Supplementary Material

Supplement to "Evaluating default policy: The business cycle matters"

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Appendix A: Data and calibration

This appendix describes the data and provides additional calibration details.

A.1 *Data*

Gross domestic product (GDP) and its components are taken from the National Income and Product Accounts (NIPA). Consumer durables are separated from consumption and added to investment. All values are converted to real values using the GDP deflator. The GDP deflator is also used to convert the nominal interest rates to real ones. For labor supply, I follow Ohanian and Raffo (2011) and take annual hours worked from the Conference Board's Total Economy Database (TED).[1] The number of households is taken from the Census Bureau's historical tables.[2]

The model was constructed to capture the salient features of Chapter 7 bankruptcy in the United States. The filing rate is thus measured as the ratio of Chapter 7 filings to the number of households. Filings data that go back to 1960 are available only on a fiscal year basis ending in June until 1990.[3] To recover the annual figure ending in December for year y prior to 1990, I use the average of y and $y+1$. To test how well this works, I compare this method's values with the known values for the period 1990–2004. This produces an R^2 of 0.985.

Three measures of debt are used in the paper because of data availability issues. First, I use net worth from the Survey of Consumer Finances (SCF). This is the closest measure of $a-x$ in the model. Second, I use the Federal Reserve Board (FRB) of Governors' measures of revolving consumer credit (as well as their charge-off rates on credit cards and interest rate on 2-year personal loans).[4] Last, I use the unsecured debt measure in Bermant and Flynn (1999) (BF). As Bermant and Flynn (1999) only look at filers,

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[1] Available at http://www.conference-board.org/data/economydatabase/#files.

[2] Available at http://www.census.gov/population/www/socdemo/hh-fam.html#ht (Table HH-1).

[3] Available at http://www.uscourts.gov/Statistics/BankruptcyStatistics.aspx#calendar in various pdf files.

[4] Available at http://www.federalreserve.gov/releases/g19/HIST/default.htm and http://www.federalreserve.gov/releases/chargeoff/chgallsa.htm. The interest rates on 2-year personal loans are slightly lower than the rates on credit cards, but the time series goes back to 1972 rather than just 1994.

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this should be a good approximation for $-a + x$, since these households typically have no assets.

Call the three debt measures SCF, FRB, and BF, respectively. In Table 1 of the main paper, the following debt measures are used: the "Debt–Output Ratio" is measured using SCF; the "Discharged Debt–Output Ratio" is measured using FRB charge-off rates times the FRB debt; the "Debt–Income of Filers" is measured using BF. In Table 2 of the main paper, "Debt" is measured using FRB. Likewise, "Discharged Debt" is measured using the FRB charge-off rate times the FRB debt. Last, in Figure 2 of the main paper, the "Debt–Output Ratio" is measured using FRB.

Figure A1 shows the annual percent of Chapter 7 filings per household for the period 1960–2012. The number of households filing drastically increased starting in 1984, leveled off in the late 1990s, experienced a spike in 2005, a sharp decrease in 2006 and 2007, and a subsequent recovery.[5] Unfortunately, the sample period drastically affects not only the level of bankruptcies but also their cyclicality and volatility. Because of this, Table A1 reports statistics for the sample periods 1960–1984, 1985–2004, 1997–2004, and 1960–2004. In addition to 1984 and 2005 being breakpoints visually, these were years

