Econometrica Supplementary Material

SUPPLEMENT TO "MICRO DATA AND MACRO TECHNOLOGY"

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APPENDIX B: DATA

THIS SECTION DISCUSSES the construction of the data.

B.1. *Economic Censuses*

We use the 1987 through 2007 Census of Manufactures to estimate plant-level elasticities of substitution and demand. We remove all Administrative Record plants because these plants do not have data on output or capital. We also eliminate a set of outliers and missing values from the data set. We first remove all plants born in the given Census year, as well as a small set of plants with missing age data. We then remove plants with zero, missing, or negative data for the equipment capital stock, structures capital stock, labor costs, value added, or materials. We also remove plants above the 99.5th percentile or below the 0.5th percentile of their two-digit SIC industry for these variables to remove plants with potential data problems. Finally, we drop plants in Alaska and Hawaii as we do not have amenity instruments for these locations.

For capital costs, we multiply capital stock measures by rental rates of capital. For the capital stock, we use the Census constructed measure of perpetual inventory capital stock, which is constructed for structures and equipment capital separately. The Census uses book values of capital together with investment histories to construct these capital stocks; thus, they will be primarily based upon book values for plants that exist only in Census years, while for large plants always sampled in the ASM, they may be based on a long time span of continuous investment histories.

The Annual Survey of Manufactures tracks about 50,000 plants over five-year panel rotations that are more heavily weighted toward large plants. We use the ASM to calculate the heterogeneity indices and materials shares. The ASM has data on plant investment over time as well as book values of the stock of capital, which are used by the Census to construct perpetual inventory measures of capital stocks. The ASM plant samples also have data on the value of non-monetary compensation given to employees, such as health care or retirement benefits, which we use to better measure payments to labor. We include non-monetary compensation as part of labor costs when we use the 2002 and 2007 Census of Manufactures, as these years include non-monetary labor compensation for all plants.

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B.2. *Rental Rates*

We define the rental rate using the external real rate of return specification of Harper, Berndt, and Wood (1989). The rental rate for industry n is defined as

$$R_{i,t} = T_{i,t}(p_{i,t-1}r_{i,t} + \delta_{i,t}p_{i,t}),$$

where $r_{i,t}$ is a constant external real rate of return of 3.5 percent, $p_{i,t}$ is the price index for capital in that industry, $\delta_{i,t}$ is the depreciation rate for that industry, and $T_{i,t}$ is the effective rate of capital taxation. We calculate $T_{i,t}$ following Harper, Berndt, and Wood (1989) as

$$T_{i,t} = \frac{1 - u_t z_{i,t} - k_{i,t}}{1 - u_t},$$

where $z_{i,t}$ is the present value of depreciation deductions for tax purposes on a dollar's investment in capital type i over the lifetime of the investment, $k_{i,t}$ is the effective rate of the investment tax credit, and u_t is the effective corporate income tax rate. We obtained $z_{i,t}$, u_t, and $k_{i,t}$ from Dale Jorgenson at the asset year level; we then used a set of capital flow tables at the asset-industry level to convert these to the industry level.

To calculate depreciation rates $\delta_{i,t}$, we take depreciation rates from NIPA at the asset level and use the capital flow tables to convert them to the industry level. Our primary source of prices of capital $p_{i,t}$ are from NIPA, which calculates separate price indices for structures and equipment capital.

The capital flow tables and investment price series depend upon the industry definition; because the United States switches from SIC basis to NAICS basis during this period, we construct separate rental price series for SIC two-digit industries and NAICS three-digit industries. Finally, when we examine aggregate factor shares, we have to aggregate all of the rental price series; we do so by calculating Tornqvist indices between equipment and structures capital for each industry, and then a Tornqvist index across rental rates for each industry. The Tornqvist indices allow for the share of equipment capital in industry capital and for the share of different industries in manufacturing capital to change over this period.

B.3. *Local Wages*

We construct measures of the local wage in order to estimate the elasticity of substitution across plants, using both worker- and establishment-level data to measure the local area wage. The primary data set that we use is the Census 5 percent samples of Americans, together with the American Community Surveys. Both of those data sets have data on wages and local area geographic location for a large sample of workers.

To obtain the local wage, we first calculate the individual wage for workers with age between 20 and 65 who are employed in the private sector as workers earning a wage or salary and who do not live in group quarters. We calculate the wage as an hourly wage, defined as total yearly wage and salary income divided by total hours worked. We use the CPI to deflate wage income, which affects the wages matched to the 2007 Census of Manufactures, as these rely on information on workers over five different years of the ACS.

We measure total hours worked as weeks worked per year multiplied by hours worked per week. We remove all individuals with zero or missing income or zero total hours worked. In 2008 and 2009, only the intervalled number of weeks worked is available.

We thus impute the number of weeks worked for individuals in 2008 and 2009 based on averages of the number of weeks worked from 2005 to 2007 from cells of the intervalled weeks worked, an indicator if the worker is female, an indicator if the worker is black, the educational attainment of the worker (as constructed below), and age (as a set of dummy variables for age intervals).

Total wage and salary income in the Population Censuses and American Community Surveys are topcoded. The topcode threshold is $140,000 in 1990, $175,000 in 2000, and the 99.5% of the state distribution of income for that year in the ACS years. For all cases, we impute the total wage and salary income to 1.5 times the topcode if the wage and salary income is topcoded, in line with Acemoglu and Angrist (2000).

Before calculating local area wages, we adjust measures of local wages for differences in worker characteristics through regressions with the individual log wage as a dependent variable. We include education through a set of dummy variables based upon the worker's maximum educational attainment, which include four categories: college, some college, high school degree, and high school dropouts. We define experience as the individual's age minus an initial age of working that depends upon their education status, and include a quartic in experience in the regression. We also have data on the race of workers and so include three race categories of white, black, and other, as well as an indicator for Hispanic origin and gender. We include six occupational categories: Managerial and Professional; Technical, Sales, and Administrative; Service; Farming, Forestry, and Fishing; Precision Production, Craft, and Repairers; and Operatives and Laborers. Finally, we include thirteen industrial categories: Agriculture, Forestry, and Fisheries; Mining; Construction; Manufacturing; Transportation, Communications and Other Public Utilities; Wholesale Trade; Retail Trade; Finance, Insurance, and Real Estate; Business and Retail Services; Personal Services; Entertainment and Recreation Services; Professional and Related Services; and Public Administration.

We then regress the local wage on all of these characteristics, with separate regressions by year. For wages matched to the 2007 Census of Manufactures which use multiple ACS years, we include year effects as well to allow the overall wage distribution to vary over time.

We then aggregate the residuals from this regression to the commuting zone level. The Population Census and ACS data only contain information on the Public Use Micro Area (PUMA) of the individual worker. Thus, we use crosswalks from Autor and Dorn (2013) in order to aggregate from the PUMA to the Commuting Zone. Since some PUMAs contain multiple commuting zones, we weight each residual wage by the multiple of the person weight in the Census or ACS and a weight that indicates the fraction of the PUMA in the given Commuting Zone. We then construct average residual wages for each commuting zone.

Because the Economic Census is conducted in different years from the Population Censuses, we match the 1987 and 1992 Censuses of Manufactures to wages from the 1990 Population Census, the 1997 and 2002 Censuses of Manufactures to wages from the 2000 Population Census, and the 2007 Census of Manufactures to the 2005–2009 American Community Surveys.

