g -column vector, and g is the total number of restrictions on B_j and Γ_j . The structural coefficients in equation j are identified if the system of equations

$$\begin{bmatrix} \Pi & I_{J+K} \\ \Phi^{B_j} & \Phi^{\Gamma_j} \end{bmatrix} \begin{bmatrix} B_j \\ \Gamma_j \end{bmatrix} = \begin{bmatrix} 0 \\ \phi_j \end{bmatrix}$$

has a single solution, or is overidentified (see Hausman (1983)). The necessary and sufficient condition for this to be true is known as the rank condition and can be written as

$$\text{rank} \begin{bmatrix} \Pi & I_{J+K} \\ \Phi^{B_j} & \Phi^{T_j} \end{bmatrix} = 3J + K.$$

We first show that the rank condition is satisfied for the supply equations when Π_{zp} has full rank. To see this, note that only one good appears in each equation, and the coefficient on the good is restricted to equal -1 . Moreover, by the multidimensional SER, instruments do not appear in supply equations. The restriction matrix for each supply equation is thus given by

$$[\Phi^{B_j} \quad \Phi^{T_j}] = \begin{bmatrix} I_J & 0_{J \times J} & 0_{J \times J} & 0_{J \times K} \\ 0_{J \times J} & 0_{J \times J} & I_J & 0_{J \times K} \end{bmatrix}.$$

Therefore, the rank condition for supply equation j is

$$\text{rank} \begin{bmatrix} \Pi_{zy} & \Pi_{zp} & I_J & 0_{J \times K} \\ \Pi_{xy} & \Pi_{xp} & 0_{J \times J} & I_K \\ I_J & 0_{J \times J} & 0_{J \times J} & 0_{J \times K} \\ 0_{J \times J} & 0_{J \times J} & I_J & 0_{J \times K} \end{bmatrix} = 3J + K.$$

The matrix on the left-hand side is $(3J + K) \times (3J + K)$. Hence, we have to prove that it has full rank. To do this, consider whether linear row-operations can be used to fully cancel out rows. If this is impossible, the matrix has full row rank, and since it is square, full rank.

Consider the partitions from top to bottom. Rows from the second partition cannot be used to cancel out rows in any of the other partitions, as it is the only partition with nonzero elements in the right-most partition. The second partition has full row rank by virtue of the fact that we have assumed that the control variables are linearly independent. It can therefore be removed from consideration. The rank restriction thus simplifies to

$$\text{rank} \begin{bmatrix} \Pi_{zy} & \Pi_{zp} & I_J & 0_{J \times K} \\ I_J & 0_{J \times J} & 0_{J \times J} & 0_{J \times K} \\ 0_{J \times J} & 0_{J \times J} & I_J & 0_{J \times K} \end{bmatrix} = 3J. \quad (\text{A.5})$$

The right-most partition no longer contributes to the rank, as it consists of zeros, and can therefore be removed from consideration as well. Furthermore, multiply the second partition from the top in equation (A.5) by $-\Pi_{zy}$ and add it to the first partition to arrive at

$$\text{rank} \begin{bmatrix} 0_{J \times J} & \Pi_{zp} & I_J \\ I_J & 0_{J \times J} & 0_{J \times J} \\ 0_{J \times J} & 0_{J \times J} & I_J \end{bmatrix} = 3J.$$

The second partition from the top has full row rank, and cannot be formed through linear combinations of the other partitions. The rank condition thus simplifies to

$$\text{rank} \begin{bmatrix} \Pi_{zp} & I_J \\ 0_{J \times J} & I_J \end{bmatrix} = 2J.$$

Now multiply the bottom partition by -1 and subtract from the top partition to arrive at

$$\text{rank} \begin{bmatrix} \Pi_{zp} & 0_{J \times J} \\ 0_{J \times J} & I_J \end{bmatrix} = 2J.$$

Both the bottom and the top partition have full row rank provided Π_{zp} has full rank. Furthermore, we clearly cannot use operations from the first partition to cancel out the second partition or vice versa. Therefore, the rank condition is satisfied.