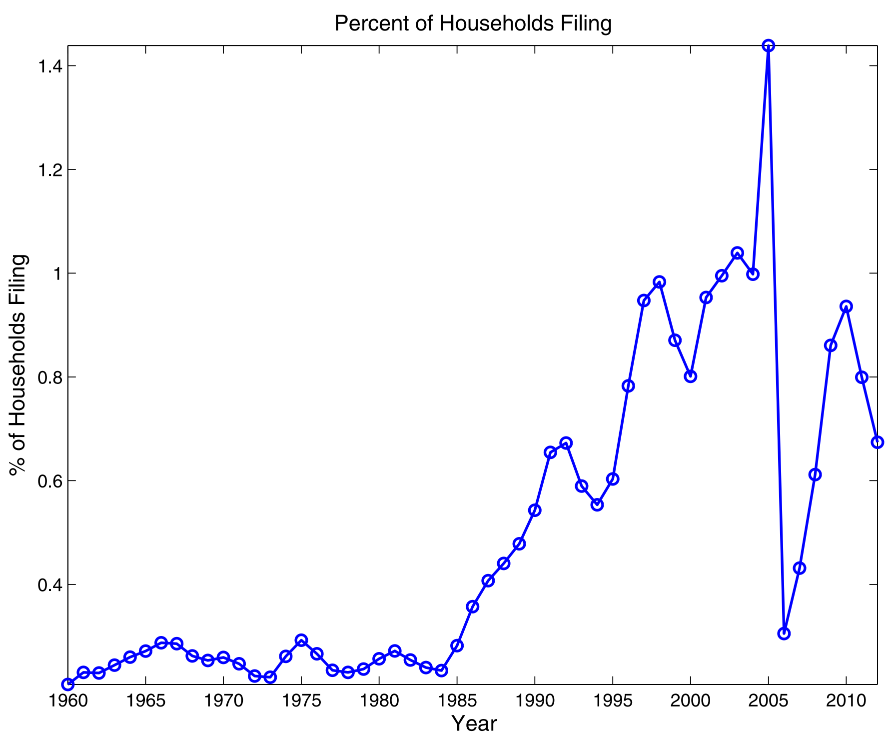

FIGURE A1. Chapter 7 filings per household (1960–2012).

[5]Several possible explanations for the drastic rise in filings have recently been evaluated by Livshits, MacGee, and Tertilt (2010). The spike in 2005 and subsequent sharp decrease is presumably due to anticipation of BAPCPA clearing out potential defaulters.

TABLE A1. Cyclical properties of Chapter 7 filings per household.

Statistic	1960–2004	1960–1984	1997–2004	1985–2004
Corr(filing, output)	−0.01	−0.31	−0.96	−0.06
St.dev.(filing) in %	10.25	7.53	8.22	11.23
St.dev.(output) in %	1.99	2.14	1.35	1.53
St.dev.(filing)/St.dev.(output)	5.14	3.51	6.11	7.33
Mean(filing*) in %	0.45	0.25	0.95	0.70

*In levels.

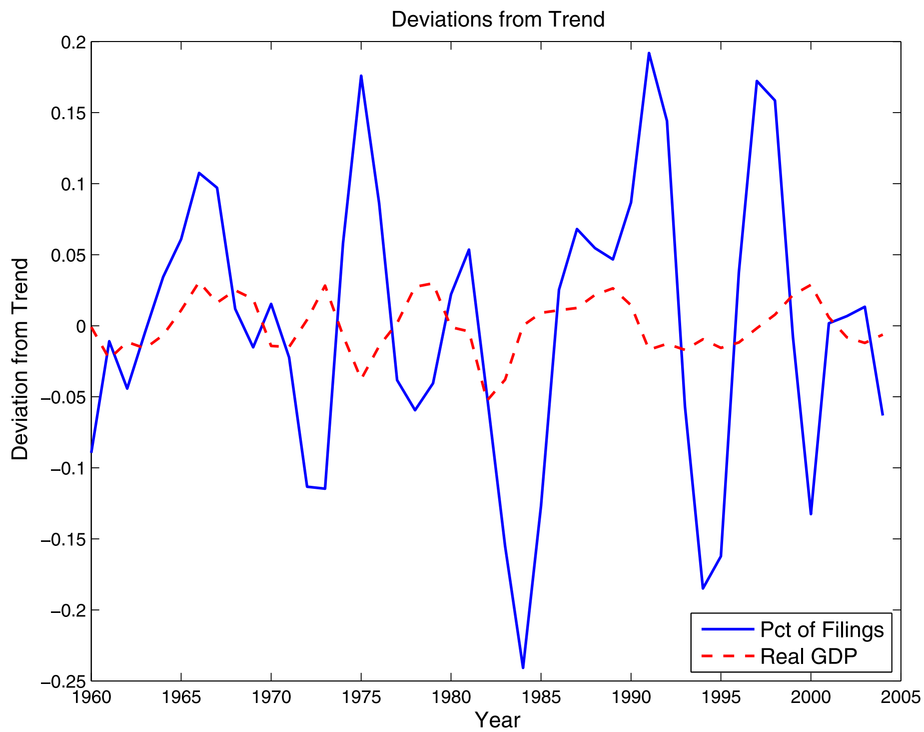

FIGURE A2. Cyclical properties of filings (1960–2004).

of substantial bankruptcy reform.[6] Bankruptcy filings are between 3.5 and 7.3 times more volatile than output and are acyclical to countercyclical, with a correlation between −0.01 and −0.96 depending on the sample period. Figure A2 plots log filings per household and log real GDP using the HP filter with parameter 100 for the sample 1960–2004. Visually, bankruptcy filings are much more volatile than output and appear to be countercyclical, but not strongly so.

[6]There were essentially two rounds of substantial bankruptcy legislation. The first round began with the *Marquette* decision in 1979, which was subsequently amended by the Bankruptcy Amendment Act of 1984. The second round began with the Bankruptcy Reform Act of 1999, which was passed by Congress but not signed into law. Subsequent revisions of this legislation resulted in BAPCPA.