The second data set that we use for our IV and panel data specifications is the Longitudinal Business Database, which contains data on payroll and employment for all U.S. establishments. We construct the establishment wage as total payroll divided by total employment. We measure the local wage as the mean log wage at the commuting zone level, after regressing the log establishment wage on indicator variables for four-digit SIC or six-digit NAICS industry codes to remove industry effects. We match the Longitudinal Business Database to its equivalent year in the Census of Manufactures.

B.4. *Instruments*

We use three different sets of instruments for the local wage in order to estimate the elasticity of substitution.

The first set of instruments are local amenities that could affect labor supply developed by Albouy, Graf, Kellogg, and Wolff (2016). They include measures of the slope, elevation, relative humidity, average dew point, average precipitation, and average sunlight for each local area. We also include multiple measures of temperature. The first measures are the number of heating degree days (HDD) and cooling degree days (CDD). HDD measures how cold a location is, and is defined as the sum of the difference between 65F and each day's mean temperature, for all days colder than 65F. CDD is a measure of how hot a location is, and is defined as the sum of the difference between each day's mean temperature and 65F, for all days warmer than 65F. In addition, we include a set of temperature day bins which bin the average number of days in a year over 30 years that the average temperature (mean of minimum and maximum temperature) lies within the bin. We include six bins of 10 degrees Centigrade each.

The amenities in Albouy et al. (2016) were collected at the PUMA level. We aggregate them to the commuting zone level by taking an average across PUMAs in the same commuting zone, weighting PUMAs by their population in the commuting zone. We do not have amenities for Alaska and Hawaii, and all specifications exclude these states.

The second instrument, from Bartik (1991), is based upon the differential impact of national-level shocks to industry employment across locations. Positive national shocks to an industry should increase labor demand and wages more in areas with high concentrations of that industry. Formally, the predicted growth rate in employment for a given location is the sum across industries of the product of the local employment share of this industry and the 5- or 10-year change in national-level employment for that industry. We use the Longitudinal Business Database, which contains all U.S. establishments, to construct these instruments.

The implicit assumption here is that changes in industry shares at the national level are independent of local manufacturing plant productivity. To help ensure that this assumption holds, we exclude manufacturing industries from the labor demand instrument. We calculate the instrument defining locations by commuting zones and industries at the SIC four-digit level, or NAICS six-digit level, depending upon the years. We drop industries with national employment of less than 100 people as likely data errors.

We also use a second set of labor demand instruments from Beaudry, Green, and Sand (2012). The first instrument is the interaction of predicted changes in industry employment shares and industry initial wage premia. The second instrument is the interaction of national changes in industry wage premia and predicted future industry employment shares. We also exclude manufacturing industries from these instruments. National wage premia for an industry are calculated as the mean log payroll to employment ratio across the entire LBD for a given year.

The main complication with constructing the Bartik and BGS instruments is that industry definitions change over time: in 1987, when industry definitions switch from 1972 industry definitions to 1987 industry definitions, and in 1997, when industry definitions switch from the 1987 SIC definitions to NAICS definitions. Thus, we often cannot construct exact 10-year instruments because industry definitions are not consistent over time. Instead, we use 10-year instruments for 1987, 1997, and 2002, and 5-year instruments and their lag for 1992 and 2007. For 1987, the instrument used is from 1977 to 1986; for 1997, from 1987 to 1997; and for 2002, from 1992 to 2001. For 1992, we use the 1982 to 1986 and 1987 to 1992 instruments. For 2007, we use the 1997 to 2001 and 2002 to 2007 instruments.

APPENDIX C: ELASTICITY ESTIMATES

C.1. *Non-CES Production Functions*

We examine the possibility of non-constant elasticities empirically by allowing elasticities to vary by quantile. Because we need to control for quantile-invariant industry fixed effects, we use the two-step approach of Canay (2011) to estimate quantile elasticities. We estimate the elasticity at the 10th through 90th quantile both among all plants and among plants within each two-digit industry for each year. The elasticity varies across quantiles in an inverse U shape, with elasticities close to zero at the bottom quantiles, a peak close to the median, and then a slight fall for high quantiles. Supplemental Appendix H.2 contains further details of the differences across quantiles and the estimation approach.

We then obtain $\bar{\sigma}_n$ by assigning each plant the elasticity for its closest conditional quantile. In Table C.I, Column (2) reports estimates of the average $\bar{\sigma}_n$ under the assumption that elasticities at each quantile are the same across industries, while column (3) allows these quantile elasticities to vary across industries. The average baseline OLS estimate across years is 0.39 (column (1)), compared to 0.45 using common quantile elasticities and 0.47 using separate quantile elasticities for each industry. Thus, the conditional quantile approach for allowing local elasticities increases our estimates of the aggregate elasticity slightly.

A second approach is to use plants' capital shares as the dependent variable instead of the logarithm of the ratio of capital cost to labor cost. Our goal is to estimate an approximation to $\bar{\sigma}_n \equiv \sum_{i\in I_n} \frac{\alpha_{ni}(1-\alpha_{ni})\theta_{ni}}{\sum_{i'\in I_n}\alpha_{ni'}(1-\alpha_{ni'})\theta_{ni'}}\sigma_{ni}$.

Consider the following regression equation:

$$\alpha_{nic} = \gamma_n + \beta_n \ln w_c + \epsilon_{nic}. \tag{C.1}$$

Here, β_n is an estimate of how the average capital share in a location covaries with relative factor prices in the location, that is,

$$\beta_n \approx \frac{\mathrm{d}\mathbb{E}[\alpha_{nic}]}{\mathrm{d}\ln w/r}. \tag{C.2}$$

TABLE C.I

NON-CES ESTIMATES OF AVERAGE PLANT CAPITAL-LABOR SUBSTITUTION ELASTICITY[a]

	(1) Baseline	(2) Quantile Sector Level	(3) Quantile Industry Level	(4) Avg Capital Share Unweighted	(5) Avg Capital Share Weighted
1987	0.43	0.46	0.47	0.45	0.56
1992	0.48	0.49	0.54	0.52	0.57
1997	0.34	0.39	0.45	0.43	0.58
2002	0.34	0.40	0.43	0.41	0.44
2007	0.38	0.52	0.45	0.40	0.38

[a]The table contains five specifications. All specifications average across separate plant elasticity of substitution for each industry using the cross-industry weights used for aggregation. In (1), we estimate a separate OLS estimate using our baseline estimation strategy as in Section 3.3. In (2) and (3), we estimate separate elasticities for the 10th to the 90th quantiles using the two-step estimation procedure of Canay (2011); (2) assumes a common estimate for all of manufacturing and (3) separate quantile elasticities for each two-digit SIC or three-digit NAICS industry. In (4) and (5), we estimate (C.1) by having the capital share as the dependent variable; (4) does not weight the data, while (5) weights plants by their total cost of capital and labor. All regressions include industry fixed effects, age fixed effects, and a multi-unit status indicator. Wages used are the average log wage for the commuting zone, computed as wage and salary income over total number of hours worked adjusted for differences in worker characteristics using the Population Censuses.

In Supplemental Appendix H.3, we show that, to a first-order approximation, the estimator $\hat{\beta}_n$ converges to a weighted average of terms $\alpha_{nic}(1-\alpha_{nic})(\sigma_{nic}-1)$:

$$\hat{\beta}_n \xrightarrow{p} \sum_c \sum_{i \in I_{nc}} \rho_{nic}\alpha_{nic}(1-\alpha_{nic})(\sigma_{nic}-1), \tag{C.3}$$

where I_{nc} are the set of plants in industry n in location c and the weights $\rho_{nic} = \frac{(\ln w_c - \overline{\ln w})^2}{\sum_{\tilde{c}} \sum_{\tilde{i} \in I_{n\tilde{c}}} (\ln w_{\tilde{c}} - \overline{\ln w})^2}$ sum to 1.

