Online Appendix for "Monte Carlo Confidence Sets for Identified Sets"

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G Additional Results for the simulations and applications

G.1 Example 1: missing data

See Appendix A.1 of the paper for the SMC implementation details for this example.

Additional simulation results: Here we present additional simulation results for the missing data example using (i) a likelihood criterion and curved prior and (ii) a continuously-updated GMM criterion and flat prior. For the "curved" prior, we take $\pi(\mu, \eta_1, \eta_2) = \pi(\mu|\eta_1, \eta_2)\pi(\eta_1)\pi(\eta_2)$ with $\pi(\eta_1) = \text{Beta}(3, 8)$, $\pi(\eta_2) = \text{Beta}(8, 1)$, and $\pi(\mu|\eta_1, \eta_2) = U[\eta_1(1-\eta_2), \eta_2 + \eta_1(1-\eta_2)]$. Figure 7 plots the marginal curved priors for η_1 and η_2.

Results for the likelihood criterion with curved prior are presented in Table 6, and are very similar to those presented in Table 1, though the coverage of percentile-based CSs is worse here for the partially identified cases ($c = 1, 2$). Results for the CU-GMM criterion and flat prior are presented in Table 7. Results for Procedures 2 and 3 are very similar to the results with a likelihood criterion and show coverage very close to nominal coverage in point point- and partially-identified cases. Here procedure 1 does not over-cover in the point-identified case because the weighting matrix is singular when the model is identified, which forces the draws to concentrate on the region in which $\eta_2 = 1$. This, in turn, means projection is no longer conservative in the point-identified case, though it is still very conservative in the partially-identified cases. Percentile CSs again under-cover badly in the partially-identified case.

G.2 Example 2: entry game with correlated shocks

See Appendix A.2 of the paper for the implementation details for this example.

	$\eta_2 = 1 - \frac{2}{\sqrt{n}}$						$\eta_2 = 1 - \frac{1}{\sqrt{n}}$						$\eta_2 = 1$ (Point ID)					
	0.90		0.95		0.99		0.90		0.95		0.99		0.90		0.95		0.99	
	$\widehat{\Theta}_\alpha$ (Procedure 1) for Θ_I																	
100	.911	—	.957	—	.990	—	.898	—	.950	—	.989	—	.985	—	.994	—	.999	—
250	.906	—	.954	—	.993	—	.899	—	.951	—	.991	—	.992	—	.997	—	1.000	—
500	.908	—	.956	—	.992	—	.910	—	.957	—	.991	—	.994	—	.998	—	1.000	—
1000	.896	—	.948	—	.989	—	.905	—	.954	—	.989	—	.996	—	.999	—	1.000	—
	$\widehat{M}_\alpha$ (Procedure 2) for M_I																	
100	.890	[.33,.67]	.948	[.31,.69]	.990	[.29,.71]	.911	[.37,.63]	.952	[.36,.64]	.992	[.33,.67]	.912	[.41,.58]	.961	[.40,.60]	.990	[.37,.63]
250	.905	[.39,.61]	.953	[.38,.62]	.991	[.36,.64]	.913	[.42,.58]	.957	[.41,.59]	.992	[.39,.61]	.917	[.45,.55]	.962	[.44,.56]	.993	[.42,.58]