A.2 *Calibration*

There are several calibration details omitted in the main text. This section fills out those details. Table A2 lists the profiles of all variables.

One omitted detail is how the sample in Karahan and Ozkan (2011) of ages 24–60 was dealt with. To obtain the shock persistences for younger and older households, I assume the cubic profile is correct. To obtain the standard deviations, I assume the profile is correct for older households, but use a different approach for younger households. Specifically, I assume the standard deviation is constant for younger households and choose it

Table A2. Parameter values for profiles.

Age	Parameters*					Age	Parameters*				
	ρ_s	θ_s	ϕ_h	γ_h	$\sigma^2_{\eta,h,1}$		ρ_s	θ_s	ϕ_h	γ_h	$\sigma^2_{\eta,h,1}$
20	0.9994	1.18	1.000	0.694	0.038	53	0.9947	1.34	1.880	0.965	0.019
21	0.9994	1.18	1.031	0.717	0.038	54	0.9942	1.32	1.852	0.961	0.021
22	0.9994	1.19	1.064	0.739	0.038	55	0.9937	1.30	1.818	0.956	0.023
23	0.9994	1.25	1.099	0.760	0.038	56	0.9931	1.28	1.777	0.950	0.025
24	0.9994	1.33	1.137	0.779	0.038	57	0.9925	1.26	1.729	0.944	0.028
25	0.9993	1.40	1.176	0.798	0.044	58	0.9919	1.24	1.673	0.938	0.030
26	0.9993	1.46	1.217	0.816	0.041	59	0.9912	1.23	1.611	0.931	0.033
27	0.9993	1.51	1.259	0.833	0.037	60	0.9905	1.22	1.540	0.923	0.036
28	0.9993	1.55	1.301	0.849	0.034	61	0.9897	1.22	1.462	0.916	0.039
29	0.9993	1.59	1.345	0.863	0.031	62	0.9888	1.22	1.375	0.907	0.042
30	0.9992	1.62	1.389	0.877	0.029	63	0.9878	1.21	1.280	0.898	0.045
31	0.9992	1.64	1.433	0.890	0.026	64	0.9867	1.21	1.176	0.889	0.048
32	0.9992	1.66	1.476	0.902	0.024	65	0.9855	1.20	1.176	0.889	0.048
33	0.9991	1.67	1.520	0.913	0.022	66	0.9841	1.20	1.176	0.889	0.048
34	0.9991	1.68	1.562	0.923	0.020	67	0.9827	1.19	1.176	0.889	0.048
35	0.9990	1.68	1.604	0.932	0.018	68	0.9813	1.19	1.176	0.889	0.048
36	0.9990	1.68	1.645	0.940	0.016	69	0.9797	1.18	1.176	0.889	0.048
37	0.9989	1.68	1.684	0.948	0.015	70	0.9780	1.18	1.176	0.889	0.048
38	0.9987	1.67	1.721	0.955	0.014	71	0.9760	1.17	1.176	0.889	0.048
39	0.9986	1.65	1.756	0.960	0.013	72	0.9738	1.16	1.176	0.889	0.048
40	0.9984	1.64	1.789	0.965	0.012	73	0.9714	1.15	1.176	0.889	0.048
41	0.9982	1.62	1.819	0.970	0.012	74	0.9687	1.15	1.176	0.889	0.048
42	0.9980	1.60	1.846	0.973	0.011	75	0.9657	1.14	1.176	0.889	0.048
43	0.9978	1.58	1.870	0.976	0.011	76	0.9622	1.13	1.176	0.889	0.048
44	0.9976	1.56	1.891	0.978	0.011	77	0.9583	1.12	1.176	0.889	0.048
45	0.9973	1.53	1.908	0.979	0.011	78	0.9540	1.11	1.176	0.889	0.048
46	0.9971	1.51	1.921	0.980	0.012	79	0.9492	1.10	1.176	0.889	0.048
47	0.9968	1.49	1.930	0.980	0.012	80	0.9438	1.09	1.176	0.889	0.048
48	0.9965	1.46	1.935	0.979	0.013	81	0.9379	1.08	1.176	0.889	0.048
49	0.9962	1.44	1.935	0.977	0.014	82	0.9312	1.06	1.176	0.889	0.048
50	0.9959	1.41	1.929	0.975	0.015	83	0.9237	1.05	1.176	0.889	0.048
51	0.9955	1.39	1.919	0.973	0.016	84	0.9154	1.04	1.176	0.889	0.048
52	0.9951	1.37	1.902	0.969	0.018	85	0.0000	1.02	1.176	0.889	0.048

*ρ_s is the conditional probability of survival; θ_s is adult-equivalent household size; ϕ_h is deterministic earnings profile; γ_h is income shock persistence; $\sigma^2_{\eta,h,1}$ is persistent shock variance.

so that the unconditional standard deviation at age 24 is the same as what Karahan and Ozkan (2011) find.