Given our estimate for β, we compute $\bar{\sigma}_n$ using $\hat{\bar{\sigma}} - 1 = \frac{\hat{\beta}}{(1-\chi)\alpha(1-\alpha)}$. Column (4) of Table C.I presents these estimates pooling across industries within the manufacturing sector.[1] The average elasticity across years is 0.44.[2]

Compared to our goal of estimating $\bar{\sigma}_n$, however, β_n weights observations by ρ_{ni} instead of the cost weights θ_{ni}. There are two differences. First, plants in locations with more extreme wages are weighted more heavily, as is typical for least squares estimators. We have verified in Monte Carlo simulations that this weighting does not lead to a significant bias; see Supplemental Appendix H.3 for details. Second, we do not weight by θ_{ni}. To address this latter concern, we estimate (C.1) weighting each observation by θ_{ni}. However, we show analytically and confirm in our Monte Carlo that this weighting also introduces an upward bias in elasticity estimates: larger plants tend to be more capital intensive which means that the weights are correlated with the error term. Thus, we view the weighted estimate as an upper bound on $\bar{\sigma}_n$. Column (5) shows that these estimates average 0.51 across years.

Given the narrow range of estimates in Table C.I, we do not believe that assuming a constant plant-level elasticity is a first-order issue for our aggregation framework.

C.2. *Dynamic Panel Estimates*

In this section, we use the panel structure of our data in order to examine how individual plants respond to changes in factor prices. This adjustment may be slow; the long-run response to a factor price change should be larger than the short-run adjustment. We therefore use dynamic panel methods to examine both the unbalanced panel (which still requires plants that exist in at least three consecutive Census years), as well as the balanced panel of plants that exist in all five Census years.[3]

[1]In Supplemental Appendix H.3, we also estimate $\bar{\sigma}_n$ separately for each industry and then take the appropriate average. In addition, we pursue an instrumental variables specification using the instruments used in Section 3.3. Estimates are quantitatively similar across specifications.

[2]The regression in column (4) of Table C.I differs from (C.1) in that it includes the controls for a vector of plant characteristics X_{nic} (detailed in Table I): $\alpha_{nic} = \beta_n \ln w_c + \gamma X_{nic} + \epsilon_{nic}$. We show in Supplemental Appendix H.3 that, in this case, the estimator converges in probability to $\hat{\beta}_n \xrightarrow{p} \sum_c \sum_{i \in I_{nc}} \rho^*_{nic}\alpha_{nic}(1-\alpha_{nic})(\sigma_{nic}-1)$, where the weights are $\rho^*_{nic} = \frac{\ln w^*_{nic}(\ln w_c - \overline{\ln w})}{\sum_{\tilde{c}} \sum_{\tilde{i} \in I_{n\tilde{c}}} \ln w^*_{n\tilde{i}\tilde{c}}(\ln w_{\tilde{c}} - \overline{\ln w})}$ and $\ln w^*_{nic}$ are the residuals from a regression of $\ln w_c$ on X_{nic}.

[3]Because our dynamic panel specification with two lags requires plants to be present for three consecutive Economic Censuses, there could be differences between the elasticity for these plants compared to the overall sample. These plants are likely to be different in some ways from the typical manufacturing plant; in particular, they may be older and larger, or belong to a multi-unit firm. We examine differences by age cohort in Supplemental Appendix H.1 in the cross-section and do not find a clear gradient of the elasticity with age. In Supplemental Appendix H.3, we find that weighting by size leads to only slightly larger estimates of the elasticity, and, as discussed in Section 3.3, estimates using multi-unit plants tend to be similar to the full sample. Thus, we suspect that any sample selection bias is small.

We estimate the following econometric model for plant i and time period t:

$$\log\frac{K_{itc}}{L_{itc}} = \rho_5 \log\frac{K_{it-5c}}{L_{it-5c}} + \rho_{10}\log\frac{K_{it-10c}}{L_{it-10c}} + \beta\log(w_{tc}/r_t) + \eta_i + \delta_t + \gamma_{n(i)}t + \epsilon_{itc}, \quad \text{(C.4)}$$

where η_i is an individual plant fixed effect, ρ_5 and ρ_{10} measure the degree of persistence in the capital-labor ratio through the five-year and ten-year lag of the capital-labor ratio, and β measures the short-run elasticity of substitution. We estimate this relationship in terms of the capital-labor ratio, and not the ratio of capital cost to labor cost, so that the long-run capital-labor elasticity is $\frac{\beta}{1-\rho_5-\rho_{10}}$.[4] Because we examine plants over time, we decompose the bias of plant i's technology into a plant fixed effect, η_i, a time fixed effect, δ_t, a three-digit industry-specific trend, $\gamma_{n(i)}t$, and a residual ϵ_{itc}.

We then use the Blundell–Bond panel data model to estimate this relationship.[5] We estimate two specifications; in the first, the wage-rental ratio is treated as exogenous after the time controls (i.e., exogenous with respect to ϵ_{itc}), while in the second specification, we use all of the instruments used earlier in Section 3.3 for the wage-rental ratio. We use local amenities as an instrument for the local wage level, while we use the Bartik and BGS shocks as instruments for both wage levels and changes. The wages we use are based on establishment data in order to match the same year as the Economic Census.

Table C.II contains the estimates of these dynamic panel models. The first two columns report estimates for the unbalanced panel, the third and fourth columns for the balanced panel, and the fifth and sixth columns for the unbalanced panel estimating the coefficients using second-step GMM. The first three rows report the lag of the capital-labor ratio, the short-run elasticity, which is the coefficient on the wage-rental rate ratio, and the long-run elasticity, which is the short-run elasticity divided by 1 minus the sum of the coefficients on the lags of the capital-labor ratio, across six specifications.

We start by estimating models with only the first lag of the capital-labor ratio, so $\rho_{10} = 0$. The coefficient on the lag of the capital-labor ratio is precisely estimated and ranges from 0.28 to 0.35, indicating substantial autocorrelation even over a 5-year time horizon. The estimates of the short-run elasticity are fairly low, ranging from 0.06 to 0.22 across specifications, indicating long-run elasticities between 0.08 and 0.31. These long-run elasticities are substantially lower than our cross-sectional estimates.

One of the main testable assumptions of the Blundell–Bond model with one lag is that there is no correlation between ϵ_{it} and ϵ_{it-10}. We can test this by examining the correlation between differenced residuals; while the autocorrelation is low (at about 0.045), we strongly reject the hypothesis that there is no correlation between errors two periods apart. Thus, we also estimate specifications including a second lag of the capital-labor ratio in the fourth through seventh rows of Table C.II. The coefficient on the first lag ranges from 0.31 to 0.34. The coefficient on the second lag is roughly one-fifth to one-fourth the

[4]We measure the labor input at a plant as the wage bill divided by the local wage.