500	.913	[.42,.58]	.957	[.41,.59]	.992	[.40,.60]	.915	[.44,.56]	.955	[.43,.57]	.993	[.42,.58]	.919	[.46,.54]	.959	[.46,.54]	.993	[.44,.56]
1000	.898	[.44,.56]	.948	[.44,.56]	.988	[.43,.57]	.898	[.46,.54]	.948	[.45,.55]	.990	[.44,.56]	.913	[.47,.53]	.954	[.47,.53]	.992	[.46,.54]
	$\widehat{M}_\alpha^\chi$ (Procedure 3) for M_I																	
100	.922	[.32,.68]	.950	[.31,.69]	.989	[.28,.72]	.912	[.37,.63]	.942	[.36,.64]	.989	[.33,.67]	.909	[.42,.58]	.941	[.40,.59]	.985	[.38,.62]
250	.910	[.39,.61]	.949	[.38,.62]	.989	[.36,.64]	.913	[.42,.58]	.951	[.41,.59]	.990	[.39,.61]	.891	[.45,.55]	.952	[.44,.56]	.992	[.42,.58]
500	.898	[.42,.58]	.952	[.41,.59]	.992	[.40,.60]	.910	[.44,.56]	.947	[.44,.56]	.990	[.42,.58]	.911	[.46,.54]	.948	[.46,.54]	.992	[.44,.56]
1000	.892	[.44,.56]	.944	[.44,.56]	.989	[.43,.57]	.892	[.46,.54]	.944	[.45,.55]	.986	[.45,.55]	.902	[.48,.52]	.941	[.47,.53]	.987	[.46,.54]
	$\widehat{M}_\alpha^{proj}$ (Projection) for M_I																	
100	.971	[.30,.70]	.988	[.29,.71]	.997	[.26,.74]	.966	[.35,.65]	.987	[.33,.67]	.997	[.31,.69]	.985	[.38,.62]	.994	[.36,.64]	.999	[.34,.66]
250	.971	[.37,.63]	.987	[.36,.64]	.999	[.34,.66]	.969	[.40,.60]	.986	[.39,.61]	.998	[.37,.62]	.992	[.42,.58]	.997	[.41,.59]	1.000	[.39,.61]
500	.975	[.41,.59]	.988	[.40,.60]	.998	[.39,.61]	.972	[.43,.57]	.989	[.42,.58]	.998	[.41,.59]	.994	[.44,.56]	.998	[.44,.57]	1.000	[.42,.58]
1000	.965	[.44,.56]	.983	[.43,.57]	.997	[.42,.58]	.966	[.45,.55]	.985	[.45,.55]	.998	[.44,.56]	.996	[.45,.55]	.999	[.45,.55]	1.000	[.44,.56]
	$\widehat{M}_\alpha^{perc}$ (Percentile) for M_I																	
100	.000	[.37,.54]	.037	[.36,.56]	.398	[.33,.59]	.458	[.40,.56]	.646	[.38,.57]	.866	[.35,.60]	.902	[.42,.58]	.951	[.40,.59]	.989	[.37,.62]
250	.000	[.41,.53]	.075	[.40,.54]	.438	[.38,.56]	.480	[.43,.54]	.653	[.42,.55]	.867	[.40,.57]	.909	[.45,.55]	.954	[.44,.56]	.992	[.42,.58]
500	.000	[.44,.52]	.098	[.43,.53]	.468	[.42,.54]	.488	[.45,.53]	.660	[.44,.54]	.878	[.43,.55]	.910	[.46,.54]	.955	[.46,.54]	.991	[.44,.56]
1000	.000	[.46,.52]	.107	[.45,.52]	.472	[.44,.53]	.483	[.47,.52]	.655	[.46,.53]	.866	[.45,.54]	.901	[.47,.53]	.948	[.47,.53]	.989	[.46,.54]
	GMS CSs for μ via moment inequalities																	
100	.810	[.34,.66]	.909	[.32,.68]	.981	[.29,.71]	.806	[.39,.61]	.896	[.37,.63]	.979	[.34,.66]	.894	[.42,.58]	.943	[.40,.60]	.974	[.39,.61]
250	.800	[.40,.60]	.897	[.39,.61]	.978	[.36,.63]	.798	[.43,.57]	.897	[.42,.58]	.976	[.40,.60]	.903	[.45,.55]	.951	[.44,.56]	.984	[.43,.57]
500	.795	[.43,.57]	.902	[.42,.58]	.980	[.40,.60]	.788	[.45,.55]	.894	[.44,.56]	.977	[.43,.57]	.905	[.46,.54]	.950	[.46,.54]	.987	[.45,.55]
1000	.785	[.45,.55]	.885	[.44,.56]	.973	[.43,.57]	.785	[.46,.54]	.884	[.46,.54]	.973	[.45,.55]	.895	[.47,.53]	.943	[.47,.53]	.986	[.46,.54]