Another detail is how the earnings profile was constructed from the estimates of Hubbard, Skinner, and Zeldes (1994). Hubbard, Skinner, and Zeldes (1994) estimate separate deterministic profiles for household heads with less than 12 years of education (NHS, no high school diploma), 12–15 years (HS, high school diploma), and 16+ years (COL, college) of education. I average the profiles of these three types for the year 1986 assuming 13% of the population is NHS, 48% is HS, and 39% is COL. This breakdown of educational attainment is from the 2010 Current Population Survey for ages 30–34.[7]

Last, the consumption equivalence profile was constructed assuming $\theta_s = f(N_s)$, where N_s is the average number of household members (the profile of which was provided by Alexander Bick and Sekyu Choi) and f is the mean equivalence scale in Fernández-Villaverde and Krueger (2007) linearly interpolated to be continuous. This leads to a consumption equivalence profile very similar to the one in Livshits, MacGee, and Tertilt (2007).

Appendix B: Computation

This section describes the computation.

B.1 *Grids*

The steady-state model is computed using a collection of standard techniques. The efficiency process is discretized using the method of Tauchen (1986). The persistent shock is discretized with 15 points, with a coverage of ± 6.25 "average unconditional standard deviations," $\bar{\sigma}_{\eta,1}/\sqrt{1-\bar{\rho}^2}$ (which is roughly ± 5 average unconditional standard deviations in recessions). The overbar denotes the numerical average across ages. The transitory shock is discretized with 5 points and a coverage of ± 5 standard deviations. The right-tail process was discretized with 10 log-spaced points. The mass on each point was also computed using Tauchen (1986) (e.g., the mass on the low point is the cdf evaluated at the midpoint of the lowest two points). The asset grid (which is really the knots of a linear interpolant) is composed of 150 points, 60 of which are strictly negative. These are unevenly spaced and concentrated close to zero. The household problem is solved using backward induction with linear splines (interpolants) representing continuation utilities and expenditure schedules. A grid search is first employed to avoid local minima, followed by a one-dimensional version of the simplex method.

B.2 *Business cycle computation*

The model with aggregate risk is computed using the method of Krusell and Smith (1998). The "moments" used are the capital–labor ratio and a parameter that controls

[7]Available at http://www.census.gov/hhes/socdemo/education/data/cps/2010/tables.html (Table 1).

the equity premium.[8] The equity premium W is defined on the domain $[0, 1]$, and it controls $\bar{q}_g$ and $\bar{q}_b$ as

$$\bar{q}_g(\mathcal{S}) = \left(R(\mathcal{G}) + \frac{W}{1-W} \frac{F(b|z)}{F(g|z)} R(\mathcal{B}) \right)^{-1}, \tag{A1}$$

$$\bar{q}_b(\mathcal{S}) = \left(\frac{(1-W)}{W} \frac{F(g|z)}{F(b|z)} R(\mathcal{G}) + R(\mathcal{B}) \right)^{-1}, \tag{A2}$$

where $R(\mathcal{S}) = 1 + r(\mathcal{S}) - \delta$. The probabilities ensure the equilibrium value of W is always close to, but slightly above, 0.5. The household problem is solved with backward induction. Linear interpolation is used to interpolate between aggregate moments (as well as in the household problem). To optimize, a grid search is used, followed by a simplex algorithm (either one-dimensional or two-dimensional, depending on the portfolio restrictions). For results to be comparable between the steady-state and business cycle versions, the same number and placement of grid points are used in both models (in both the asset and efficiency direction).

The laws of motion take the form

$$\begin{bmatrix} K'/N' \\ W' \end{bmatrix} = \begin{bmatrix} \Gamma_{11,zz'} & \Gamma_{12,zz'} & 0 \\ \Gamma_{21,zz'} & \Gamma_{22,zz'} & 0 \end{bmatrix} \begin{bmatrix} 1 \\ K/N \\ W \end{bmatrix}. \tag{A3}$$

Here the subscript zz' means the coefficients are z- and z'-dependent. Note that W does *not* influence the one-step-ahead forecast. This allows the equity premium to vary, *holding fixed all other prices*. The resulting laws of motion are very accurate for both the experiments in the main paper and the robustness tests.