For demand equations, the additional restrictions come from the multidimensional RER and the assumption that Π_{zp}^τ has full rank. The matrix of restrictions on demand equations can be written as

$$[\Phi^{B_j} \quad \Phi^{I_j}] = \begin{bmatrix} I_J & 0_{J \times J} & 0_{J \times J} & 0_{J \times K} \\ 0_{J \times J} & I_J & -I_J & 0_{J \times K} \end{bmatrix}.$$

The rank condition is hence given by

$$\text{rank} \begin{bmatrix} \Pi_{zy} & \Pi_{zp} & I_J & 0_{J \times K} \\ \Pi_{xy} & \Pi_{xp} & 0_{J \times J} & I_K \\ I_J & 0_{J \times J} & 0_{J \times J} & 0_{J \times K} \\ 0_{J \times J} & I_J & -I_J & 0_{J \times K} \end{bmatrix} = 3J + K.$$

Applying the same operations as above, we can simplify this to

$$\text{rank} \begin{bmatrix} \Pi_{zp} & I_J \\ I_J & -I_J \end{bmatrix} = 2J.$$

Finally, add the bottom partition to the top partition to arrive at

$$\text{rank} \begin{bmatrix} \Pi_{zp} + I_J & 0_{J \times J} \\ I_J & -I_J \end{bmatrix} = 2J.$$

This rank condition is satisfied under the assumption that Π_{zp}^τ has full rank.

As the proof shows, the rank condition for supply (demand) equations is satisfied if Π_{zp} (Π_{zp}^τ) has full rank. Therefore, strong identification requires that the null hypothesis that Π_{zp} (Π_{zp}^τ) has insufficient rank can be rejected. This proves the proposition. *Q.E.D.*

Our main result from Section 2 thus carries over to a setting with multiple goods when we assume the SER and the MRER. Two qualifications apply. First, as in the single-good setting, there must be exogenous variation in the tax rate. However, an additional requirement with multiple goods is that the variation in the tax rate for each good must be independent. Second, the reduced-form matrices Π_{zp} , Π_{zp}^τ must have full rank. As in the single-good case, this implies that the incidence of each tax must be shared between the demand and supply side. If the incidence of a particular tax is paid for by one side, this implies that either Π_{zp} or Π_{zp}^τ does not have full rank. However, an additional restriction in the multiple-good case is that the variation in prices caused by each tax must be linearly independent. If two of the tax instruments under consideration have exactly the same tax incidence, this condition is violated. The two qualifications jointly imply that the condition that Π_{zp} , Π_{zp}^τ have full rank may be difficult to satisfy in real-world settings with multiple goods. The null hypothesis that matrices Π_{zp} , Π_{zp}^τ have insufficient rank can be tested empirically using the F -statistic outlined in Sanderson and Windmeijer (2016).

APPENDIX B: TESTING THE RER

In this section, we show how to test the RER with an additional instrument.¹ We start from the basic model outlined in Section 2 of our note, and assume the SER is satisfied. It may potentially be possible to test both the RER and SER when three instruments are available, but we leave this to future research. In addition, we assume the first instrument z_{it} equals the log of the gross-of-tax rate $z_{it} \equiv \log(1 + \tau_{it})$.

To derive a test for the RER, we distinguish between two cases. Either the second instrument z_{it}^2 shifts the demand curve and can thus be excluded from the supply equation, or it shifts the supply curve and can thus be excluded from the demand equation. Initially, assume z_{it}^2 can be excluded from the demand curve. In that case, equations (1), (2) can be written as

$$\begin{aligned} y_{it} &= \varepsilon^S p_{it} + \gamma^2 z_{it}^2 + \Gamma^S x_{it} + \nu_{it}^S, \\ y_{it} &= \varepsilon^D p_{it} + \gamma z_{it} + \Gamma^D x_{it} + \nu_{it}^D. \end{aligned}$$