Table 6: Missing data example: average coverage probabilities for Θ_I and M_I and average lower and upper bounds of CSs for M_I across 5000 MC replications. Procedures 1–3, Projection and Percentile are implemented using a likelihood criterion and **curved prior**.

	$\eta_2 = 1 - \frac{2}{\sqrt{n}}$						$\eta_2 = 1 - \frac{1}{\sqrt{n}}$						$\eta_2 = 1$ (Point ID)					
	0.90		0.95		0.99		0.90		0.95		0.99		0.90		0.95		0.99	
							$\widehat{\Theta}_\alpha$ (Procedure 1) for Θ_I											
100	.910	—	.951	—	.989	—	.888	—	.938	—	.972	—	.914	—	.945	—	.990	—
250	.908	—	.958	—	.994	—	.901	—	.952	—	.988	—	.904	—	.950	—	.990	—
500	.912	—	.957	—	.992	—	.923	—	.965	—	.993	—	.904	—	.954	—	.990	—
1000	.909	—	.955	—	.989	—	.912	—	.963	—	.995	—	.911	—	.953	—	.989	—
							$\widehat{M}_\alpha$ (Procedure 2) for M_I											
100	.921	[.32,.68]	.965	[.30,.70]	.993	[.27,.73]	.913	[.37,.63]	.949	[.35,.65]	.991	[.32,.68]	.914	[.42,.58]	.945	[.40,.60]	.990	[.37,.63]
250	.911	[.39,.61]	.956	[.38,.62]	.993	[.36,.64]	.913	[.42,.58]	.953	[.41,.59]	.993	[.39,.61]	.904	[.45,.55]	.950	[.44,.56]	.990	[.42,.58]
500	.908	[.42,.58]	.954	[.41,.59]	.991	[.40,.60]	.912	[.44,.56]	.953	[.44,.57]	.991	[.42,.58]	.904	[.46,.54]	.954	[.46,.54]	.990	[.44,.56]
1000	.904	[.44,.56]	.953	[.44,.56]	.989	[.43,.57]	.902	[.46,.54]	.950	[.45,.55]	.990	[.44,.56]	.911	[.48,.52]	.953	[.47,.53]	.989	[.46,.54]
							$\widehat{M}_\alpha^\chi$ (Procedure 3) for M_I											
100	.921	[.32,.68]	.949	[.31,.69]	.989	[.28,.72]	.913	[.37,.63]	.946	[.35,.64]	.989	[.32,.68]	.914	[.42,.58]	.945	[.40,.60]	.990	[.37,.63]
250	.911	[.39,.61]	.951	[.38,.62]	.992	[.36,.64]	.914	[.42,.58]	.952	[.41,.59]	.993	[.39,.61]	.886	[.45,.55]	.950	[.44,.56]	.991	[.42,.58]
500	.909	[.42,.58]	.950	[.41,.59]	.990	[.40,.60]	.913	[.44,.56]	.948	[.44,.57]	.990	[.42,.58]	.904	[.46,.54]	.947	[.46,.54]	.989	[.44,.56]
1000	.902	[.44,.56]	.950	[.44,.56]	.989	[.43,.57]	.901	[.46,.54]	.950	[.45,.55]	.988	[.45,.55]	.902	[.48,.52]	.948	[.47,.53]	.988	[.46,.54]
							$\widehat{M}_\alpha^{proj}$ (Projection) for M_I											
100	.977	[.29,.71]	.990	[.28,.72]	.999	[.24,.76]	.970	[.34,.66]	.988	[.33,.67]	.998	[.30,.70]	.914	[.42,.58]	.945	[.40,.60]	.990	[.37,.63]
250	.972	[.37,.63]	.992	[.36,.64]	.999	[.34,.66]	.976	[.40,.60]	.992	[.39,.61]	.999	[.37,.64]	.904	[.45,.55]	.950	[.44,.56]	.990	[.42,.58]
500	.975	[.41,.59]	.988	[.40,.60]	.998	[.39,.61]	.979	[.43,.57]	.992	[.42,.58]	1.000	[.40,.60]	.904	[.46,.54]	.954	[.46,.54]	.990	[.44,.56]
1000	.972	[.44,.57]	.986	[.43,.57]	.998	[.42,.58]	.975	[.45,.55]	.991	[.44,.56]	.999	[.43,.57]	.911	[.48,.52]	.953	[.47,.53]	.989	[.46,.54]
							$\widehat{M}_\alpha^{perc}$ (Percentile) for M_I											
100	.399	[.38,.62]	.665	[.36,.64]	.939	[.31,.68]	.642	[.41,.59]	.800	[.39,.61]	.954	[.35,.65]	.912	[.42,.58]	.945	[.40,.60]	.990	[.37,.63]
250	.386	[.42,.58]	.642	[.41,.59]	.914	[.38,.62]	.641	[.44,.56]	.804	[.43,.57]	.954	[.41,.59]	.903	[.45,.55]	.950	[.44,.56]	.990	[.42,.58]
500	.384	[.44,.56]	.639	[.44,.57]	.911	[.42,.58]	.638	[.46,.54]	.803	[.45,.55]	.952	[.43,.57]	.905	[.46,.54]	.953	[.46,.54]	.989	[.44,.56]
1000	.392	[.46,.54]	.647	[.45,.55]	.908	[.44,.56]	.651	[.47,.53]	.803	[.46,.54]	.950	[.45,.55]	.909	[.47,.53]	.954	[.47,.53]	.989	[.46,.54]
							GMS CSs for μ via moment inequalities											
100	.806	[.34,.66]	.908	[.32,.68]	.978	[.29,.71]	.803	[.39,.61]	.900	[.37,.63]	.976	[.34,.66]	.899	[.42,.58]	.946	[.40,.60]	.975	[.39,.62]
250	.787	[.40,.60]	.898	[.39,.61]	.979	[.37,.64]	.798	[.43,.57]	.898	[.42,.58]	.978	[.40,.60]	.896	[.45,.55]	.946	[.44,.56]	.981	[.43,.57]
500	.788	[.43,.57]	.896	[.42,.58]	.978	[.40,.60]	.786	[.45,.55]	.898	[.44,.56]	.976	[.43,.57]	.898	[.46,.54]	.950	[.46,.54]	.985	[.45,.56]
1000	.789	[.45,.55]	.892	[.44,.56]	.979	[.43,.57]	.800	[.46,.54]	.893	[.46,.54]	.977	[.45,.55]	.906	[.47,.53]	.950	[.47,.53]	.986	[.46,.54]

Table 7: Missing data example: average coverage probabilities for Θ_I and M_I and average lower and upper bounds of CSs for M_I across 5000 MC replications. Procedures 1–3, Projection and Percentile are implemented using a **CU-GMM criterion** and flat prior.

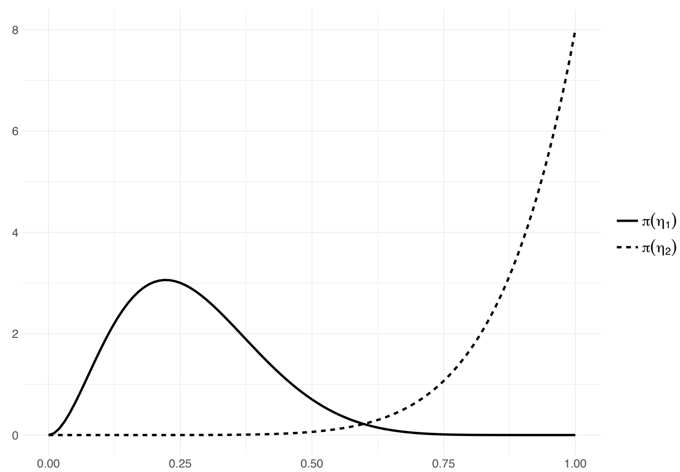

Figure 7: Missing data example: Marginal "curved" priors for η_1 (solid line) and η_2 (dashed line).