Because the computational burden is heavy, especially in terms of memory usage, the number of points used for the aggregate moments is kept to a minimum: five in the K/N direction linearly spaced within $\pm 15\%$ of the steady-state value and three in the W direction placed at 0.50, 0.51, and 0.52. Because the natural borrowing limit and welfare depend on the worst possible scenario that can occur and on how quickly it can be reached, the bounds and the number of grid points used can influence the results. The bounds I have chosen I believe are reasonable, especially given U.S. history, where the capital-output ratio declined by some 20% in the Great Depression and hours worked per household has seen declines of 4% since 1960 (relative to trend).[9]

The model is simulated nonstochastically as in Young (2010) for 5000 periods, and the first 250 periods are discarded. At each point in the simulation, the bond market must be cleared. Because the equity premium is included in the aggregate state space, this involves linearly interpolating the asset policies to find an equilibrium W.

[8]The use of the capital–labor ratio rather than capital and labor separately significantly reduces the difficulty of the problem and reduces error from highly unlikely states (e.g., very low capital, very high labor) being included in household expectations. Including an equity premium as a state variable is fairly common in the literature (see, for instance, Storesletten, Telmer, and Yaron (2007)) because it speeds up the simulation.

[9]Author's calculations. Hours worked per household are calculated as discussed in Appendix A. The capital–output ratio is constructed from NIPA data using the current cost net stock of fixed assets divided by nominal GDP.

B.3 *Computing the transition*

For computing transitions between steady states, the following algorithm is used. Let a superscript of "old" denote a variable associated with the original steady state and "new" with the new one. Let a superscript of t denote the time period.

1. Fix T, the length of the transition period.
2. Take $K^1 = K^{\text{old}}$ and guess on $\{K^t\}_{t=2}^T$, a level of the capital stock for each period in the transition.
3. Compute $\{\bar{q}^t, r^t, w^t\}_{t=1}^T$ associated with the capital stock sequence.
4. Using backward induction with $V^T := V^{\text{new}}$, compute the value function, policy functions, and price schedules for each time period. To compute V^t and a'^t, one needs q^t, which is known since it depends only on d^{t+1} and factor prices.
5. Using forward induction, compute μ^{t+1} as a function of μ^t and a'^t for each $1 \leq t \leq T-1$.
6. Compute the implied capital stock for each t and call it $\tilde{K}^t$.
7. If $\max_{2\leq t\leq T} |K^t - \tilde{K}^t| < \epsilon_K$, go to step 8. Otherwise, go to step 2 with an updated guess on the capital stock of $\theta_K K^t + (1-\theta_K)\tilde{K}^t$ for $2 \leq t \leq T$.
8. If $|\tilde{K}^T - K^{\text{new}}| < \delta_K$, STOP. Otherwise, increase T and go to step 1.

A transition length of $T = 50$ was sufficient when using $\delta_K = 0.001$ and $\epsilon_K = 0.0001$. The differences in the capital–output ratio of BAPCPA and FS are fairly small, so $\theta_K = 0.8$ worked without trouble.

B.4 *Business cycle welfare calculations*

Welfare in the business cycle is computed as follows. In the Krusell and Smith (1998) method, one replaces the aggregate state variables with "moments" m, and the aggregate law of motion maps moments and shocks into moments tomorrow. Linear interpolation in the moments, which performs a convex combination of adjacent values on a grid, effectively induces a probability distribution over the moments tomorrow, $F(m'|m, z, z')$ with $\sum_{m'} F(m'|m, z, z') = 1$ and $F(m'|m, z, z') \geq 0$.[10] Hence, there is an implied Markov chain $F(m', z'|m, z)$. The computational equivalent of the G_X ergodic distribution in Section 4.1 is $F(m, z)$, the invariant distribution of the implied Markov chain.

An interesting alternative to using this unconditional $F(m, z)$ distribution is to consider

$$\frac{\sum_s \hat{F}(s|z) \int_e V_{\text{FS}}(0, e, s, 0; \mathcal{S}) \hat{f}(e|s, z)\, de}{\sum_s \hat{F}(s|z) \int_e V_{\text{NFS}}(0, e, s, 0; \mathcal{S}) \hat{f}(e|s, z)\, de} - 1, \tag{A4}$$

[10]This is only the case for interpolation. Extrapolation would result in $F(m'|m, z, z') < 0$.

which is a function of $\mathcal{S}$. One can compute the mean and standard deviation of this measure over the business cycle. If this is done, the average is close to the unconditional measure in the paper. Specifically, the average gain of moving from FS to NFS is -0.125% (the unconditional measure is -0.128%). The standard deviation is 0.110%.[11]

Appendix C: Robustness

This section conducts many robustness checks. I have tried to include the most important ones, but the source code can be used for additional checks. Although there are many bankruptcy regimes in the paper, only FS and NFS are considered, as they are the most different from one another and the cost of computing these checks is quite large.

