SUPPLEMENT TO "CONSUMPTION DYNAMICS DURING RECESSIONS"

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APPENDIX C: Model Extensions

IN THIS SECTION, WE EXTEND OUR MODEL to include two features which are particularly important for housing markets: rental markets and collateralized borrowing.

C.1. *Rental Markets*

In this section, we show that the inclusion of rental markets does not change any of the conclusions from our model. We choose not to include rental markets in our benchmark model for several reasons: (1) Since we want to use a broad measure of durables as our benchmark that encompasses all durable goods subject to lumpy adjustment costs, we must necessarily aggregate different durable goods that are somewhat different into one good. Overall, consumer durable spending is a larger fraction of total household durable spending than is residential investment, so it makes sense for our benchmark calibration to reflect something closer to consumer durable markets than housing markets. While rental markets are clearly important for housing, rental markets are not particularly important for consumer durables such as automobiles and furniture. (2) Our estimation procedure is based on the concept of the "gap" between a household's current durable holdings and those it would choose if it temporarily faced no adjustment costs. While this gap is well-defined for durable owners, it is not well-defined for renters. In general, even if temporarily facing no adjustment costs, households may still choose not to purchase. (3) Finally, even for renters, in reality moving is not costless. In order to remain computationally feasible, our extension with rental markets assumes that households can costlessly adjust their durable holdings when renting. Thus, this model likely underestimates true frictions to durable adjustment. Nevertheless, we now show that including this extension has only small effects on our results.

We extend the benchmark model by allowing households to rent durables. These durables depreciate fully each period and are more expensive due to the presence of higher depreciation, but they are not subject to adjustment costs. The household value function is then:

$$V(a_{-1}, d_{-1}, \eta) = \max[V^{\text{adjust}}(a_{-1}, d_{-1}, \eta), V^{\text{noadjust}}(a_{-1}, d_{-1}, \eta),$$
$$V^{\text{rent}}(a_{-1}, d_{-1}, \eta)],$$

with

$$V^{\text{rent}}(a_{-1}, d_{-1}, \eta) = \max_{c,d,a} \frac{\left[c^v d^{1-v}\right]^{1-\gamma}}{1-\gamma} + \beta E_\varepsilon V\left(a, 0, \eta'\right),$$

s.t.

$$c = wh\eta(1-\tau) + (1+r)a_{-1} + d_{-1}(1-\delta_d)$$
$$\quad - r^d d - a - f^d(1-\delta_d)d_{-1} - f^t wh\eta,$$

$$a > -(1-\theta)d,$$

$$\log \eta' = \rho_\eta \log \eta + \varepsilon \quad \text{with} \quad \varepsilon \sim N(0, \sigma_\eta),$$

where r^d is the rental rate on durables and the 0 in the continuation value reflects the fact that households who choose to rent today will start the following period with no durable holdings. The expressions for $V^{\text{adjust}}(a_{-1}, d_{-1}, \eta)$ and $V^{\text{noadjust}}(a_{-1}, d_{-1}, \eta)$ are unchanged. In addition to estimating the other parameters of the model, we pick the value of r^d to target an owner occupancy rate of 0.65. We show the estimated distribution and hazard for the model with rental markets in Figures S.1 and S.2. Just as in our baseline model, we are able to well explain the empirical data.
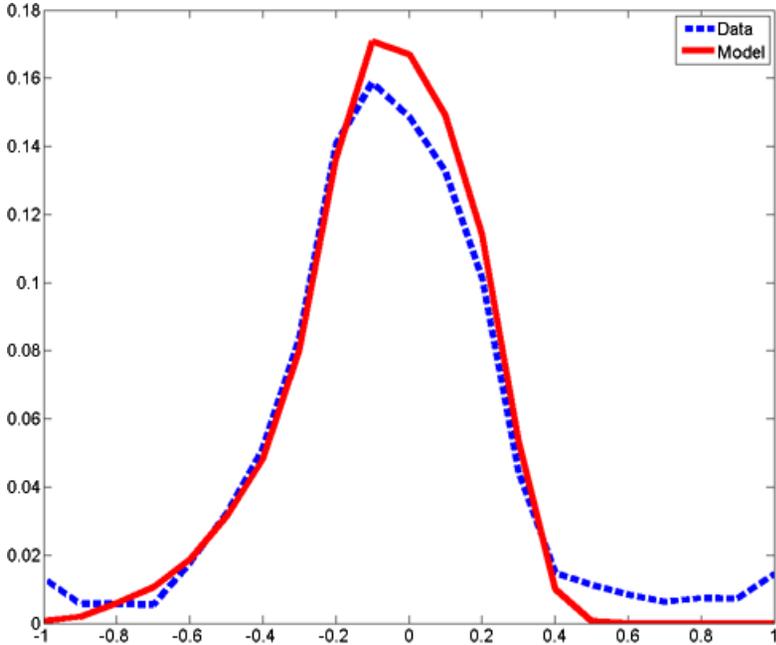