The RER implies that $\gamma = \varepsilon^D$. Without z_{it}^2 , this cannot be tested because the structural coefficients in the demand equation are not identified, unless we impose that $\gamma = \varepsilon^D$. However, with an additional instrument, γ and ε^D can be estimated independently. To do this, first instrument the price before taxation using both instruments:

$$p_{it} = \pi_{zp} z_{it} + \pi_{z^2p} z_{it}^2 + \Pi x_{it} + \xi_{it}.$$

Second, use instrumented prices, \hat{p}_{it} , to estimate the structural coefficients in the demand equation:

$$y_{it} = \varepsilon^D \hat{p}_{it} + \gamma z_{it} + \Gamma^D x_{it} + \nu_{it}^D.$$

The second stage provides consistent estimates for both ε^D and γ under the assumption that the second instrument can be excluded from the demand equation. In the second-stage equation, the RER, $\gamma = \varepsilon^D$, can therefore be tested using a standard Wald test. Rejecting the null hypothesis implies that the RER does not hold.

As a second case, assume that z_{it}^2 shifts demand rather than supply. In that case, the demand-supply system is given by

$$y_{it} = \varepsilon^S p_{it} + \Gamma^S x_{it} + \nu_{it}^S, \tag{B.1}$$

$$y_{it} = \varepsilon^D p_{it} + \gamma z_{it} + \gamma^2 z_{it}^2 + \Gamma^D x_{it} + \nu_{it}^D. \tag{B.2}$$

Here we make use of the following proposition which follows from the RER:

PROPOSITION 2: *If, and only if, the RER holds, the system of equations (B.1), (B.2) can be rewritten as*

$$y_{it} = \varepsilon^S p_{it}^\tau - \varepsilon^S z_{it} + \Gamma^S x_{it} + \nu_{it}^S, \tag{B.3}$$

$$y_{it} = \varepsilon^D p_{it}^\tau + \gamma^2 z_{it}^2 + \Gamma^D x_{it} + \nu_{it}^D. \tag{B.4}$$

¹Earlier empirical tests for whether the RER holds in the context of tax salience, and tax avoidance, have been developed in, respectively, Chetty, Looney, and Kroft (2009) and Kopczuk et al. (2016).

PROOF: Suppose the RER holds such that $\varepsilon^D = \gamma$. We then immediately arrive at (B.3), (B.4) when we substitute the definition of p_{it}^τ , $p_{it} = p_{it}^\tau - z_{it}$, into equations (B.1), (B.2). This proves the “if” part of the proposition. Now suppose (B.1), (B.2) can be rewritten as (B.3), (B.4). Substitute $p_{it}^\tau = p_{it} + z_{it}$ into equations (B.3), (B.4). This system of equations only equals (B.1), (B.2) if the RER holds such that $\varepsilon^D = \gamma$. This proves the “only if” part of the proposition. *Q.E.D.*

We cannot test the RER itself, because we do not have an instrument that can be excluded from the demand equation. However, by Proposition 2, testing the RER is exactly equivalent to testing whether, in (B.3), the coefficient on z_{it} equals $-\varepsilon^S$. This second hypothesis can be tested because we have an additional instrument that can be excluded from the supply equation.

To perform the test, first instrument p_{it}^τ using both instruments:

$$p_{it}^\tau = \pi_{zp}^\tau z_{it} + \pi_{z^2p}^\tau z_{it}^2 + \Pi x_{it} + \xi_{it}^\tau.$$

Then use instrumented prices to estimate the supply equation:

$$y_{it} = \varepsilon^S \hat{p}_{it}^\tau + \gamma^S z_{it} + \Gamma^S x_{it} + \nu_{it}^S,$$

where we allow the coefficient on the instrument γ^S to differ from $-\varepsilon^S$. In this second-stage equation, the RER can be tested through a Wald test with the null hypothesis $\varepsilon^S = -\gamma^S$. Rejecting the null hypothesis implies that the RER does not hold.