C.1 *An alternative debt calibration*

The benchmark calibration used debt statistics from Chatterjee, Corbae, Nakajima, and Ríos-Rull (2007), which are based on net worth measures from the Survey of Consumer Finances (SCF). When one uses debt measures based on revolving consumer credit, as Livshits, MacGee, and Tertilt (2007) do, the debt statistics are much larger. As a robustness check, I recalibrated the model to match a debt–output ratio of 6.22 (the average revolving consumer credit to GDP ratio from 1995 to 2004; authors' calculations) and a population in debt of 17.6 (the population with zero or negative net worth in 2001 according to Wolff (2010)), with all other targets the same. The results are presented in Table A3. The fit is not as good as in the baseline calibration. Even with the flexible default cost structure $\chi(e) = \chi_0 - \chi_1 e^{-1}$, the model struggles to match both debt and default rates.

This alternative calibration has, quite naturally, a massive impact on average borrowing limits as can be seen in Figure A3. Now the average borrowing limit is typically higher in FS than in NFS, with or without aggregate risk. Additionally, there is a noticeable contraction in credit for middle-aged households.

Table A4 reports the welfare gain of implementing NFS along with other statistics for FS and NFS. Given how this calibration changes borrowing limits, it should not be surprising that the welfare of NFS relative to FS is now much lower. Without aggregate risk, the welfare gain of moving from FS to NFS is -4.76%, that is, a large welfare loss. With aggregate risk, the welfare gain is even lower at -5.12%. The mechanism by which it is lower is the same as in the main text: High default costs result in a contraction in credit, and the NFS contraction is larger than the FS.

[11] In fact, NFS looks better in recessions. The principle reason is simply that households want to borrow in recessions (since earnings are expected to be higher in the future), and NFS has larger borrowing limits.

TABLE A3. Statistics for a high-debt calibration.

Statistic	Data	Model	Parameter	Value
Targeted Statistics				
Capital–output ratio	3.08	3.07	β	0.956
Debt–output ratio × 100*	6.22	4.41	χ_0	0.974
Population filing (%)	0.93	0.63	χ_1	0.563
Population in debt (%)	17.6	20.8	π_{rl}	0.213
Earnings share of top 20%	60.2	59.7	$\underline{u}$	0.454
Earnings mean–median	1.57	1.84	$\overline{u}$	27.95
Wealth mean–median	4.03	4.00	ξ	0.183
Untargeted Statistics				
Wealth share of top 20%	81.7	73.0		
Wealth share of 4th quintile	12.2	22.0		
Wealth share of 3rd quintile	5.0	5.8		
Wealth share of 2nd quintile	1.3	0.5		
Wealth Gini	0.80	0.73		
Earnings share of 4th quintile	22.9	16.3		
Earnings share of 3rd quintile	13.0	11.1		
Earnings share of 2nd quintile	4.0	7.6		
Earnings Gini	0.61	0.54		
Average interest on debt (%)	12.7	4.1		
Discharged debt–output ratio × 100*	0.32	0.31		
Discharged $-a$–output ratio × 100*	0.32	0.07		
Debt–income of filers	1.62	2.08		
Debt–income of below-median filers	1.66	2.10		
Debt–income of above-median filers	1.58	1.98		
Percentage of filers below median	68.8	90.7		
Population with $d = 1$, any h (%)		0.77		
Right-tail population filing		0.00		
Right-tail debt–output ratio × 100		0.93		

Note: Model debt is measured as $-a + x$, a filing is measured as $h = 0$ and $d = 1$, and discharged debt is measured as $-a + x$ when $h = 0$ and $d = 1$. Statistics marked with an asterisk (*) have debt in the data measured with revolving consumer credit.

C.2 *No expenditure shocks*

As suggested in the main text, expenditure shocks play an important role in evaluating bankruptcy policy. In fact, in terms of insurance coefficients, NFS dominates FS except with respect to expenditure shocks. This section explores what happens if there are no expenditure shocks. All other parameters are kept at their baseline values.[12] Note that NFS without expenditure shocks now completely eliminates default, and the model reduces to a standard incomplete markets environment with a natural borrowing limit.

[12]An earlier version of this paper (available by request) has no expenditure shocks in the baseline. The main difference in results is with respect to BAPCPA. As discussed in the text, BAPCPA does very poorly at insuring against these shocks. As a consequence, the long-run implications for filing rates in BAPCPA with and without expenditure shocks are different.