G.3 Airline entry game application

See Appendix A.3 of the paper for the implementation details for this application.

Illustrating convergence of the SMC algorithm: To illustrate convergence of the SMC algorithm, Figures 10 and 11 present Q-Q plots of the profile QLR $PQ_n(M(\theta^b))$ for each parameter against the average quantiles across six independent runs of the algorithm. We report Q-Q plots for the profile QLR rather than the raw draws themselves because it is the posterior quantiles of the profile QLR that are used to compute critical values for CSs using Procedure 2. The Q-Q plots show the profile QLR draws used to compute the critical values for the CSs align closely with draws obtained from independent runs of the algorithm. Moreover, Table 8 shows that recomputing the CSs using the independent runs adjust the endpoints at most by around 10^{-3}. Table 8 also reports the average endpoints of Percentile CSs obtained across independent runs of the SMC algorithm and shows that these align closely with the Percentile CSs obtained from the original draws. The standard deviation of Procedure 2 CS endpoints is less than 0.009 for Δ_{OA} an less than 0.003 for all other parameters across independent runs.

G.4 Trade flow application

See Appendix A.4 of the paper for the implementation details for this application.

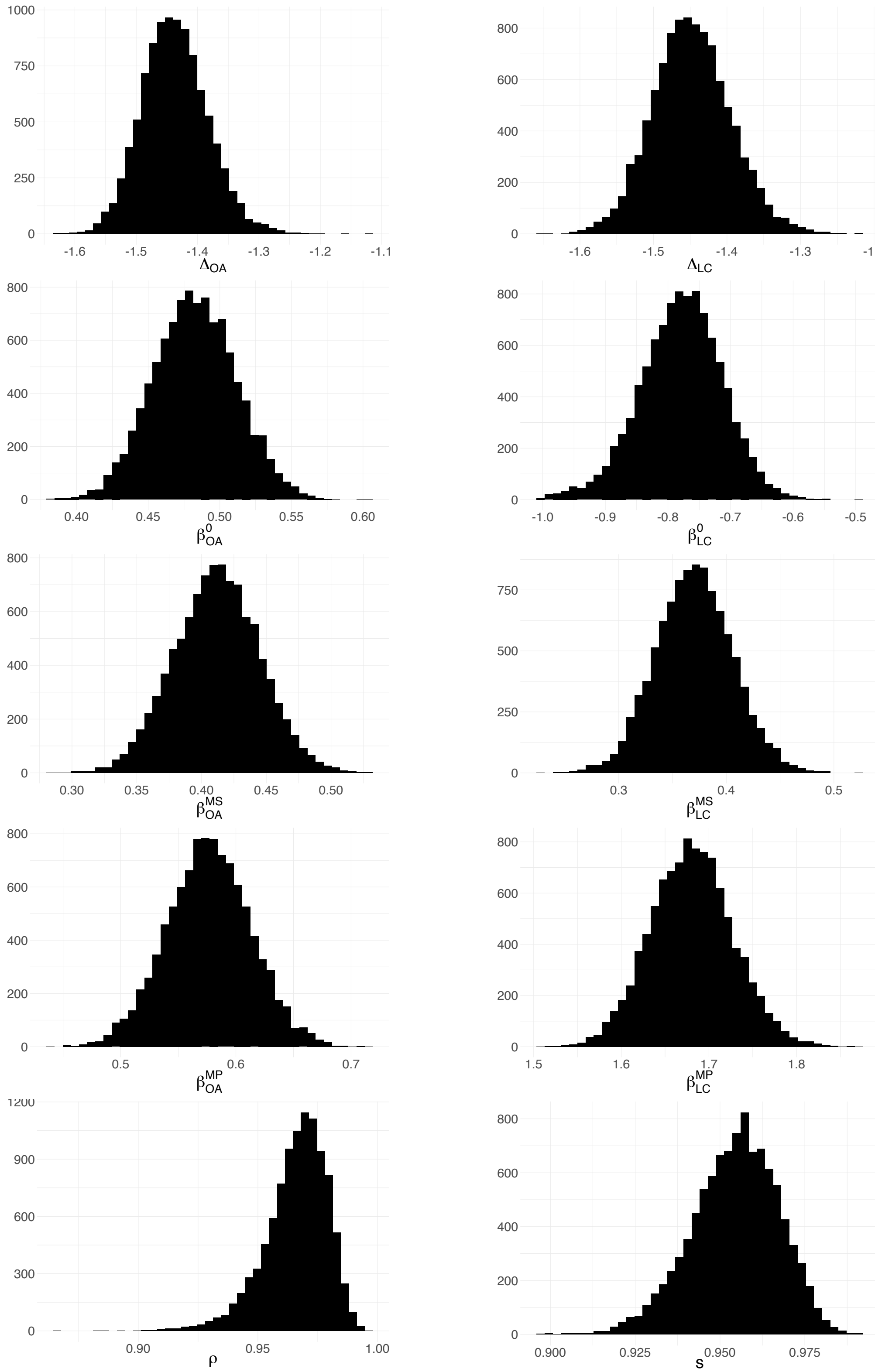

Figure 8: Airline entry game: histograms of the SMC draws for Δ_{OA}, Δ_{LC}, β^0_{OA}, β^0_{LC}, β^{MS}_{OA}, β^{MS}_{LC}, β^{MP}_{OA}, β^{MP}_{LC}, ρ and s for the **fixed-$\boldsymbol{s}$ model**.