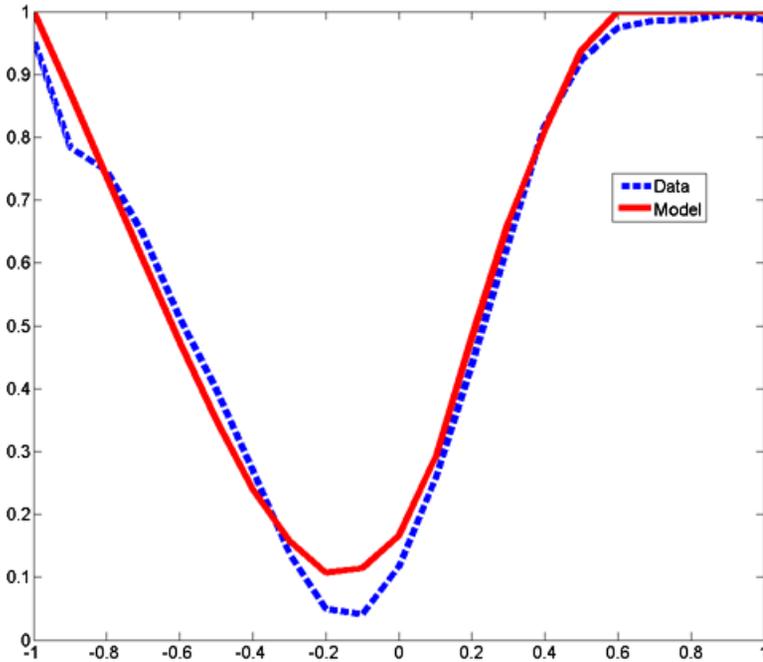FIGURE S.1.—Gap distribution in model with rental markets.

FIGURE S.2.—Predicted and actual hazard: model with rental markets.

After estimating the rental market model, we explore its aggregate impli-
cations. Given gaps and hazards in the PSID data from 1999 to 2011, we can
use (1) to compute an implied responsiveness across time. Again we find large
variation that is strongly procyclical.

In addition, we can calculate the true impulse responses to shocks in our
structural model. Just as in the benchmark model, the model with rental mar-
kets delivers a strong state-dependent IRF. Figure S.3 shows the IRF on im-
pact and Figure S.4 shows the full impulse response to an aggregate income
shock. Both figures show that the response to an aggregate income shock
in a boom period (1999) is much stronger than the same income shock in
a recession (2009). For brevity, we do not show results for other aggregate
shocks or for alternative measures of responsiveness, but they deliver similar
results.

C.2. *Collateralized Borrowing*

In this section, we explore the implications of a second extension that is
important for understanding some durable markets: the role of collateralized
borrowing. In our benchmark model, we assume that $\theta = 1$, so that households
cannot borrow against their durables. In this section, we relax that assumption.
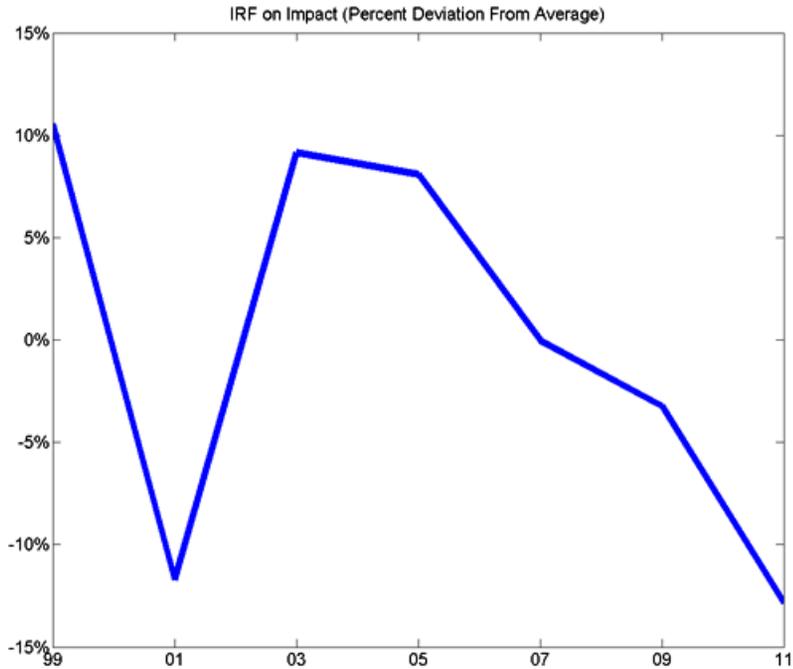
FIGURE S.3.—IRF on impact: model with rental markets.

Since collateralized borrowing is particularly important for housing, we think of this robustness check as one that applies more to housing markets than to broad durable spending.

In this extension, we assume that households can borrow up to 80% of the value of their durables; that is, we assume that $\theta = 0.2$ so that households must put down a 20% downpayment. The reason that we do not include collateralized borrowing in our benchmark specification is because technical considerations force us to admit this borrowing in a way that we view as somewhat empirically unrealistic. Since it is not feasible to have a separate adjustment cost on durable equity together with a fixed cost of durable adjustment, we must assume that durable equity can be adjusted costlessly. That is, the collateralized borrowing model which is numerically feasible must allow for costless refinancing. Clearly, households in the real world cannot costlessly extract equity from their durables and do not refinance continuously. Thus, the specification with collateralized borrowing with frictionless equity adjustment substantially overstates households' ability to smooth idiosyncratic earnings shocks and substantially understates the illiquidity of durable wealth.