[5]Blundell–Bond uses system GMM with two equations. One moment condition is based on differencing (C.4) and then instrumenting with lagged terms, so $E[Z_{it-5}(\epsilon_{it} - \epsilon_{it-5})] = 0$. The second moment condition uses (C.4) directly but differences the instruments, so $E[(Z_{it} - Z_{it-5})\epsilon_{it}] = 0$. For example, in the differenced equation, we would instrument for the change in the capital-labor ratio with lagged values of the capital-labor ratio, while in the levels equation we would instrument for the lag of the capital-labor ratio with lagged values of changes in the capital-labor ratio. We also examined estimates using the Arellano–Bond model, which only uses the differenced equation. Unfortunately, with the Arellano–Bond model we have very little power to estimate the capital-labor elasticity in specifications where we instrument for wages, although we obtain similar estimates of the coefficient of the lagged capital-labor ratio.

TABLE C.II

DYNAMIC PANEL ESTIMATES OF THE PLANT CAPITAL-LABOR SUBSTITUTION ELASTICITY[a]

	(1) No Inst	(2) All Inst	(3) No Inst	(4) All Inst	(5) No Inst	(6) All Inst
Lag	0.28 *(0.003)*	0.31 *(0.004)*	0.34 *(0.004)*	0.35 *(0.004)*	0.28 *(0.007)*	0.31 *(0.006)*
SR Elasticity	0.13 *(0.05)*	0.07 *(0.07)*	0.12 *(0.05)*	0.06 *(0.08)*	0.22 *(0.12)*	0.08 *(0.08)*
LR Elasticity	0.18	0.10	0.18	0.08	0.31	0.11
Lag	0.31 *(0.004)*	0.32 *(0.005)*	0.34 *(0.004)*	0.34 *(0.005)*	0.31 *(0.008)*	0.33 *(0.007)*
Second Lag	0.06 *(0.005)*	0.07 *(0.005)*	0.08 *(0.005)*	0.07 *(0.006)*	0.07 *(0.008)*	0.07 *(0.007)*
SR Elasticity	0.18 *(0.04)*	0.21 *(0.09)*	0.15 *(0.04)*	0.36 *(0.14)*	0.27 *(0.09)*	0.21 *(0.08)*
LR Elasticity	0.29	0.34	0.26	0.61	0.43	0.35
Balanced	No	No	Yes	Yes	No	No
Two Step	No	No	No	No	Yes	Yes

[a]The table contains six specifications. In (1) and (2), we examine an unbalanced panel of plants in the Census of Manufactures for at least three consecutive Censuses between 1987 and 2007, while (3) and (4) examine the balanced panel. The first four specifications use one-step GMM, while (5) and (6) use two-step GMM on the unbalanced panel. All specifications estimate the Blundell–Bond model, either assuming that the wage-rental rate ratio is exogenous, or instrumenting for it using amenity, Bartik, and BGS instruments. Instruments are as defined in the text. All specifications also include year effects and time trends for the three-digit NAICS industry reported in 1997 as controls for biased technical change. The wage is the average log wage for the commuting zone, computed as payroll/number of employees at the establishment level using the LBD. The rental rate is the average rental rate between structures and equipment, weighting each by their respective capital stock. Standard errors, in parentheses, are clustered at the commuting zone level.

magnitude of the second lag, ranging from 0.06 to 0.08, but remains strongly significant. Thus, dynamic adjustment does occur beyond a 5-year time horizon. However, the sharp reduction in the magnitude of the second lag gives us confidence that we do not need to include additional lags of the capital-labor ratio.

The short-run elasticities in the specifications with two lags of the capital-labor ratio are considerably higher than those with one lag, with estimates between 0.15 and 0.36. These short-run elasticities imply long-run elasticities between 0.26 and 0.61.

Apart from the estimate of 0.61, however, these long-run elasticities remain slightly below most of the estimates of the cross-sectional elasticity.

APPENDIX D: ENTRY AND EXIT

This section studies an economy with entry and exit by introducing entry and overhead costs. In doing this, we must address several issues. First, we have to specify which expenditures are measured in our data. For example, entry costs incurred before production are likely not measured in our data. Second, we have to specify the factor content of entry and overhead costs. In this section, we study several variations of these assumptions. For each, we derive an upper bound for the aggregate elasticity of substitution and show that it is equal to or slightly above our baseline estimate, as well as a lower bound using our dynamic panel estimates.

Entry and Overhead Costs Use Final Output

Consider an economy with a continuum of entrepreneurs. Each entrepreneur can draw a random technology τ from an exogenous distribution with CDF $T(\tau)$ by paying an entry cost of f^E units of final output. After observing the draw, she can operate a plant with the production function $F_\tau(K, L, M)$ if she is willing to pay an overhead cost of f^O units of final output. Each production function F_τ is assumed to exhibit constant returns to scale.

We assume here that the overhead cost is measured as an expenditure on intermediate inputs in our data, but the entry cost is not.

For a plant with technology τ, let c^v_τ be the unit cost associated with the production function F_τ. Entrepreneurs follow a cutoff rule and operate the plant if variable profit outweighs the fixed operating cost. Free entry implies that cost of a productivity draw equals the expected profit, $Pf^E = \int \max\{(p_\tau - c^v_\tau)y_\tau - Pf^O, 0\}\, dT(\tau)$, where p_τ and y_τ are the optimal choices of price and quantity.

Let E_τ be an indicator of whether plant τ chooses to operate. Should the plant enter, we denote its capital share by α_τ and its expenditure on capital and labor as a fraction of the average expenditure by $\theta_\tau = \frac{rK_\tau + wL_\tau}{\int [rK_{\tilde\tau} + wL_{\tilde\tau}]E_{\tilde\tau}\, \mathrm{d}T(\tilde\tau)}$. Thus, the aggregate capital share can be expressed as $\alpha = \int \alpha_\tau \theta_\tau E_\tau \, dT(\tau)$. We show in Supplemental Appendix M.1 that the aggregate capital labor elasticity is given by

$$\sigma^{\text{agg}} - 1 = \frac{1}{\alpha(1-\alpha)} \int \frac{\mathrm{d}\alpha_\tau}{\mathrm{d}\ln w/r} \theta_\tau E_\tau \, \mathrm{d}T(\tau) + \frac{1}{\alpha(1-\alpha)} \int \frac{\mathrm{d}E_\tau}{\mathrm{d}\ln w/r} (\alpha_\tau - \alpha)\theta_\tau \, dT(\tau) + \chi \bar{s}^M(\bar\zeta - 1) + \chi\big(1 - \bar{s}^M\big)(\varepsilon - 1), \tag{D.1}$$

where $\chi \equiv \frac{\int (\alpha_\tau - \alpha)^2 \theta_\tau E_\tau \, \mathrm{d}T(\tau)}{\alpha(1-\alpha)}$, s^M_τ is the share of observed expenditures (including both the operating cost and variable costs) spent on intermediate materials, and ζ_τ captures substitution between intermediate and primary inputs, defined to satisfy $(\alpha - \alpha_\tau)(\zeta_\tau - 1)\frac{\mathrm{d}\ln \frac{s^M_\tau}{1-s^M_\tau}}{\mathrm{d}\ln w/r}$, $\bar{s}^M \equiv \frac{\int (\alpha_\tau - \alpha)(\alpha_\tau - \alpha^M)\theta_\tau s^M_\tau \, \mathrm{d}T(\tau)}{\int (\alpha_\tau - \alpha)(\alpha_\tau - \alpha^M)\theta_\tau \, \mathrm{d}T(\tau)}$, $\bar\zeta \equiv \frac{\int (\alpha_\tau - \alpha)(\alpha_\tau - \alpha^M)\theta_\tau s^M_\tau \zeta_\tau \, \mathrm{d}T(\tau)}{\int (\alpha_\tau - \alpha)(\alpha_\tau - \alpha^M)\theta_\tau s^M_\tau \, \mathrm{d}T(\tau)}$, and $\alpha^M \equiv \frac{\mathrm{d}\ln P/w}{\mathrm{d}\ln r/w}$.