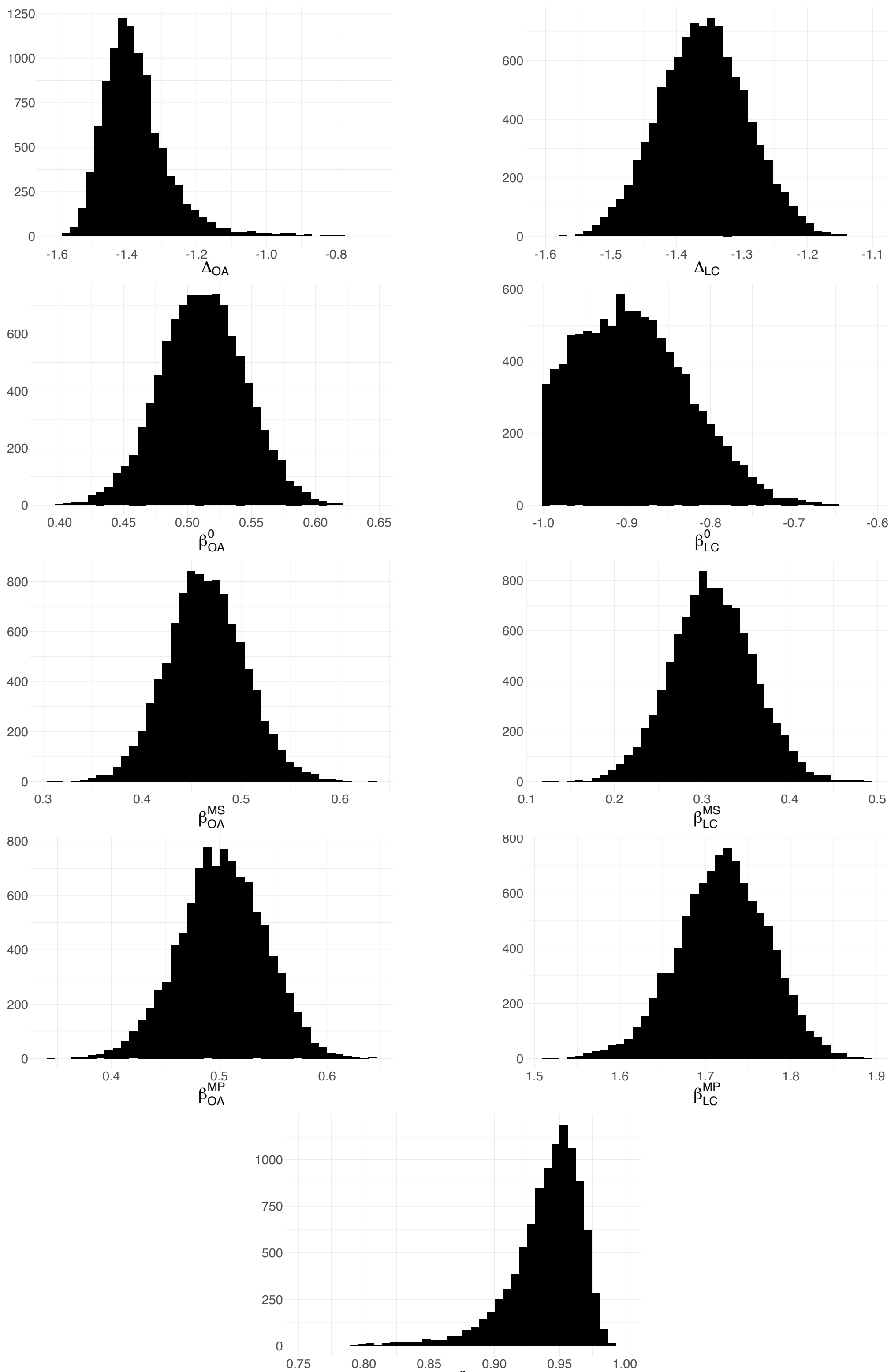

Figure 9: Airline entry game: histograms of the SMC draws for Δ_{OA}, Δ_{LC}, β^0_{OA}, β^0_{LC}, β^{MS}_{OA}, β^{MS}_{LC}, β^{MP}_{OA}, β^{MP}_{LC}, ρ for the **full model**.