Nevertheless, our basic message goes through: even with costless equity adjustment, durable responsiveness remains procyclical. To solve the model with
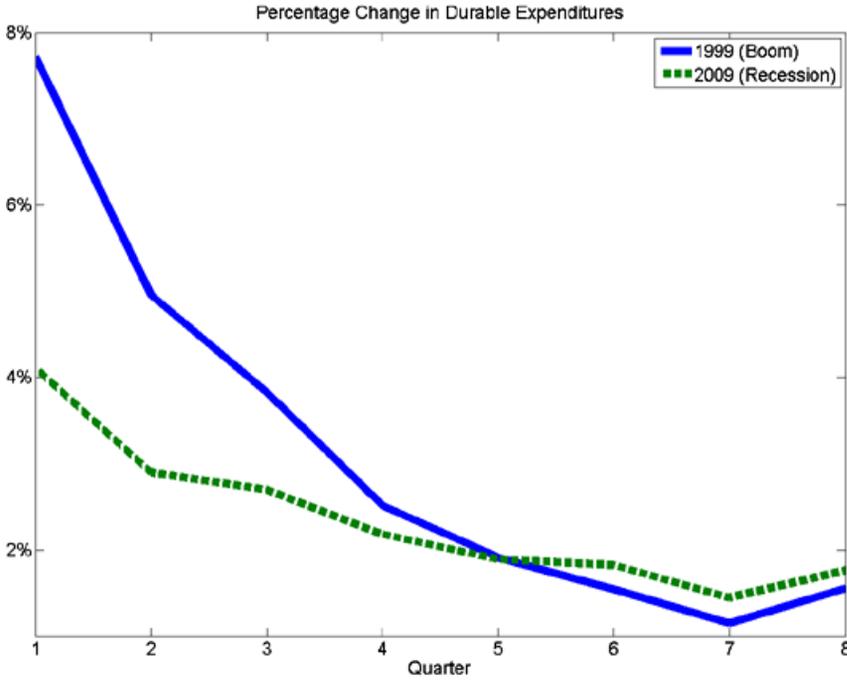
FIGURE S.4.—IRF in boom and bust: model with rental markets.

$\theta = 0.2$, we reformulate the problem using "voluntary equity" as in Diaz and Luengo-Prado (2010). This rectangularizes the constraint set of households and simplifies the solution of the model. The reformulated model is solved identically to the model in our baseline results. Figures S.5 and S.6 show that the estimated gaps and hazards are a good fit in the model and data.

Figures S.7 and S.8 show that we again get an impulse response function that is strongly procyclical: during booms, aggregate durable spending is much more responsive to income shocks than during recessions.

APPENDIX D: MODEL SOLUTION AND NUMERICAL METHODS

We describe the solution for the model with fixed costs in general equilibrium. The solution for the model with no fixed costs and partial equilibrium models is similar. A representative firm rents capital and labor and its first order conditions pin down these prices:

$$w_t = (1 - \alpha) Z_t K_t^{\alpha} H^{1-\alpha},$$
$$r_t = \alpha Z_t K_t^{\alpha-1} H^{1-\alpha} - \delta_k,$$

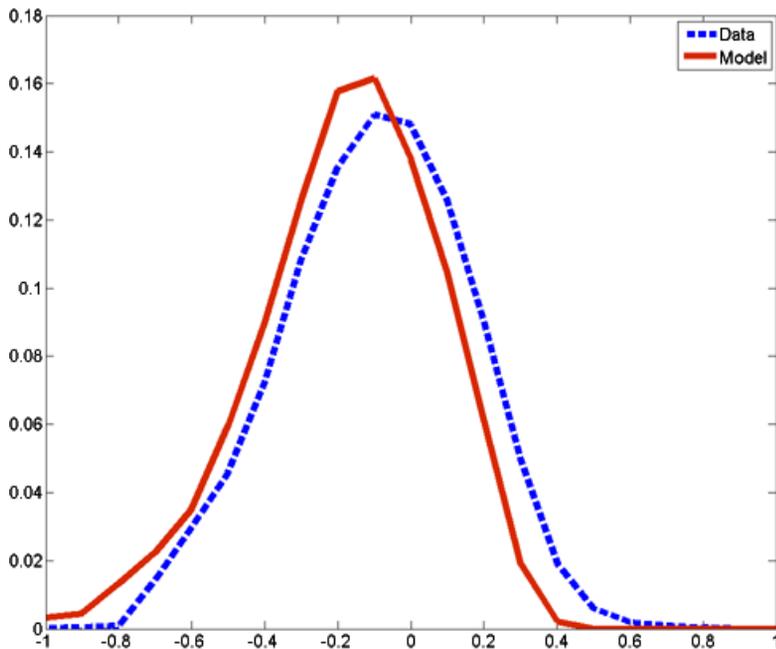FIGURE S.5.—Gap distribution in model with collateral.

where, in equilibrium, aggregate variables satisfy

$$K_t = \int a_{t-1}^i,$$

$$D_t = \int d_t^i,$$

$$C_t = \int c_t^i,$$

$$A_t = \int A(d^i, d_{-1}^i),$$

$$H = \int h \eta_t^i,$$

together with an aggregate resource constraint:

$$C_t + D_t + K_{t+1} + A_t$$
$$= Z_t K_t^\alpha H^{1-\alpha} + (1 - \delta_k)K_t + (1 - \delta_d)D_{t-1}.$$
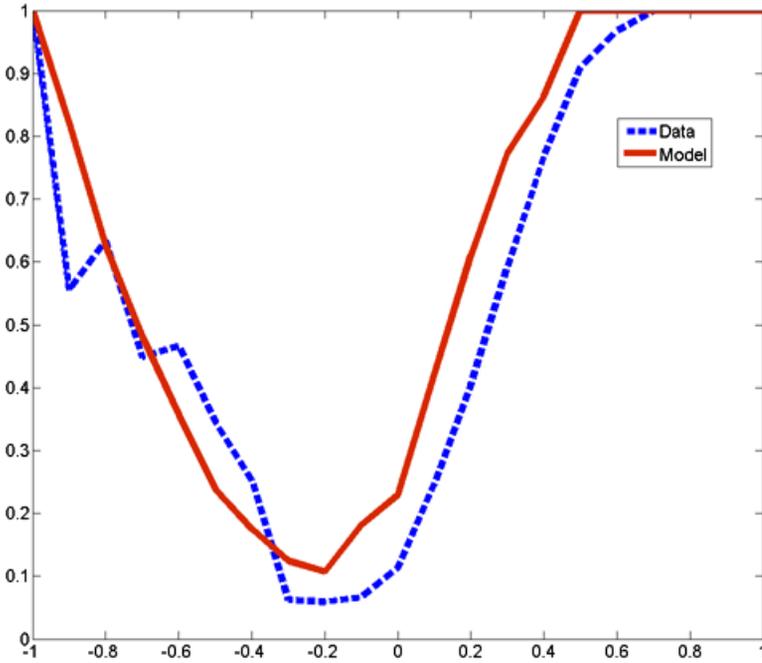
FIGURE S.6.—Hazard in model with collateral.

Aggregate productivity evolves as an AR process:

$$\log Z_t = \rho_Z \log Z_{t-1} + \xi_t.$$

Solving the household problem requires forecasting aggregate prices and thus the aggregate capital stock, which is determined by the continuous distribution of household states, so as usual, solving the model requires making computational assumptions. Following Krusell and Smith (1998), we conjecture that after conditioning on aggregate productivity, aggregate capital is a linear function of current aggregate capital:[1]

$$K_{t+1} = \gamma_0(Z) + \gamma_1(Z)K_t.$$

Given these assumptions, the household's recursive problem is given by

$$V(a_{-1}, d_{-1}, \eta; Z, K)$$
$$= \max\left[V^{\text{adjust}}(a_{-1}, d_{-1}, \eta; Z, K), V^{\text{noadjust}}(a_{-1}, d_{-1}, \eta; Z, K)\right],$$

[1]The forecasting rule might also depend on the previous durable stock. An earlier version of this paper found that this added little explanatory power and had substantial computational cost.
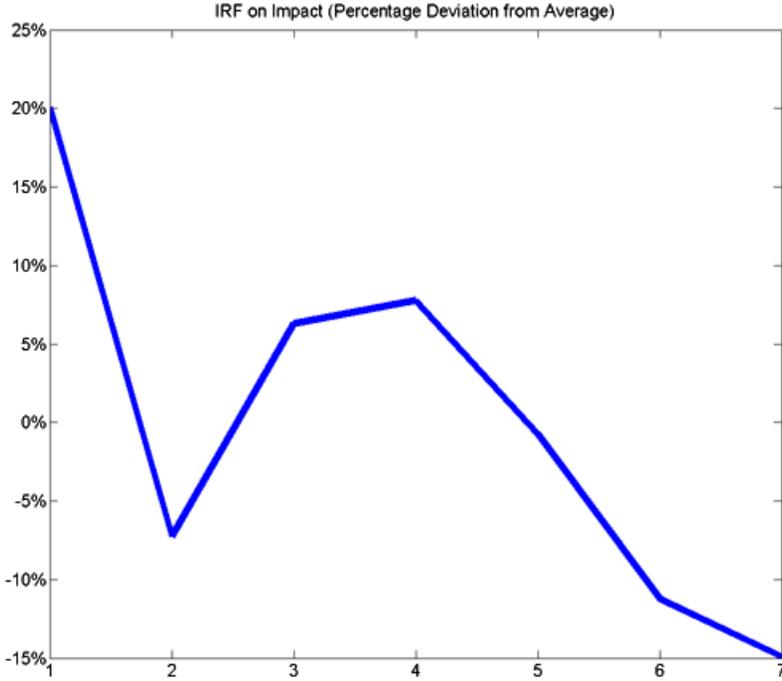
FIGURE S.7.—IRF on impact: model with collateral.

with

$$V^{\text{adjust}}(a_{-1}, d_{-1}, \eta; Z, K)$$

$$= \max_{c,d,a} \frac{\left[c^v d^{1-v}\right]^{1-\theta}}{1 - \theta} + \beta E_{\varepsilon,\xi} V(a, d, \eta'; Z', K'),$$

s.t.

$$c = wh\eta + (1 + r)a_{-1} + d_{-1}(1 - \delta_d) - d$$
$$- a - F^d(1 - \delta_d)d_{-1} - F^t wh\eta,$$

$a > 0$;   equilibrium conditions and prod. processes,

$$V^{\text{noadjust}}(a_{-1}, d_{-1}, \eta; Z, K)$$

$$= \max_{c,a} \frac{\left[c^v d^{1-v}\right]^{1-\theta}}{1 - \theta} + \beta E_{\varepsilon,\xi} V\big(a, d_{-1}\big(1 - \delta_d(1 - \chi)\big), \eta'; Z', K'\big),$$

s.t.

$$c = wh\eta + (1 + r)a_{-1} - \delta_d \chi d_{-1} - a,$$

$a > 0$;   equilibrium conditions and prod. processes.