The first term captures within-plant substitution between capital and labor. The second term captures the change in the aggregate capital share due to entry and exit; an increase in the wage induces labor-intensive plants to exit and capital-intensive plants to enter. The third term captures substitution between primary and intermediate inputs. The final term captures changes in plants' scales; an increase in the wage causes capital-intensive plants to expand relative to labor-intensive plants.

With this formula in hand, we now show that our baseline estimate of the aggregate elasticity of substitution is larger than the true aggregate elasticity. First, our estimated micro elasticity of substitution—in particular the estimate of $\bar\sigma$ derived using α_τ as the dependent variable in (C.1)—incorporates the first two terms of (D.1), capturing both within-plant substitution and changes due to entry and exit. At root, our cross-sectional estimates capture how the average capital share varies with the local wage, and changes in this average reflect both the intensive and extensive margins. In Supplemental Appendix M.2, we provide more detail, discuss how selection causes an upward bias similar to the weighted regressions in column 5 of Table C.I, and confirm these findings using Monte Carlo simulations in Supplemental Appendix M.4.

Second the estimated micro elasticity of substitution between intermediate and primary inputs, $\hat\zeta$, reported in Table III, is larger than $\bar\zeta$. $\bar\zeta$ captures only the intensive margin—substitution within plants—while $\hat\zeta$ uses cross-sectional variation and incorporates both the intensive and extensive margins.

Finally, our baseline strategy overstates how a plant's scale responds to a change in its marginal cost because part of this cost—the overhead cost—is fixed. Formally, we had estimated this response from plants' ratios of revenue to cost (in our baseline model, this was a function of the elasticity of demand, $\frac{\varepsilon}{\varepsilon-1}$). Here, cost includes both variable and overhead components: $\frac{\hat\varepsilon_\tau}{\hat\varepsilon_\tau - 1} = \frac{\frac{\varepsilon}{\varepsilon-1} c^v_\tau y_\tau}{c^v_\tau y_\tau + c^O f^O} < \frac{\varepsilon}{\varepsilon-1}$, or $\varepsilon < \hat\varepsilon_\tau$.

Together, these imply that our baseline estimate is an upper bound for the true aggregate elasticity. Using the restrictions that $\hat{\varepsilon} > 1$ and $\bar{\zeta} > 0$, the intensive margin effect (the first term in (D.1)) provides a conservative lower bound on the aggregate elasticity. We use our dynamic panel estimates to compute this lower bound. The implied range averages $[0.35, 0.65]$ across years.[6]

We also explore alternative assumptions about the factor content of overhead costs. If overhead costs used a plant's own output, then the upper bound for the aggregate elasticity is the same and the lower bound is slightly lower, and can be found by setting $\varepsilon = 0$. This averages 0.30 across years. If overhead costs used labor, the aggregate elasticity would include an extra term which captures the contribution of changes in the composition of expenditures between variable and overhead costs. However, this term is negative and quantitatively negligible, so the upper bound is the same as when the overhead cost uses final output, and the lower bound is slightly lower.

Foregone Labor

We now instead assume that both entry costs and overhead costs require the entrepreneur's labor, but these costs—the opportunity cost of the entrepreneur's time—do not appear on the plant's wage bill. In such a world, the measured capital share $\hat{\alpha}$, based on measured expenditures on labor and capital, differs from α, the true capital share incorporating unmeasured labor. Entry and overhead costs in unmeasured labor mean that the measured capital share is above the true capital share, so $\hat{\alpha} > \alpha$.

We then define two aggregate elasticities: $\hat{\sigma}^{\text{agg}} - 1 \equiv \frac{\mathrm{d}\ln \frac{\hat{\alpha}}{1-\hat{\alpha}}}{\mathrm{d}\ln w/r}$ captures changes in measured factor usage, while $\sigma^{\text{agg}} - 1 = \frac{\mathrm{d}\ln \frac{\alpha}{1-\alpha}}{\mathrm{d}\ln w/r}$ captures changes in true factor usage. $\hat{\sigma}^{\text{agg}}$ is relevant for questions about changes in national accounts, whereas σ^{agg} is relevant for questions such as the welfare cost of capital taxation. In practice, we show that the two elasticities are fairly close. The measured share elasticity $\hat{\sigma}^{\text{agg}}$ corresponds to our baseline estimate, while the resource-based elasticity σ^{agg} is slightly higher than our baseline estimate.

To see the connection between the two, we define the following objects: $V \equiv \int c_\tau^v y_\tau \times E_\tau \,\mathrm{d}T(\tau)$ and $O \equiv \int wf^O E_\tau \,\mathrm{d}T(\tau)$ be average expenditure on variable inputs and average payment of the operating cost among those that pay the entry cost. In addition, let $\hat{s}^M$ be the aggregate share of measured expenditures spent on intermediate inputs. Per entrant, the average expenditure on capital can be expressed as $\hat{\alpha}(1 - \hat{s}^M)V$, while the average expenditure on labor is $wf^E + O + (1 - \hat{\alpha})(1 - \hat{s}^M)V$. The underlying capital share is thus

$$\alpha = \frac{\hat{\alpha}(1 - \hat{s}^M)V}{wf^E + O + (1 - \hat{s}^M)V}.$$

Free entry requires that $wf^E = \frac{1}{\varepsilon-1}V - O$. Together, these yield

$$\alpha = \frac{(1 - \hat{s}^M)}{\frac{1}{\varepsilon - 1} + (1 - \hat{s}^M)}\hat{\alpha}. \tag{D.2}$$

[6]The lower bound uses an intensive margin micro elasticity of substitution of 0.34 from column (2) of Table C.II. To derive the upper bound, we compute the aggregate elasticity in each year using our baseline formula but using the estimated cross-sectional elasticity from column (4) of Table C.I.

In Supplemental Appendix M.3, we show that $\hat{\sigma}^{\text{agg}}$ corresponds to our baseline estimate. We also show that differentiating (D.2) and rearranging yields

$$\sigma^{\text{agg}} - \hat{\sigma}^{\text{agg}} = \frac{\dfrac{\mathrm{d}\ln(1-\hat{s}^M)}{\mathrm{d}\ln w/r} + \hat{\alpha}(1-\hat{\sigma}^{\text{agg}})}{1+(1-\hat{\alpha})(1-\hat{s}^M)(\varepsilon-1)}.$$

We then estimate that $\sigma^{\text{agg}} - \hat{\sigma}^{\text{agg}}$ averages 0.072 across all industries in all years.[7] A small positive gap between σ^{agg} and $\hat{\sigma}^{\text{agg}}$ is in line with our Monte Carlo analysis described in Supplemental Appendix M.4.

APPENDIX E: SHIFTS IN THE TECHNOLOGICAL FRONTIER

Shifts in factor prices may induce changes in the technological frontier, as outlined by Acemoglu (2002). Holding the technological frontier fixed, an increase in the wage would change the economy's capital-labor ratio. This would change the size of the market for innovations that complement each factor, and the subsequent adjustment of the technological frontier could amplify or dampen the initial wage increase.