APPENDIX C: THE RAMSEY EXCLUSION RESTRICTION AND THE SUFFICIENT STATISTICS APPROACH

Our approach allows researchers to estimate both the supply and the demand elasticity using exogenous variation in a single tax rate as their instrument. As such, if the RER holds, our approach overcomes the simultaneity bias inherent to structural welfare analysis mentioned in the motivating quote to our paper by Chetty (2009b). Chetty (2009b) considered an alternative approach to overcoming simultaneity bias: the sufficient statistics approach. The sufficient statistics approach specifies structural models of welfare analysis, and expresses the key formulae of the model in terms of reduced-form elasticities. The relevant reduced-form elasticities can often be estimated using only one instrument, which allows the sufficient statistics approach to overcome simultaneity bias.

In this appendix, we contrast our approach to the sufficient statistics approach, and show that the underlying assumption to overcoming simultaneity bias in both cases is the RER. As such, if there is simultaneity, and the RER holds, welfare analysis requires only one instrument, independent of the approach. If the RER does not hold, welfare analysis requires two instruments, again independent of the approach.

To show this, note that the fundamental result underlying the sufficient statistics approach is the seminal analysis by Harberger (1964a, 1964b). In his analysis, Harberger allows the tax rate to affect both prices and quantities, thus allowing for simultaneity. He shows that the excess burden of a tax can nevertheless be calculated on the basis of the reduced-form elasticity between the traded quantity and the tax rate. His formula for the derivative of the excess burden of a tax with respect to the tax rate, expressed in coefficients of our model, is given by

$$\frac{dEB}{d\tau} = \pi_{zy}\tau. \tag{C.1}$$

The right-hand side of the equation can be calculated by estimating π_{zy} in equation (3), requiring only one instrument.

The easiest method of deriving Harberger's formula is to start with the supply-demand system (1), (2), and to impose the SER and the RER. Subsequently, the excess burden of the tax is given by the formula for the area of the deadweight loss triangle. The formula for the marginal excess burden of a tax is then derived by taking the derivative of the area of the deadweight loss triangle with respect to the tax rate (e.g., Gruber (2016) provides a textbook version of this proof).

In this appendix, we also derive Harberger's formula, but reverse the order of the proof. In particular, we do first impose the SER, but only impose the RER in the last step of the proof.² The purpose of reversing the order is to show that Harberger's formula fails to hold if the RER is not imposed.

Hence, the starting point to derive the deadweight loss is the supply-demand system given in equations (1), (2), where we assume the SER such that $\eta = 0$, and choose the instrument $z_{it} = \log(1 + \tau_{it})$. We consider the deadweight loss of a tax $\tau_{it} = \tau$, and for simplicity use the approximation $\log(1 + \tau) \approx \tau$. The deadweight loss triangle for this tax is graphically depicted in Figure C.1.

To calculate the size of the triangle, note that, with respect to $\tau_{it} = 0$, demand has shifted down by $\frac{\gamma}{\varepsilon^D} \tau$. The traded quantity has decreased by $\frac{\varepsilon^S \gamma}{\varepsilon^S - \varepsilon^D} \tau$. Hence, the area of the deadweight loss triangle is given by

$$EB = \frac{1}{2} \frac{\varepsilon^S \gamma^2 \tau^2}{(\varepsilon^S - \varepsilon^D) \varepsilon^D}.$$

The marginal excess burden can be found by differentiating the above expression with respect to τ :

$$\frac{dEB}{d\tau} = \frac{\varepsilon^S \gamma^2 \tau}{(\varepsilon^S - \varepsilon^D) \varepsilon^D} = \pi_{zy} \frac{\gamma}{\varepsilon^D} \tau. \quad (C.2)$$

As can be seen, the expression depends on the reduced-form elasticity π_{zy} as well as on the ratio between the structural parameters $\frac{\gamma}{\varepsilon^D}$. The reduced-form elasticity can be estimated using a single instrument, but separately identifying $\frac{\gamma}{\varepsilon^D}$ requires an additional instrument.³

After we impose the RER such that $\gamma = \varepsilon^D$, we arrive at Harberger's formula (C.1). Hence, the reduced-form elasticity π_{zy} is *only* a sufficient statistic for welfare analysis when the RER holds. The above implies that calculating the excess burden of a tax using sufficient statistics requires two instruments unless the RER applies.