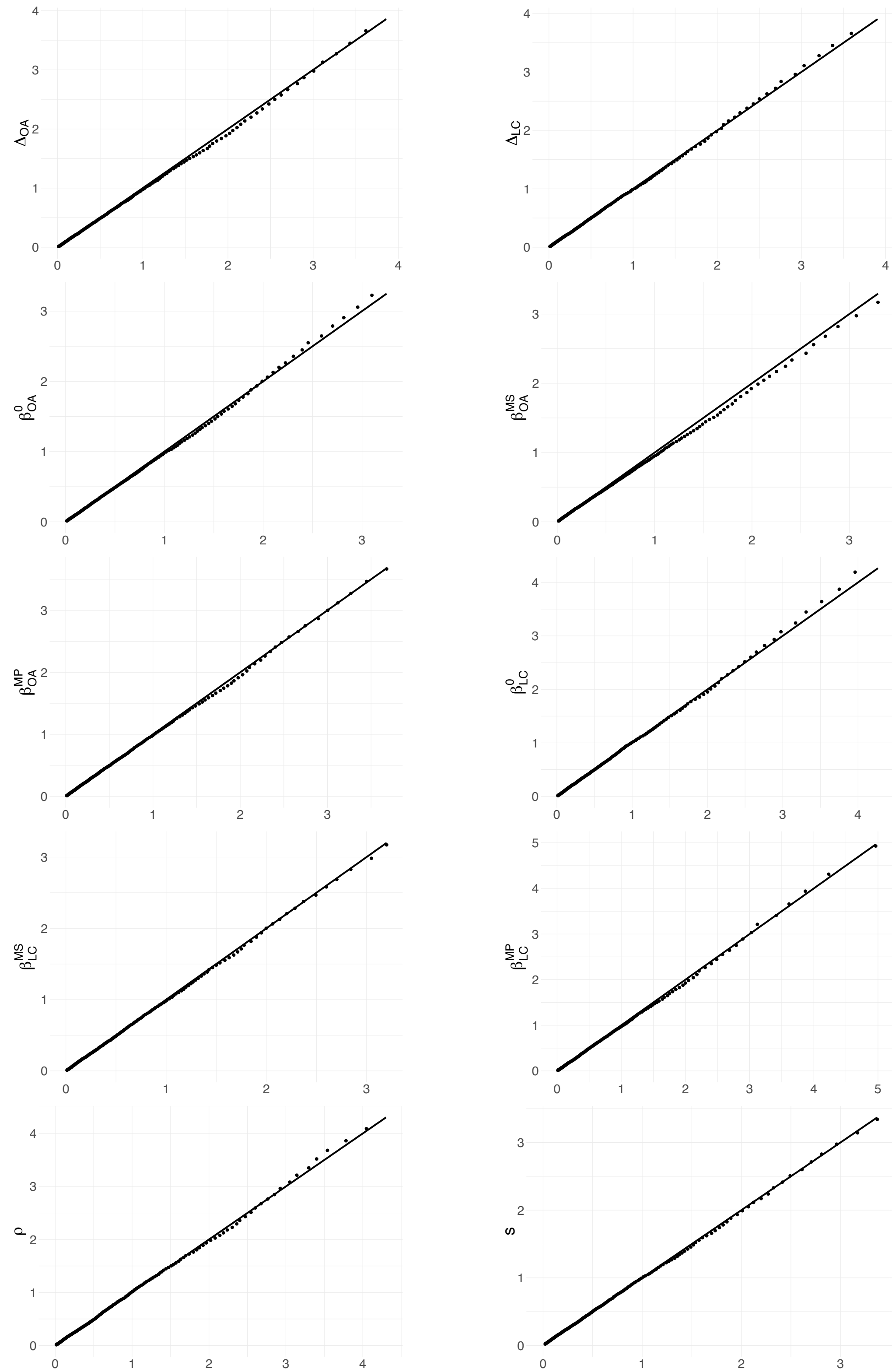

Figure 10: Airline entry game: QQ plots of posterior draws for $PQ_n(M(\theta^b))$ (not the raw parameter draws) for subvector inference on each parameter against the average quantiles obtained from an additional six independent runs of the algorithm for the **fixed-s model**.