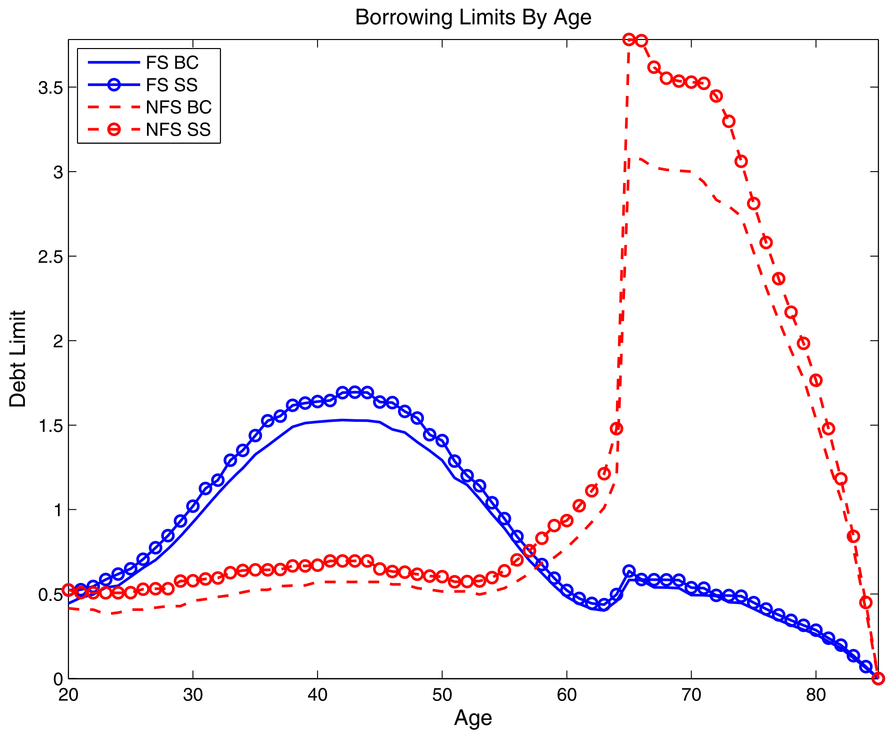

FIGURE A3. Average borrowing limits by age for a high-debt calibration.

Table A4 reports statistics and welfare without expenditure shocks. Note that the welfare gain of moving from FS to NFS is now very large but is significantly reduced once aggregate risk is added. There is a similar story with the debt statistics where the debt–output ratio in NFS falls by 45% once aggregate risk is added. These results suggest that the levels of the welfare gain and debt associated with NFS are not robust, but how aggregate risk affects them is.

C.3 *Guaranteed earnings prior to retirement*

While in most studies the use of a natural borrowing limit rather than some exogenously fixed limit is of secondary importance, here a natural limit occurs as the consequence of implementing NFS. Consequently, it is extremely important. There are then three issues to consider, namely, what is the limit in the theory, in the computation, and in the data? In the theory, the use of a log-efficiency process implies efficiency, and hence labor income, can be arbitrarily close to zero prior to retirement. However, because some labor income is *guaranteed* in retirement, the natural limit will not be zero as long as $\kappa_G > 0$. In the computation, the log process is discretized using a large support.[13] This makes the

[13]Specifically, the support is $\pm 6.25\bar{\sigma}_{\eta,1}/\sqrt{1-\bar{\gamma}^2}$ for the persistent shock and $\pm 5\bar{\sigma}_{\varepsilon}$ for the transitory shock, where an overbar denotes the numerical average across ages. Another justification for using a large coverage is that the estimates omit the very poor.

Table A4. Robustness to calibrations and expenditure shocks.

	Benchmark	High Debt	No Exp.
FS → *NFS*			
Welfare gain SS (%)	0.21	−4.76	4.22
Welfare gain BC (%)	−0.13	−5.12	3.05
FS			
K/Y SS	3.08	3.07	3.13
K/Y BC	3.09	3.09	3.14
Debt/Y SS	0.61	4.41	0.34
Debt/Y BC	0.56	3.74	0.28
Population in debt SS	7.22	20.82	8.86
Population in debt BC	6.83	19.48	7.69
Population filing SS	0.97	0.63	0.15
Population filing BC	0.98	0.68	0.18
NFS			
K/Y SS	2.99	3.03	3.06
K/Y BC	3.00	3.04	3.10
Debt/Y SS	2.11	1.82	6.80
Debt/Y BC	1.47	1.32	3.69
Population in debt SS	15.15	12.26	28.94
Population in debt BC	12.78	10.11	23.33
Population filing SS	0.64	0.35	0.00
Population filing BC	0.63	0.36	0.00

lowest efficiency realization prior to retirement close to zero. Last, Carroll (1992) documents that noncapital household income in the data, *including* transfer income, falls to (or very close to) zero between 0.30% and 0.65% of the time for working-age households.