Percentage Change in Durable Expenditures

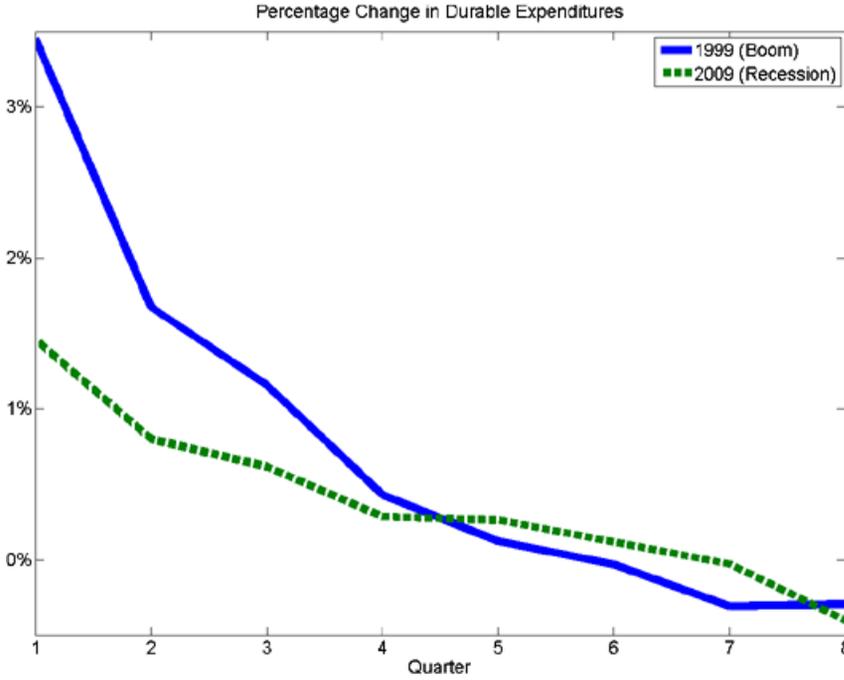FIGURE S.8.—IRF boom and bust: model with collateral.

We begin by substituting the budget constraint into the utility function to eliminate non-durable consumption as a choice-variable. We discretize η and Z using the algorithm of Tauchen (1986). Furthermore, we note that, conditional on adjusting, households do not care separately about the value of a_{-1}, d_{-1} and care only about their net-cash-on-hand $x = (1+r)a_{-1} + d_{-1}(1 - \delta_d) - f^d(1-\delta_d)d_{-1} - f^t wh\eta$, so we can eliminate one state-variable and rewrite the value function when adjusting as

$$\widetilde{V}^{\text{adjust}}(x_{-1}, \eta; Z, K) = \max_{c,d,a} \frac{\left[c^v d^{1-v}\right]^{1-\theta}}{1-\theta} + \beta E_{\varepsilon, \xi} V\left(a, d, \eta'; Z', K'\right),$$

s.t.

$$c = wh\eta + x_{-1} - d - a.$$

Since the choice when adjusting is two-dimensional, it takes substantially longer to find the optimal policy for a given state than it does to solve for the policy when not adjusting. Thus, eliminating a state-variable from this problem dramatically speeds calculations. Given these value functions, we approx-

imate $\widetilde{V}^{\mathrm{adjust}}(:, \eta; Z, :)$ and $V^{\mathrm{noadjust}}(:, :, \eta; Z, :)$ as multilinear[2] functions in the continuous idiosyncratic states and one continuous aggregate state. Initializing the grid for aggregate capital requires knowledge of the steady-state level of capital, so before solving the model with aggregate shocks, we solve for the steady-state of the model. The solution method is similar and simpler than the solution with aggregate shocks, so we only describe the latter.

Given an initial guess for the value functions and transition function, we solve for the optimal two-dimensional policy functions using a Nelder–Meade algorithm initialized from three different starting values to reduce the problems of finding local maxima in the policy function. The values of adjusting and not adjusting are compared, to generate the overall policy function and to update the overall value function. We iterate until the separate value functions change by less[3] than 0.001. Once the value functions have converged, we then solve for the optimal policy function an additional time on a finer grid, to use for simulation.

We then simulate a panel of households and compute the evolution of the aggregate capital stock to update the aggregate transition rule $K' = \gamma_0(Z) + \gamma_1(Z)K$. We then repeat the above procedure until the coefficients in the value function change by less than 1%. Once the transition rule has converged, aggregate forecasts are highly accurate, with $R^2 > 0.999$. We have experimented with including the aggregate durable stock in the transition rule and found that it did little to improve forecasts, at considerable additional computational cost.

For the benchmark results partial equilibrium results, we use 132 grid points each for interpolating a_{-1} and d_{-1}, 100 grid points for approximating x_{-1}, and we discretize our shocks using 7 grid points for idiosyncratic productivity and 21 grid points for the aggregate shock. In the general equilibrium model, we use 25 grid points for interpolating a_{-1} and d_{-1}, 15 grid points for aggregate productivity, 7 grid points for idiosyncratic productivity, and 5 grid points for interpolating K.

We construct a finer grid with 90 grid points for a_{-1} and d_{-1} to compute the final policy function used for simulation in the GE model and a grid with 400 grid points for our partial equilibrium model. Thus, our fine policy function must be solved for approximately 3 million grid points in GE and their associated expectations. In partial equilibrium, our policy function uses almost 20 million grid points. (A large advantage of partial equilibrium is that since we need not solve for the aggregate transition rule, we can make the inner solution of the model more accurate.) Our simulation uses 10,000 households for 3000 periods with an initial burn-in of 250 periods.

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[2]We have experimented with cubic spline interpolation and have found that the speed advantages of linear interpolation appear to be worth potential decreases in accuracy (especially since fixed costs imply that the value functions may not be well approximated by cubic splines).

