Econometrica Supplementary Material

SUPPLEMENT TO "PREFERENCE MONOTONICITY AND INFORMATION AGGREGATION IN ELECTIONS"

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APPENDIX: OMITTED PROOFS

THIS APPENDIX refers to equations, lemmas, and theorems in the main text (in print) by the respective numbers as assigned in the text.

PROOF OF LEMMA 4: First, assume that $V_A > t(A,\beta) > t(B,\beta) > V_B$ for every $\beta \in (0,1)$. Then, from Lemma 2, we have $\Theta(\beta) = \theta^*(\beta)$ for all $\beta \in (0,1)$. Moreover, from Corollary 1, we have $V_A > t(A,\beta) > \theta^*(\beta) > t(B,\beta) > V_B$. Note that $t(A,\beta) > t(B,\beta)$ implies that $\int_{\beta_b}^{\beta_a} f_u(z)\,dz > \int_{\beta_b}^{\beta_a} f_d(z)\,dz$. This has to be true as $\beta \to 0$ and $\beta \to 1$, which implies that $f_u(\beta) > f_d(\beta)$ at $\beta \in \{0,1\}$. From Lemma 2, it follows that $\Theta(0) = (0, V_B]$ and $\Theta(1) = [V_A, 1]$. Therefore, for any consequential rule θ, $\mathcal{B}(\theta) = \{\beta : \theta \in \Theta(\beta)\}$ consists of beliefs in the open interval $(0,1)$. For all such beliefs, $t(A,\beta) > \theta^*(\beta) > t(B,\beta)$, that is, P wins only in state A. For each P-trivial rule $\theta < V_B$, $\mathcal{B}(\theta) = \{0\}$. Since $t(A,0) = t(B,0) = V_B > \theta$ for such rules, P wins in both states. Similarly, for each Q-trivial rule $\theta > V_A$, $\mathcal{B}(\theta) = \{1\}$. Since $t(A,1) = t(B,1) = V_A < \theta$ for such rules, Q wins in both states.

Next, consider some regular β such that $t(A,\beta) < t(B,\beta)$ and consider the voting rule $\theta^*(\beta)$. By Corollary 1, $t(A,\beta) < \theta^*(\beta) < t(B,\beta)$, and the outcome is as described in the lemma. Similarly, for a regular β such that $t(A,\beta) = t(B,\beta) = t$, there is an equilibrium sequence with induced prior converging to β for all $\theta \in (0,1) \setminus \{t\}$, and the outcome is as detailed in the lemma. *Q.E.D.*

PROOF OF LEMMA 5: Suppose SPM holds. We have $t(A,\beta) - t(B,\beta) = (q_A - q_B)\gamma_I[\int_{\beta_b}^{\beta_a} h(\mu)\,d\mu] > 0$ by SPM. Moreover, $\frac{dt(S,\beta)}{d\beta} = q_S h(\beta_a) + (1 - q_S)h(\beta_b) > 0$. Since $t(S,\beta)$ is strictly monotonic, $t(S,0) = V_B$, and $t(S,1) = V_A$, we must $t(S,\beta) \in (V_B, V_A)$ for all $\beta \in (0,1)$ and $S \in \{A, B\}$.

Next assume that SPM fails. Since $V_A > V_B$, it cannot be the case that $f_u(\mu) \le f_d(\mu)$ for all $\mu \in (0,1)$. By continuity of f_u and f_d and by the assumption that $f_u(\mu)$ cannot be equal to $f_d(\mu)$ for any open interval, it must be the case that there are three numbers $0 < r < s < t < 1$ such that $h(s) = 0$, and either (i) $h(\mu) > 0$ in the interval (r,s) and $h(\mu) < 0$ in the interval (s,t) or (ii) $h(\mu) < 0$ in the interval (r,s) and $h(\mu) > 0$ in the interval (s,t). Without loss of generality, we consider the first case.

To show that there exists $\{q_A, q_B\}$ that leads to equal vote shares in the two states for some regular β, consider $q_A = \frac{1}{2} + \varepsilon$ and $q_B = \frac{1}{2} - \varepsilon$ for $0 < \varepsilon \le \frac{1}{2}$. Notice that for a given $\beta \in (0,1)$, $|\beta - \beta_s|$ is strictly increasing in ε and

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$\beta_a > \beta > \beta_b$. Now consider any $\beta_1 \in (r, s)$ and $\beta_2 \in (s, t)$. There must be some $\overline{\varepsilon} > 0$ such that for all $\varepsilon < \overline{\varepsilon}$, the following is true: at $\beta = \beta_1$, both β_a and β_b lie in (r, s), and at $\beta = \beta_2$, both β_a and β_b lie in (s, t). Define $Z(\beta) \equiv t(A, \beta) - t(B, \beta) = \varepsilon\gamma_I(\int_{\beta_b}^{\beta_a} h(\mu)\, d\mu)$. It is easy to see that $Z(\beta_1) > 0$ and $Z(\beta_2) < 0$. Since $Z(\beta)$ is continuous in β, for every $\varepsilon < \overline{\varepsilon}$, we must have some β_e such that $Z(\beta_\varepsilon) = 0$. Moreover, for $\beta = \beta_\varepsilon$, since $Z(\beta_\varepsilon) = 0$, it must be true that $\beta_b < s < \beta_a$. Therefore, $Z'(\beta_\varepsilon) = \varepsilon\gamma_I(\frac{d\beta_a}{d\beta} h(\beta_a) - \frac{d\beta_b}{d\beta} h(\beta_b)) < 0$. This establishes the uniqueness of β_ε in the range $[\beta_1, \beta_2]$ for the each $\varepsilon < \overline{\varepsilon}$. Now we have $Z(\beta) > 0$ for $[\beta_1, \beta_\varepsilon)$ and $Z(\beta) < 0$ for $(\beta_\varepsilon, \beta_2]$. Therefore, for each $\varepsilon < \overline{\varepsilon}$, the signal precision $(\frac{1}{2} + \varepsilon, \frac{1}{2} - \varepsilon)$ leads to a regular β_ε that satisfies $t(A, \beta_\varepsilon) = t(B, \beta_\varepsilon)$. *Q.E.D.*

PROOF OF PROPOSITION 1: First, consider some $\beta \in (0, 1)$ such that $t(A, \beta) = t(B, \beta)$. In other words, $\int_{\beta_b}^{\beta_a} h(t)\, dt = 0$. Since $\beta \in (0, 1)$, we have $\frac{d\beta_s}{d\beta} > 0$ for $s \in \{a, b\}$. After some algebra, we can show $\frac{d\beta_a}{d\beta} / \frac{d\beta_b}{d\beta} = \frac{\beta_a(1-\beta_a)}{\beta_b(1-\beta_b)}$ for any $\{q_A, q_B\}$. Now, $\frac{dt(A,\beta)}{d\beta} - \frac{dt(B,\beta)}{d\beta} = (q_A - q_B)\gamma_I(h(\beta_a)\frac{d\beta_a}{d\beta} - h(\beta_b)\frac{d\beta_b}{d\beta}) \neq 0$ by Assumption A3. Thus, Assumption A3 guarantees that if, for some $\beta \in (0, 1)$, we have $t(A, \beta) = t(B, \beta)$, then it must be the case that β is regular. The rest of the proof follows from Lemma 4 and the observation that $t(A, \beta) \lesseqgtr t(B, \beta) \Leftrightarrow (F(\beta_a, u) - F(\beta_b, u)) \lesseqgtr (F(\beta_a, d) - F(\beta_b, d))$. *Q.E.D.*

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