The sufficient statistics literature has made substantive progress since Harberger's analysis. However, a large number of articles assume prices are not affected by taxation (Feldstein (1999), Saez (2001), Slemrod (2001), Gruber and Saez (2002), Chetty (2009a), Doerrenberg, Peichl, and Siegloch (2017), Keen and Slemrod (2017)), thus ruling out simultaneity by assumption. If there is indeed no simultaneity, reduced-form and structural coefficients are equivalent.

²If we do not impose the SER, it becomes complicated to even define the excess burden of a tax, because a tax on the demand side can then apparently have direct welfare consequences for the supply side.

³Note that there continues to exist a representation of the excess burden of a tax that only involves reduced-form elasticities (see, e.g., Chetty, Looney, and Kroft (2009) for a derivation). However, the excess burden depends on two reduced-form elasticities that can only be identified using two instruments.

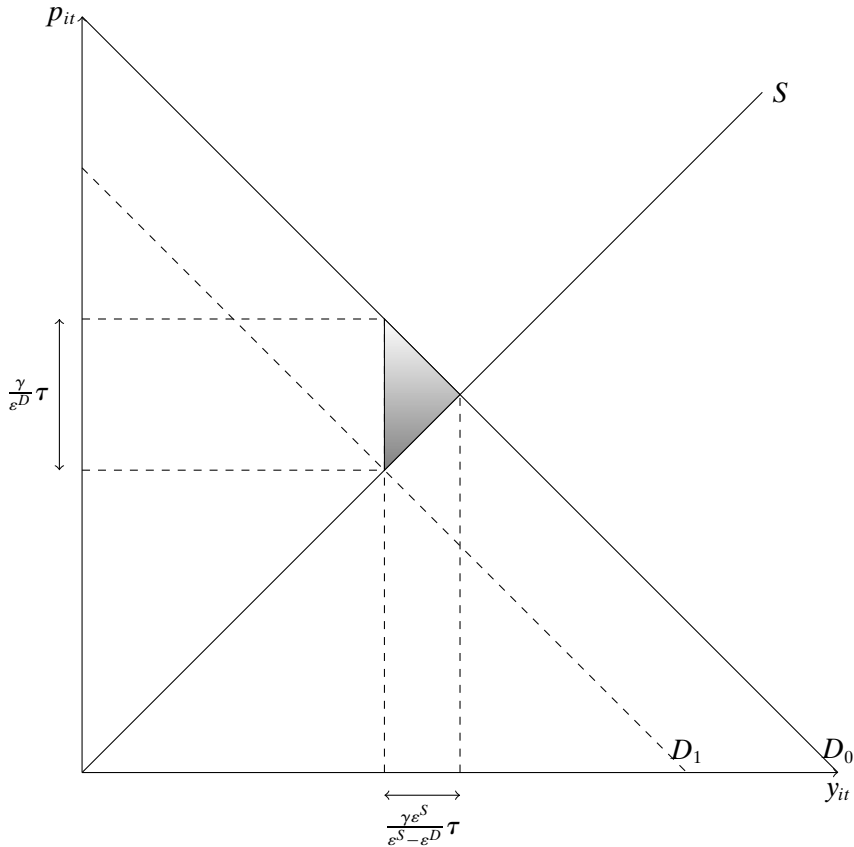


FIGURE C.1.—The deadweight loss of a tax.

To our knowledge, all papers in the sufficient statistics literature that do allow for simultaneity either assume the RER holds (Hendren (2016)), or require at least two instruments for welfare analysis (Chetty, Looney, and Kroft (2009), Saez, Matsaganis, and Tsakloglou (2012), Kopczuk et al. (2016)). Therefore, it appears to be the RER, rather than the approach to welfare analysis, that determines how many instruments are required. In settings where the RER holds, one instrument is sufficient for welfare analysis. If the RER does not hold, welfare analysis requires (at least) two instruments.

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