[3]Finer converge values did not appear to affect the results.

In order to simulate U.S. time-series data and compute impulse response functions in our models, we feed aggregate shocks into the model picked to match U.S. data. For example, if we want to calculate the impulse response of the economy to an income shock in 1990q1, we perform the following exercise. First, compute HP filtered log real GDP for the actual U.S. economy: $Y_{1990q1}, \ldots, Y_{2013q4}$. Second, simulate a burn-in period of random aggregate income shocks. Then feed the model aggregate shocks $Y_{1990q1}, \ldots, Y_{2013q4}$ from 1960q1–2013q4 and compute implied durable investment $\text{ID}_{1960q1}^{\text{noimpulse}}, \ldots, \text{ID}_{2013q4}^{\text{noimpulse}}$. Repeat the simulation with the same sequence of shocks but assume that there is an additional 1% impulse to income in 1990q1 which dies off at rate 0.87 and compute the sequence of investment rates $\text{ID}_{1990q1}^{\text{withimpulse}}, \ldots, \text{ID}_{2013q4}^{\text{withimpulse}}$. This delivers an estimate of the impulse response function to an income shock in 1990q1: $\text{IRF}_{t=1}^{1990q1} = \log \text{ID}_{1990q1}^{\text{withimpulse}} - \log \text{ID}_{1990q1}^{\text{noimpulse}}$, $\text{IRF}_{t=2}^{1990q1} = \log \text{ID}_{1990q2}^{\text{withimpulse}} - \log \text{ID}_{1990q2}^{\text{noimpulse}}$, etc. Finally, to account for random sampling error in both the distribution of idiosyncratic shocks and the random path of past aggregate shocks, we repeat this 250 times and average the results across simulations.

APPENDIX E: TIME-SERIES EVIDENCE

In this section, we argue that time-series data on durable spending provide additional support for our theoretical model with fixed costs of durable adjustment. We first show that the model with fixed costs of durable adjustment delivers standard business cycle moments that better fit the data than a frictionless model with durable consumption. Since these moments do not condition on the state of the business cycle, we refer to our model as better matching *unconditional* business cycle moments. While the frictionless model is not a good fit to the data, it is straightforward to introduce convex adjustment costs into an RBC model to perfectly match the unconditional behavior of the model with fixed adjustment costs. However, we next show that even though an RBC model with convex adjustment costs and a model with fixed costs of adjustment have observationally equivalent unconditional behavior, they have very different implications for the *conditional* behavior of durable spending over the business cycle and that U.S. time-series data strongly support the model with fixed costs.

Table S.I reports the unconditional business cycle moments from our model and how they compare to data.[4] In addition, we report results for representative agent RBC models with and without convex adjustment costs, which we

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[4]See Appendix A for data definitions. We define durable expenditures as NIPA durable expenditures + residential investment. The BEA treats durable and residential investment differently, including housing services in GDP while excluding durable services. In both our model and data analysis, we define GDP as consumer durable expenditures + private domestic investment + non-durable expenditures (excluding housing services).

TABLE S.I

BUSINESS CYCLE STANDARD DEVIATIONS (RELATIVE TO Y)

	Data	RBC	RBC W/Adj.	W/Fixed Costs
Durable	3.04	19.56	2.58	2.58
Non-Durable	0.57	0.41	0.63	0.68
Investment	2.24	8.16	2.36	2.36

describe in Appendix F. Clearly, the representative agent model with friction-less durable adjustment is a poor fit to the data. The volatility of both capital investment and durable spending is substantially too large, while the volatility of non-durable consumption is too low.

The reason that investment in the frictionless RBC model is too volatile is because this model features the comovement problem identified in Greenwood and Hercowitz (1991) and further explored in Fisher (1997). A change in productivity changes the relative returns to saving in productive capital versus durables. An increase in productivity makes it more valuable to shift saving into productive capital, and the additional output produced can later be used to finance durable consumption. This generates a strong negative correlation between durable expenditures and investment in productive capital in the models with no adjustment costs and increases the volatility of both variables. The introduction of adjustment costs breaks this comovement problem and substantially dampens the volatility of investment. In addition, fixed costs of adjustment make a fraction of household wealth illiquid, which amplifies the volatility of non-durable consumption for the reasons explored in Kaplan and Violante (2014).

The poor fit of frictionless multisector RBC models is well known, so it is not surprising that we reach a similar conclusion. Moreover, quadratic adjustment costs can substantially improve the fit of the representative agent model.[5] The third column of Table S.I shows that we can pick adjustment costs in an RBC model to generate exactly the same volatility of investment and durable spending as in the model with fixed costs of adjustment.[6] This shows that while adjustment costs are important for matching the volatility of durable spending in the data, the form of the adjustment costs cannot be identified from unconditional business cycle movements: quadratic adjustment costs and fixed costs

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[5]Smooth adjustment costs can be microfounded in various ways: Gomme, Kydland, and Rupert (2001) added time-to-build to a disaggregated model, while Davis and Heathcote (2005) introduced a fixed factor of production.

[6]Since we only pick two adjustment cost parameters, we can directly target the volatility of durable and capital investment, and we get a slightly different number for the volatility of non-durable consumption. (But this value is close enough that again it would not provide any direct identification of the adjustment cost specification.)

of adjustment can produce exactly the same dampening of durable spending, on average.