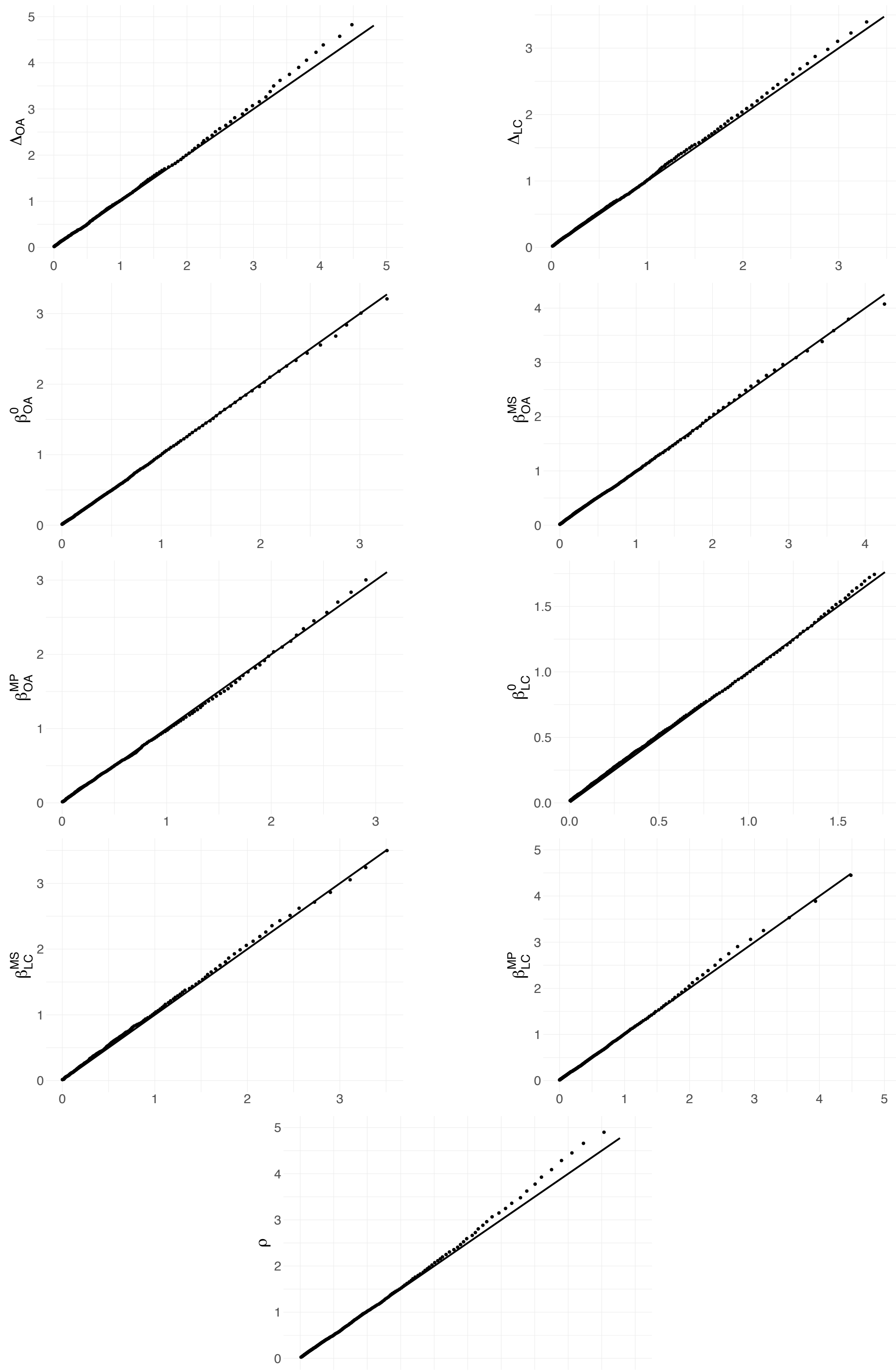

Figure 11: Airline entry game: QQ plots of posterior draws for $PQ_n(M(\theta^b))$ (not the raw parameter draws) for subvector inference on each parameter against the average quantiles obtained from an additional six independent runs of the algorithm for the **full model**.