We characterize the technological frontier as a set of intermediate input varieties that complement capital and a set that complement labor. The two sets can be respectively aggregated into two bundles, $M_K \equiv (\int_0^{N_K} M_K(j)^{\frac{\varphi-1}{\varphi}}\, dj)^{\frac{\varphi}{\varphi-1}}$ and $M_L \equiv (\int_0^{N_L} M_L(j)^{\frac{\varphi-1}{\varphi}}\, dj)^{\frac{\varphi}{\varphi-1}}$, so that the state of technology can be summarized by the measure of varieties of each type, N_K and N_L.

Plant i produces its output using capital, labor, intermediate inputs that complement capital and labor, as well as a third intermediate input that does not complement either factor. It is convenient to describe i's production function using a nested structure:

$$Y_i = F_i(Y_{Ki}, Y_{Li}, M_{0i}),$$

with $Y_{Ki} \equiv K_i^{\psi} M_{Ki}^{1-\psi}$ and $Y_{Li} \equiv L_i^{\psi} M_{Li}^{1-\psi}$.

Each variety of intermediate input is produced by a monopolist by using ϱ units of the final good aggregate, so no capital or labor is used. Monopolists compete monopolistically and thus set a price of $\frac{\varphi}{\varphi-1}\varrho P$. The unit cost of the input bundle that complements factor $x \in \{K, L\}$ is thus $q_x = (\int_0^{N_x} (\frac{\varphi}{\varphi-1}\varrho P)^{1-\varphi}\, dj)^{\frac{1}{1-\varphi}} = \frac{\varphi}{\varphi-1}\varrho P N_x^{\frac{1}{1-\varphi}}$.

Aggregate factor shares depend on relative factor prices and the technological frontier, N_K and N_L. We now distinguish between the short-run aggregate elasticity which holds the technological frontier fixed and the long-run elasticity which includes shifts in the frontier. These two elasticities are related by

$$\underbrace{\frac{\mathrm{d}\ln\dfrac{\alpha}{1-\alpha}}{\mathrm{d}\ln w/r}}_{\sigma^{\text{agg,LR}}-1} = \frac{\partial\ln\dfrac{\alpha}{1-\alpha}}{\partial\ln N_K}\frac{\mathrm{d}\ln N_K}{\mathrm{d}\ln w/r} + \frac{\partial\ln\dfrac{\alpha}{1-\alpha}}{\partial\ln N_L}\frac{\mathrm{d}\ln N_L}{\mathrm{d}\ln w/r} + \underbrace{\frac{\partial\ln\dfrac{\alpha}{1-\alpha}}{\partial\ln w/r}}_{\sigma^{\text{agg,SR}}-1}. \tag{E.1}$$

[7]Since the overhead cost is unmeasured, we have $\hat{\varepsilon} = \varepsilon$. We compute $\frac{\mathrm{d}\ln(1-\hat{s}^M)}{\mathrm{d}\ln w/r}$ directly after showing that it is closely related to the expression for ζ_n^N in Proposition 2. In most years, it is slightly negative and is always close to zero.

We show in Supplemental Appendix O that the effect of changes in the technological frontier on factor shares is characterized by

$$\frac{\partial \ln \frac{\alpha}{1-\alpha}}{\partial \ln N_K} = \frac{1}{\psi}\frac{1-\psi}{\varphi-1}\left(\sigma^{\text{agg,SR}}-1\right), \tag{E.2}$$

$$\frac{\partial \ln \frac{\alpha}{1-\alpha}}{\partial \ln N_L} = -\frac{1}{\psi}\frac{1-\psi}{\varphi-1}\left(\sigma^{\text{agg,SR}}-1\right). \tag{E.3}$$

For intuition, suppose that capital and labor are complements in the short run. The creation of varieties that complement capital reduces the relative cost of the capital aggregate Y_K, leading plants to reduce their relative expenditures on capital because Y_K and Y_L are complements.

To study shifts in the technological frontier, we specify its determinants in greater detail. A fixed mass Y of scientists invent new varieties and license their inventions to monopolists. Scientists can direct their research toward one of the two types of intermediate varieties. If a scientist devotes effort to finding new varieties that complement $x \in \{K, L\}$, then new varieties arrive at Poisson rate γN_x^{τ}, with $\tau < 1$.[8] Existing varieties become useless at rate δ. At an interior steady state, scientists must be indifferent about devoting effort to each type of innovation. We show in Supplemental Appendix O that the long-run technological frontier is characterized by $N_K^{1-\tau} = \alpha\frac{\gamma Y}{\delta}$ and $N_L^{1-\tau} = (1-\alpha)\frac{\gamma Y}{\delta}$. Differentiating with respect to relative factor prices gives

$$\frac{\mathrm{d}\ln N_K}{\mathrm{d}\ln w/r} = \frac{1}{1-\tau}(1-\alpha)\left(\sigma^{\text{agg,LR}}-1\right),$$

$$\frac{\mathrm{d}\ln N_L}{\mathrm{d}\ln w/r} = -\frac{1}{1-\tau}\alpha\left(\sigma^{\text{agg,LR}}-1\right).$$

Plugging these equations and (E.2) and (E.3) into (E.1) and rearranging gives

$$\sigma^{\text{agg,LR}} - 1 = \frac{1}{1+\frac{1}{\psi}\frac{1-\psi}{\varphi-1}\frac{1}{1-\tau}\left(1-\sigma^{\text{agg,SR}}\right)}\left(\sigma^{\text{agg,SR}}-1\right). \tag{E.4}$$

Because $\frac{1}{\psi}\frac{1-\psi}{\varphi-1}\frac{1}{1-\tau} > 0$, if $\sigma^{\text{agg,SR}} < 1$, then $\sigma^{\text{agg,LR}}$ is between $\sigma^{\text{agg,SR}}$ and 1. If $\sigma^{\text{agg,SR}} < 1$, an increase in wages initially raises the relative expenditure on labor. This induces the creation of varieties that complement labor, reducing the relative cost of the aggregate

[8] $\tau < 0$ implies that as more varieties are discovered, new varieties are harder to find. $\tau > 0$ would capture positive spillovers from past research. $\tau = 1$ would deliver endogenous growth in the number of varieties. We abstract from growth because it would require a number of additional assumptions about how plant-level technologies and the distribution of plants evolve over time. In Supplemental Appendix O, we study an alternative specification with spillovers across types of varieties, so that the arrival rate of new varieties that complement capital and that complement labor are $\gamma N_K^{\tau_1} N_L^{\tau_2}$ and $\gamma N_L^{\tau_1} N_K^{\tau_2}$ per unit of research, respectively. We impose $\tau_1 + \tau_2 < 1$ to avoid perpetual growth. We show that the relationship between the long-run and short-run elasticities of substitution described in (E.4) is identical with the exception that τ is replaced by $\tau_1 + \tau_2$.

Y_{Li}. Since Y_{Li} and Y_{Ki} are complements, plants shift expenditures away from Y_{Li} and hence away from labor, dampening the initial shift in factor shares.[9]

APPENDIX F: ADJUSTMENT FRICTIONS AND DISTORTIONS

Section 2 showed that the relative importance of within-plant substitution and reallocation depends upon the variation in cost shares of capital. In that environment, this variation came from non-neutral differences in technology. On the other hand, as the recent misallocation literature emphasizes, some of this heterogeneity may be due to adjustment costs or other distortions. What are the implications for the aggregate elasticity of substitution if differences in capital shares came from distortions? To answer this question, we first characterize the aggregate elasticity in terms of how plants change their input expenditures in response to permanent changes in factor prices, without taking a stand on how those choices are made.