However, we now argue that the volatility of durable expenditures *conditional* on the aggregate state of the business cycle can provide additional identification that supports the presence of fixed adjustment costs. In particular, aggregate durable expenditures are systematically more volatile during expansions than they are during slumps. This arises naturally in the model with fixed costs of durable adjustment, since we have shown that that model generates a procyclical IRF, but not in models with convex adjustment costs. Beyond supporting the fixed cost specification, we believe this result is interesting in its own right for optimal policy design. Since policies are not implemented randomly over the business cycle, conditional responses are likely to be more informative for the effects of policy than are unconditional responses.

To show that the volatility of durable expenditures rises during booms, we follow Bachmann, Caballero, and Engel (2013) and estimate a two-stage time-series model. In the first stage, we estimate an AR process for durable expenditures. Then, in the second stage, we regress the absolute value of the residuals from the first stage on the average of lagged durable expenditures to assess whether residual variance is different during booms than it is during recessions.

We now show that durable expenditures exhibit conditional heteroscedasticity, rising in booms and falling in recessions. As in Bachmann, Caballero, and Engel (2013), we assume that our series of interest can be described by an AR process:

$$x_t = \sum_{j=1}^{p} \phi_j x_{t-j} + \sigma_t e_t,$$

where $x_t \equiv \frac{I_D}{D}$ is durable expenditures divided by the durable stock,[7] $e_t \sim$ i.i.d. with zero mean and unit variance, and

$$\sigma_t = \alpha + \eta \overline{x}_{t-1},$$

$$\overline{x}_{t-1} = \frac{1}{k} \sum_{j=1}^{k} x_{t-j}.$$

That is, we allow the variance of the residuals in the AR process for durable expenditures to vary with past durable expenditures. This specification implies that the impulse response of x to e on impact at time t is given by $\alpha + \eta \overline{x}_{t-1}$. If $\eta = 0$, then the impulse response of x to e does not vary with past durable

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[7]The ratio of durable expenditures to the stock is stationary, while durable expenditures are not.

TABLE S.II

CONDITIONAL HETEROSCEDASTICITY DATA

Series	Total Dur. Exp.	Resid	Cons. Dur.	Non Dur.	GDP	TFP	FF
η	0.05	0.03	0.04	−0.003	0.002	−0.11	0.03
t-η	2.63	2.04	1.52	−1.33	0.84	−1.40	0.83
Bootstrap p-value ($\eta > 0$)	0.007	0.03	0.04	0.83	0.25	0.86	0.56
$\pm(\sigma_{max}/\sigma_{min})$	2.82	2.52	1.67	1.85	1.25	1.61	3.89
$\pm(\sigma_{95}/\sigma_{5})$	1.82	1.65	1.59	1.39	1.20	1.35	1.12
No. obs.	192	192	192	192	192	192	630

expenditures, while $\eta > 0$ implies that the IRF rises with lagged expenditures.

We estimate the time-series model using quarterly data on $\frac{I_D}{D}$ from 1960 to 2010. The estimation follows a two-stage procedure. In the first stage, we estimate the AR process via OLS to obtain residuals ε_t. The second stage then estimates by OLS η using

$$|\varepsilon_t| = \left(\frac{2}{\pi}\right)^{1/2}(\alpha + \eta\overline{x}_{t-1}) + \text{error}.$$

We repeat the estimation for all combinations of $p, k \leq 12$ and choose the best fit, p^*, k^* using AIC. For more details on the time-series model, see Bachmann, Caballero, and Engel (2013).[8] Table S.II contains the time-series estimates. Both total durable expenditures as well as residential investment exhibit strongly significant[9] $\eta > 0$. The estimated $\eta > 0$ for consumer durables, but it is only marginally significant. While $\eta > 0$ implies that there is a statistically significant increase in the IRF with lagged expenditures, it does not imply that the increase is economically significant. In the fourth and fifth rows of Table S.II, we report statistics that show that there is quantitatively large variation in the IRF across time. The maximum IRF is 2.82 times larger than the minimum IRF, while the 95th percentile is 1.82 times higher than the 5th percentile. Thus, the estimated heteroscedasticity is both statistically and economically significant.

It is important to note that while we have interpreted conditional heteroscedasticity as a time-varying impulse response to aggregate shocks with a constant variance, an alternative interpretation is that aggregate shocks themselves are larger during booms than during recessions. We test for this by

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[8]In addition to the model we presented, their paper presents an alternative time-series model. We obtained similar results for this model, so for brevity we did not report these results.

[9]The bootstrap p-value row constructs bootstrapped p-values for $\eta > 0$, accounting for the fact that errors in the first-stage estimation increase the standard errors in the second stage.

TABLE S.III

CONDITIONAL HETEROSCEDASTICITY MODELS

Series	RBC	RBC Adj. Costs	W/ Fixed Costs
η	-0.03	0.01	0.15
t-η	-0.52	0.62	3.30
$\pm(\sigma_{max}/\sigma_{min})$	1.19	1.17	2.22
$\pm(\sigma_{95}/\sigma_5)$	1.12	1.12	1.73

performing the same regressions on Baxter–King bandpass filtered GDP.[10] Table S.II shows that, as in the estimates in Bachmann, Caballero, and Engel (2013), there is no evidence of conditional heteroscedasticity for GDP. Presumably the aggregate shocks hitting Y should be similar to the aggregate shocks hitting D, so we interpret our results as evidence that it is not the shocks that drive heteroscedasticity, and is rather the mechanism that translates those shocks into durable expenditures that drives our estimates. As further evidence for this point, we also estimate the time-series model for changes in TFP[11] as well as the Federal Funds rate.[12]

In addition to these empirical estimates, we also compute heteroscedasticity estimates for our simulated models. Since we know that the true shocks in the model follow an AR(1) process, we report benchmark results restricted $k = 1$, $p = 1$, but results are not sensitive to this restriction. Table S.III shows that the frictionless model does not generate procyclical IRFs. If anything, the model without fixed costs implies $\eta < 0$. In contrast, the model with fixed costs exhibits conditional heteroscedasticity that is in line with the empirical estimates. The estimated $\eta > 0$, and the time-variation in the impulse response on impact, are similar to those in the data.