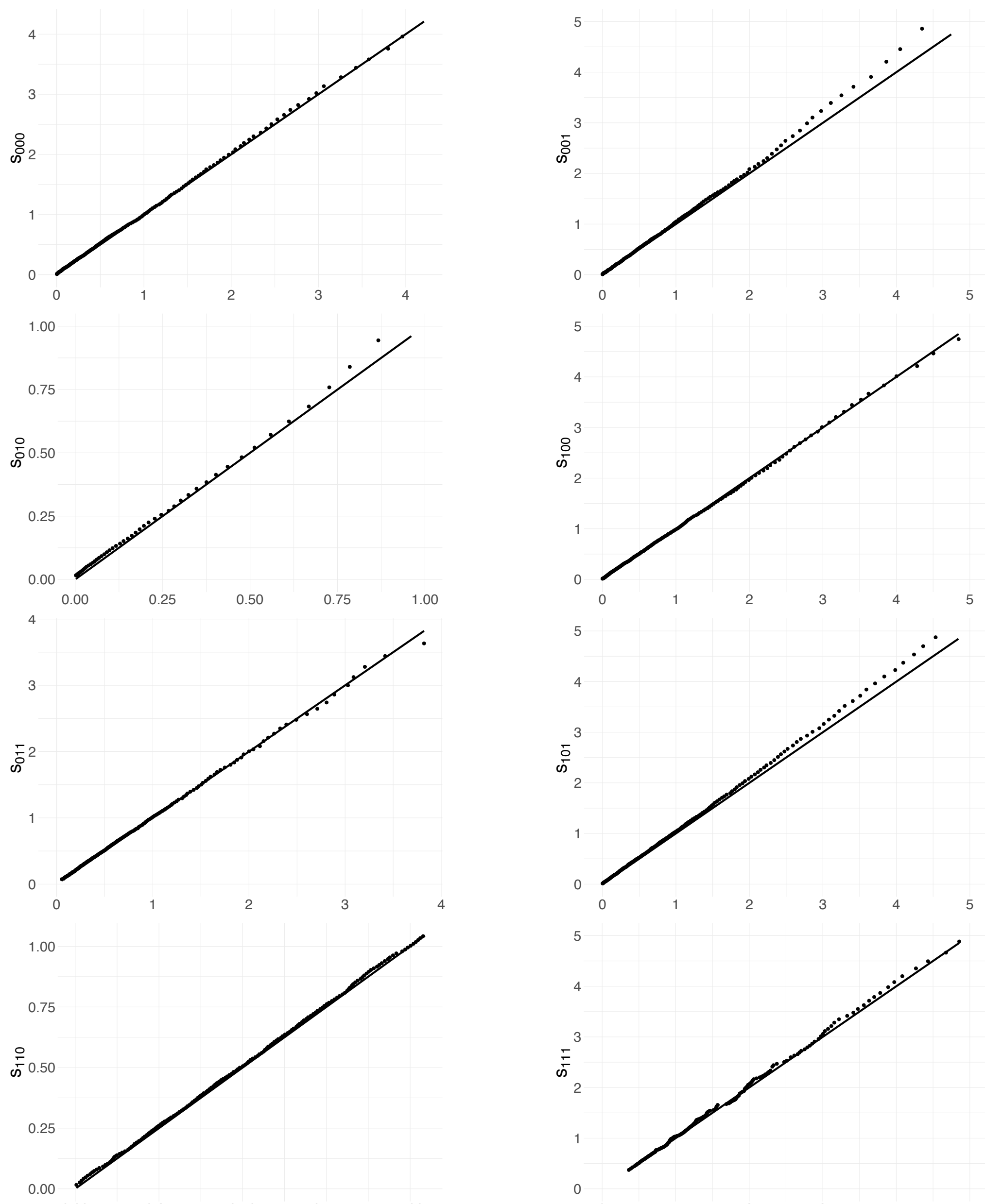

Figure 11: Airline entry game: QQ plots of posterior draws for $PQ_n(M(\theta^b))$ (not the raw parameter draws) for subvector inference on each parameter against the average quantiles obtained from an additional six independent runs of the algorithm for the **full model**.

	Full model				Fixed-s model			
	Procedure 2		Percentile		Procedure 2		Percentile	
	Reported	Check	Reported	Check	Reported	Check	Reported	Check
Δ_{OA}	[−1.599,−1.178]	[−1.605,−1.162]	[−1.515,−1.117]	[−1.518,−1.083]	[−1.563,−1.335]	[−1.564,−1.336]	[−1.536,−1.326]	[−1.536,−1.326]
Δ_{LC}	[−1.527,−1.218]	[−1.527,−1.217]	[−1.489,−1.225]	[−1.495,−1.226]	[−1.567,−1.343]	[−1.570,−1.342]	[−1.548,−1.339]	[−1.552,−1.337]
β^0_{OA}	[0.443,0.581]	[0.446,0.581]	[0.447,0.578]	[0.448,0.577]	[0.431,0.551]	[0.431,0.551]	[0.427,0.540]	[0.424,0.539]
β^{MS}_{OA}	[0.365,0.539]	[0.366,0.537]	[0.389,0.544]	[0.389,0.544]	[0.347,0.479]	[0.347,0.478]	[0.348,0.477]	[0.348,0.476]
β^{MP}_{OA}	[0.413,0.581]	[0.413,0.580]	[0.424,0.579]	[0.423,0.580]	[0.479,0.641]	[0.480,0.641]	[0.504,0.648]	[0.503,0.648]
β^0_{LC}	[−1.000,−0.729]	[−1.000,−0.729]	[−0.993,−0.751]	[−0.994,−0.748]	[−0.910,−0.627]	[−0.909,−0.622]	[−0.917,−0.655]	[−0.928,−0.656]
β^{MS}_{LC}	[0.226,0.431]	[0.225,0.433]	[0.220,0.405]	[0.220,0.406]	[0.299,0.443]	[0.299,0.443]	[0.303,0.442]	[0.300,0.441]
β^{MP}_{LC}	[1.591,1.868]	[1.593,1.868]	[1.615,1.821]	[1.610,1.818]	[1.573,1.790]	[1.573,1.788]	[1.590,1.776]	[1.590,1.777]
ρ	[0.874,0.986]	[0.868,0.987]	[0.867,0.977]	[0.857,0.977]	[0.938,0.990]	[0.938,0.990]	[0.935,0.986]	[0.934,0.986]
s	—	—	—	—	[0.926,0.980]	[0.926,0.980]	[0.927,0.977]	[0.926,0.977]
s_{000}	[0.587,0.964]	[0.586,0.965]	[0.572,0.934]	[0.566,0.934]	—	—	—	—
s_{001}	[0.812,1.000]	[0.807,1.000]	[0.797,0.995]	[0.782,0.995]	—	—	—	—
s_{010}	[0.000,1.000]	[0.000,1.000]	[0.018,0.828]	[0.016,0.830]	—	—	—	—
s_{100}	[0.637,0.998]	[0.638,0.998]	[0.612,0.990]	[0.604,0.990]	—	—	—	—
s_{011}	[0.916,1.000]	[0.916,1.000]	[0.915,0.999]	[0.915,0.999]	—	—	—	—
s_{101}	[0.491,0.920]	[0.475,0.928]	[0.449,0.799]	[0.396,0.799]	—	—	—	—
s_{110}	[0.000,1.000]	[0.000,1.000]	[0.042,0.986]	[0.041,0.985]	—	—	—	—
s_{111}	[0.942,1.000]	[0.942,1.000]	[0.941,0.999]	[0.939,0.999]	—	—	—	—

Table 8: Airline entry game: 95% CSs for structural parameters computed via Procedure 2 and Percentile methods. Columns headed "reported" are the intervals presented in Table 4. Columns headed "check" are the average lower and upper endpoints of the interval computed over an additional six independent runs of the SMC algorithm.