To examine how each plant's input use would change with factor prices, we identify each plant with a history of shocks, h, which include shocks to demand and productivity. Let $H(h)$ be the distribution of histories, so the aggregate capital share is $\alpha = \int \alpha_h \theta_h \, dH(h)$. We define σ_h and ζ_h as the local response of plant h's relative factor expenditures to a change in factor prices, so $\sigma_h - 1 \equiv \frac{\mathrm{d}\ln \frac{\alpha_h}{1-\alpha_h}}{\mathrm{d}\ln w/r}$ and $(\alpha^M - \alpha_h)(\zeta_h - 1) = \frac{\mathrm{d}\ln \frac{s_h^M}{1-s_h^M}}{\mathrm{d}\ln w/r}$. We make no assumption about how these objects are related to h's production function; σ_h and ζ_h simply reflect how h's choices would change with different factor prices.

As in our baseline, we characterize σ^{agg} by differentiating each side of $\alpha = \int \alpha_h \times \theta_h \, dH(h)$. In Section 2, we used Shephard's lemma to characterize plants' changes in scale. Here, we do not presume that K_h, L_h, and M_h minimize the plant's static cost, so we cannot make use of the envelope theorem. Instead, as we show in Supplemental Appendix N.1, differentiating yields

$$\sigma^{\mathrm{agg}} = (1-\chi)\bar{\sigma} + \chi \bar{s}^M \bar{\zeta} + \int \frac{(\alpha_h - \alpha)\theta_h}{\alpha(1-\alpha)} \left[s_h^K \frac{d\ln K_h}{d\ln w/r} + s_h^L \frac{d\ln L_h}{d\ln w/r} + s_h^M \frac{d\ln M_h}{d\ln w/r} \right] dH(h), \tag{F.1}$$

where $\bar{\sigma}$, $\bar{\zeta}$, χ, and $\bar{s}^M$ are defined as before, and s_h^K, s_h^L, and s_h^M are plant h's respective cost shares of capital, labor, and materials.

Exogenous Wedges. We first study an environment motivated by Hsieh and Klenow (2009) and Restuccia and Rogerson (2008) in which plants behave as if there are plant-specific taxes on each input. While plant i's cost of capital, labor, and intermediate inputs are r, w, and q, respectively, it behaves as if these costs were $(1+\tau_{Ki})r$, $(1+\tau_{Li})w$, and $(1+\tau_{Mi})q$. We assume that the distortions themselves do not change with relative factor prices.

We use a perturbation approach to characterize how misallocation affects the aggregate elasticity. The perturbation parameter ν refers to an economy in which the wedges are $1+\nu\tau_{Ki}$, $1+\nu\tau_{Li}$, and $1+\nu\tau_{Mi}$. Thus, a frictionless economy corresponds to $\nu = 0$,

[9]Acemoglu (2003) studied a model with $\tau = 1$ and shows that long-run factor shares are fixed. While we have imposed the restriction $\tau < 1$ to abstract from growth, we can recover Acemoglu's (2003) result in the limit of $\tau \nearrow 1$.

while the economy with misallocation corresponds to $\nu = 1$. Taking a first-order approximation around $\nu = 0$, the difference between the true underlying aggregate elasticity for the manufacturing sector and our baseline estimate is

$$\sigma^{\text{agg}} - \hat{\sigma}^{\text{agg}} \approx \sum_i \frac{(\alpha_i^* - \alpha^*)\theta_i^*}{\alpha^*(1-\alpha^*)} \left\{ \begin{array}{c} \alpha_i^*(1-\alpha_i^*)(\tau_{Ki} - \tau_{Li})(\varepsilon - \sigma_i^*) \\ + [\alpha_i^* \tau_{Ki} + (1-\alpha_i^*)\tau_{Li} - \tau_{Mi}] s_i^{M*}(1 - s_i^{M*})(\alpha_i^* - \alpha^*)(\varepsilon - \zeta_i^*) \end{array} \right\},$$

where variables with stars are the values in the undistorted economy with $\nu = 0$.

If all dispersion in factor shares were due to wedges rather than to changes in technology so that $\alpha_i^* = \alpha^*$, to a first-order approximation, our baseline estimate would recover the true aggregate elasticity. In order to proceed beyond this special case, we must model specific mechanisms through which endogenous wedges would covary with plants' technologies and with factor prices. We therefore turn to explicit models of adjustment costs.

Adjustment Costs. In this section, we study a class of capital adjustment frictions that nests time-dependent frictions such as Calvo (1983) and Taylor (1980) as well as time-to-build adjustment frictions. Formally, we parameterize capital adjustment frictions by $\{\bar{\Gamma}_j\}_{j=0}^{\infty}$. If a plant is able to choose capital freely in period t, $\bar{\Gamma}_j$ is the probability that that choice determines capital in period $t + j$.[10] The fraction of plants in the cross-section whose usage of capital was determined by a choice made j periods ago is $\Gamma_j \equiv \frac{\bar{\Gamma}_j}{\sum_{j'=0}^{\infty} \bar{\Gamma}_{j'}}$. A plant with technology τ can produce with the production function $F(\cdot; \tau)$. We assume that τ follows a Markov process with a stationary distribution $T(\tau)$, and that initial conditions are such that each plant's time since last adjustment, j, is independent of its technology, τ.

We make two simplifying assumptions: First, plants do not discount the future, which we believe is a reasonable approximation given the horizon of adjustment frictions. Second, a plant with technology τ produces with the CES production function $F(K, L; \tau) \equiv [(A_\tau K)^{\frac{\sigma-1}{\sigma}} + (B_\tau L)^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma}{\sigma-1}}$.

Plants may choose capital only occasionally, so a decision today may determine future input usage. Without adjustment frictions, a plant would tailor its inputs to match its technology and demand period by period. With adjustment frictions, a plant can only match its capital to its shocks in expectation. This difference will affect how plants' scales change with a change in relative factor prices. Thus, a plant's choice of capital satisfies $\mathbb{E}_t[\sum_{j=0}^{\infty} \Gamma_j (MRPK_{i,t+j} - r)]$, where $MRPK_{i,t+j}$ is i's marginal revenue product of capital, whereas its choice of labor is a static decision that satisfies $MRPL_{i,t+j} = w$. To find how plants' input usage changes with relative factor prices, we can simply differentiate these equations with respect to relative factor prices.

Compared to our baseline model, the model with adjustment costs makes different inferences about how plants' scales change when factor prices change, given the same data. In the baseline model, Shephard's lemma implies that the change in a plant's scale when the cost of capital falls is proportional to its actual capital share. Thus, variation in

[10]For Calvo-style adjustment frictions for which ν is the probability that a plant is able to adjust each period, $\bar{\Gamma}_j = (1-\nu)^j$. For Taylor-style adjustment frictions in which capital can be adjusted every j^* periods, $\bar{\Gamma}_j = 1\{j < j^*\}$. For time-to-build adjustment frictions in which capital chosen today will not be operational j^* periods later, $\bar{\Gamma}_j = 1\{j = j^*\}$.

capital shares in the cross section reflects scope for reallocation across plants in response to changes in factor prices.