While fixed costs induce procyclical impulse response functions, they need not be the only mechanism that can generate these dynamics. Another possible explanation for a procyclical IRF is the presence of collateral constraints. If collateral constraints tighten during recessions, then durable expenditures may respond less to shocks during recessions than during booms. We have tested for this using the models with collateral constraints in Appendix C. However, we find that this mechanism is quantitatively weak. This is partially because the response to time-varying credit conditions is solely one-sided. A tightening of credit conditions has a direct effect on households that want to increase durables, but has no direct effect on households that want to sell durables.

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[10]Unlike expenditure rates, GDP is nonstationary and so must be filtered. Using alternative filters did not substantively change the results.

[11]Available at http://www.frbsf.org/economics/economists/jfernald/quarterly_tfp.xls.

[12]To increase the sample size, we use monthly FF rates. While we could also use FF residuals or surprises, it is likely that the actual rate is more relevant for durable purchases, as households should respond to both the anticipated and unanticipated component.

This substantially dampens the scope for time-varying IRFs. Furthermore, we find no empirical support for such asymmetric heteroscedasticity.

APPENDIX F: REPRESENTATIVE AGENT RBC MODEL

We describe here a representative agent version of our durable model with quadratic adjustment costs. The time-series properties of this model are compared to our model with fixed costs in Appendix E. The planner problem is given by

$$\max_{C_t, D_{t+1}, K_{t+1}} E \sum \beta^t \left(\frac{\left[(C_t)^v (D_{t+1})^{1-v} \right]^{1-\gamma} - 1}{1-\gamma} \right),$$

$$C_t + D_{t+1} + K_{t+1} = Z_t K_t^\alpha H^{1-\alpha} + (1 - \delta_k) K_t + (1 - \delta_d) D_t$$

$$- \frac{c_k}{2} \left(\frac{K_{t+1}}{K_t} - 1 \right)^2 K_t - \frac{c_d}{2} \left(\frac{D_{t+1}}{D_t} - 1 \right)^2 D_t.$$

The RBC model has the following first-order conditions:

$$C_t: \quad v C_t^{v-1} D_t^{1-v} \left[C_t^v D_t^{1-v} \right]^{-\gamma} = \lambda_t,$$

$$K_{t+1}: \quad \lambda_t \left(1 + c_k \left(\frac{K_{t+1}}{K_t} - 1 \right) \right)$$

$$= \beta E \left[\lambda_{t+1} \left(\alpha \frac{Y_{t+1}}{K_{t+1}} + (1 - \delta_k) \right. \right.$$

$$\left. \left. + c_k \frac{(K_{t+2} - K_{t+1})}{K_{t+1}} + \frac{c_k}{2} \frac{(K_{t+2} - K_{t+1})^2}{K_{t+1}^2} \right) \right],$$

$$D_t: \quad \lambda_t \left(1 + c_d \left(\frac{D_{t+1}}{D_t} - 1 \right) \right)$$

$$= (1 - v) D_t^{-v} C_t^v \left[C_t^v d_t^{1-v} \right]^{-\gamma} + \beta E \left[\lambda_{t+1} (1 - \delta_d) \right.$$

$$\left. + c_d \frac{(D_{t+2} - D_{t+1})}{D_{t+1}} + \frac{c_d}{2} \frac{(D_{t+2} - D_{t+1})^2}{D_{t+1}^2} \right].$$

The steady-state equations (fixing $Z = 1$) are then given by

$$H = 1/3,$$
$$Y = K^\alpha H^{1-\alpha},$$
$$\lambda = v C^{v-1} D^{1-v} \left[C^v D^{1-v} \right]^{-\gamma},$$

$$1 = \beta\left[\left(\alpha\frac{Y}{K} + (1 - \delta_k)\right)\right],$$

$$\lambda = (1 - v)D^{-v}C^{v}\left[C^{v}D^{1-v}\right]^{-\gamma} + \beta\lambda(1 - \delta_d),$$

$$C = ZK^{\alpha}H^{1-\alpha} - \delta_k K - \delta_d D.$$

Solving for the steady-state gives

$$\frac{C}{D} = \frac{v\left(1 - \beta(1 - \delta_d)\right)}{1 - v},$$

$$K = \left[\frac{1}{\alpha}\left[\frac{1}{\beta} - (1 - \delta_k)\right]\right]^{1/(\alpha-1)},$$

$$Y = H^{1-\alpha}K^{\alpha},$$

$$D = \frac{Y - \delta_k K}{\dfrac{v\left(1 - \beta(1 - \delta_d)\right)}{1 - v} + \delta_d}.$$

We pick all parameters of the model to be identical to the benchmark model with fixed costs of durable adjustment. In the frictionless model, $c_k = c_d = 0$, and in the model with adjustment costs, we pick these parameters to reproduce the volatility of durable expenditures and capital investment in our benchmark model.

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