With adjustment costs, the variation in capital shares in the cross-section can be decomposed into variation within non-adjustment spells and variation across non-adjustment spells. Like in the baseline model, the variation across spells measures the scope for reallocation: a permanent increase in the relative cost of labor causes plants that expect to be more capital intensive to choose capital at the beginning of the spell and to choose labor during the spell so that they expand more, on average, than plants that expect to be labor intensive. In contrast, a plant is limited in how it can reallocate resources from capital-intensive states to labor-intensive states within a spell.

If shocks tend to be Hicks-neutral, we can show analytically that $\sigma^{\text{agg}} < \hat{\sigma}^{\text{agg}}$. That is, given the same data, the model with adjustment frictions infers less scope than the baseline model for reallocation between capital-intensive and labor-intensive plants/states. If shocks are non-neutral, this may not hold.[11] We cannot measure the bias of technology shocks. Therefore, to assess the sign of the overall bias, we examine four scenarios: shocks are purely Hicks-neutral; shocks are purely labor-augmenting; shocks are purely capital-augmenting; shocks to A and B are perfectly negatively correlated. For each we take a second-order approximation around a fixed-technology benchmark, and approximate the distribution of $\bar{\alpha}(\tau)$ using the empirical distribution of capital shares. As we detail in Supplemental Appendix N.2, the actual aggregate elasticity would be lower than our baseline estimate in all four cases.

The magnitude of the bias is increasing in the within-spell variation in technology. To gauge the magnitude, we posit that the cross-sectional distribution of technology was generated by an autoregressive process. As an upper bound, we study a case in which technology has no persistence—the IID case—which maximizes within-spell variation in technology. In that case, the difference between our baseline estimate and the true aggregate elasticity is 0.026.

Plant-Specific Prices. Last, we consider an environment in which plants pay idiosyncratic prices for their inputs. Formally, plant i pays factor prices $r_i = (1 + \tau_{Ki})r$, $w_i = (1 + \tau_{Li})w$, and $q_i = (1 + \tau_{Mi})q$, where the plant-specific input-price premium might reflect compensating differentials or supplier markups. For example, our identification of the plant-level elasticity of substitution relies on plants in different locations facing different wages. We define the aggregate elasticity of substitution as how factor shares respond to relative factor prices:[12]

$$\sigma^{\text{agg}} - 1 = \frac{\mathrm{d}\ln \dfrac{\alpha}{1-\alpha}}{\mathrm{d}\ln w/r},$$

where $\alpha \equiv \frac{\sum_i r_i K_i}{\sum_i r_i K_i + w_i L_i}$ and the derivative is taken holding fixed the input price premia, $\{\tau_{Ki}, \tau_{Li}, \tau_{Mi}\}$.

In this context, the aggregate elasticity of substitution is exactly the same as our baseline expression in Proposition 1, provided that we define the shares in terms of factor payments

[11] For example, if shocks tend to be purely capital augmenting, plants that expect to be extremely labor intensive ($\bar{\alpha}(\tau) < 0.15$) will make choices of labor that are sufficiently different across states that the model with adjustment costs infers more scope for reallocation than the baseline model.

[12] In this environment, changes in the capital-labor ratio do not map directly into changes in factor compensation, so $\frac{\mathrm{d}\ln K/L}{\mathrm{d}\ln w/r} - 1 \neq \frac{\mathrm{d}\ln \frac{\alpha}{1-\alpha}}{\mathrm{d}\ln w/r}$.

that include plant-specific prices. Thus, as long as expenditures are measured correctly, no modifications are necessary.

APPENDIX G: LOCAL VERSUS NATIONAL ELASTICITIES

In this section, we examine several reasons why the response of plants' capital-labor ratios to local factor prices might differ from the response to a national change.[13] Our identification strategy has focused on studying how plant capital-labor ratios respond to local factor prices. There are a few reasons why this may differ from plants' response to a nationwide change in the wage. It is, of course, the latter which is relevant for an aggregate elasticity at the national level. While these issues are seldom discussed in the literature, they are relevant for *any* estimate of an elasticity of substitution at a level of aggregation smaller than the entire world.

Sorting. Our estimates do not account for the possibility that plants select locations in response to factor prices. To see why this might matter, consider the following extreme example: Suppose plants cannot adjust their factor usage but can move freely. Then we would expect to find more labor-intensive plants in locations with lower wages. A national increase in the wage would not, however, change any plant's factor usage. Thus, to the extent that this channel is important, our estimated elasticity will overstate the true elasticity.

Plants' ability to sort across locations likely varies by industry. We would expect industries in which plants are more mobile to be more clustered in particular areas. This could depend, for example, on how easily goods can be shipped to other locations. Raval (2019) addressed this by looking at a set of ten large four-digit industries located in almost all MSAs and states. These are industries for which we would expect sorting across locations to be least important. The leading example of this is ready-mixed concrete; because concrete cannot be shipped very far, concrete plants exist in every locality. Elasticities for these industries are similar to the estimates for all industries in our baseline, with average elasticities of 0.40 for 1987, 0.51 for 1992, and 0.38 for 1997.

Within-Firm Coordination. In our baseline model, we assumed that each plant independently selects its factor intensity in response to local factor prices. It is possible, however, that a firm that operates plants in many locations might derive some scale economy by operating all of its plants at similar capital-labor ratios. If this is the case, then a change in factor prices in one location would, by altering the choices of these multi-unit firms, affect factor intensities in other locations. A potential problem for our approach is that, in such an environment, comparing factor intensities across locations would not reveal the full extent of substitution in response to factor prices.[14] Thus our estimate would understate the true elasticity. One can gauge the importance of this channel by estimating an elasticity of substitution among the subset of plants that belong to multi-unit firms. If this channel is important, one would expect that plants in multi-unit firms would respond less to their local wages than standalone plants. However, column 4 of Table II indicates that the estimated elasticity among this subset is higher than our OLS estimates, suggesting that this channel is not of first-order importance.

[13] The distinction has been emphasized recently in the debate about the size of government spending multipliers; see Beraja, Hurst, and Ospina (2019).

[14] As an extreme example, suppose the economy consisted of a single firm that operated in two locations and chose a common capital-labor ratio for its two plants. Our methodology would never uncover differences in capital-labor ratios across locations no matter how much the firm adjusted this common ratio.

The Technological Frontier. A similar issue may arise when we consider changes in the technological frontier. A change in factor prices in one location might induce the creation of intermediates that favor particular technologies. The interpretation of our estimate depends on whether those intermediates are available nationwide (as assumed in Section 4.3), or only locally.

Assessment. One way to gauge the importance of these mechanisms is to ask how factor shares in a location respond to the wage in nearby locations. For example, plants that move are more likely to move to nearby locations. To the extent that moving across locations is important, a decrease in the wage in a nearby location would induce the most labor-intensive plant to move to that nearby location, raising the average local capital share. Similarly, a decline in the wage in a nearby location may induce the creation of intermediate inputs that favor capital. To the extent that the intermediates are also available locally, these would also favor capital locally, and lower the average local capital share.

A regression of local factor shares on local wages and the average wages of all other locations in the state can reveal the impact of these mechanisms. Note that it can only reveal the net impact of all of them; we cannot distinguish whether neither mechanism matters or that they both matter but offset each other. Fortunately, it is the net effect that is relevant for informing us about the gap between the cross-sectional differences and national changes. When we run this regression, we cannot reject that the net impact of both mechanisms is zero. See Supplemental Appendix H.4 for these results.[15]

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[15]As discussed above, such a regression cannot pick up any induced changes in the technological frontier that are truly global.

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