While a low minimum value for earnings during working life is thus reasonable to use, I explore the robustness of the results to this assumption by guaranteeing larger earnings. Specifically, given the efficiency distribution of the benchmark economy, I replace any values less than a threshold τ with τ and renormalize so that $N = 1$ in steady state. I consider four different thresholds $\tau \in \{0.008, 0.058, 0.127, 0.233\}$. These represent a lower bound of roughly \$500, \$3500, \$7600, and \$14,000, taking average household labor income to be \$60,000, and are the values considered in Athreya (2008).

Table A5 reports the results. As τ increases, so does the welfare and the debt associated with NFS. In contrast, FS statistics are virtually unchanged. However, for all the τ considered, the welfare gain and debt associated with NFS fall when aggregate risk is added.

These results can be easily explained. The natural borrowing limit is changed by aggregate risk through fluctuating factor prices. The effect is largest when discounting plays a large role. When τ is low, most guaranteed earnings are in retirement; when τ is high, guaranteed earnings are always present. In this latter case, discounting plays

Table A5. Robustness to guaranteed income.

$\tau =$	0.008	0.058	0.127	0.233
FS $\to$ *NFS*				
Welfare gain SS (%)	0.21	2.77	3.00	3.76
Welfare gain BC (%)	−0.13	2.05	2.52	3.49
FS				
K/Y SS	3.08	3.08	3.08	3.08
K/Y BC	3.09	3.09	3.09	3.08
Debt/Y SS	0.61	0.62	0.61	0.66
Debt/Y BC	0.56	0.56	0.56	0.60
Population in debt SS	7.22	7.26	7.25	9.06
Population in debt BC	6.82	6.83	6.88	8.11
Population filing SS	0.97	1.00	0.98	0.95
Population filing BC	0.98	0.98	0.98	0.97
NFS				
K/Y SS	2.99	2.94	2.94	2.89
K/Y BC	3.00	2.96	2.95	2.91
Debt/Y SS	2.11	8.14	9.08	15.32
Debt/Y BC	1.47	5.55	7.05	13.62
Population in debt SS	15.15	24.50	25.20	28.46
Population in debt BC	12.78	21.48	23.14	27.57
Population filing SS	0.64	0.64	0.64	0.64
Population filing BC	0.63	0.65	0.64	0.64

a small role. Also, higher values of τ truncate downside risk, which is precisely what bankruptcy is designed to do. Overall, these results suggest that, to the extent earnings are guaranteed in working life, it would be substantially welfare improving to implement NFS.

C.4 *Retirement schemes*

In the benchmark calibration, labor income in retirement is composed of a guaranteed fraction $\kappa_G = 0.15$ of average earnings and a fraction $\kappa_F = 0.35$ of earnings from the last period of working life. The average replacement rate is roughly 50% because $\kappa_G + \kappa_F = 0.5$. However, as already discussed, it is not the replacement rate that really matters but how much of it is guaranteed. I now examine the robustness of the results to alternative replacement schemes (κ_G, κ_F) subject to keeping $\kappa_G + \kappa_F = 0.5$.

Table A6 records the results. Overall, welfare in levels is not greatly affected by these changes. Also, in each case, aggregate risk causes the welfare and the debt associated with NFS to fall, with the largest reductions in debt occurring for $\kappa_G = 0.5$. These results agree with the argument developed in the paper: A large amount of guaranteed retirement earnings results in a large natural borrowing limit that is substantially affected by fluctuating factor prices.

TABLE A6. Robustness of results to alternative retirement schemes.

$(\kappa_G, \kappa_F) =$	(0.00, 0.50)	(0.30, 0.20)	(0.50, 0.00)
FS → *NFS*			
Welfare gain SS (%)	−0.59	0.07	−0.01
Welfare gain BC (%)	−0.65	−0.22	−0.30
FS			
K/Y SS	3.17	3.02	2.97
K/Y BC	3.18	3.02	2.97
Debt/Y SS	0.61	0.61	0.61
Debt/Y BC	0.55	0.57	0.60
Population in debt SS	8.02	7.06	6.88
Population in debt BC	7.05	6.59	6.43
Population filing SS	0.88	1.01	1.04
Population filing BC	0.91	1.05	1.14
NFS			
K/Y SS	3.09	2.92	2.86
K/Y BC	3.10	2.93	2.87
Debt/Y SS	0.93	2.52	3.84
Debt/Y BC	0.77	1.86	2.77
Population in debt SS	10.21	16.11	18.05
Population in debt BC	9.20	13.80	15.58
Population filing SS	0.59	0.67	0.70
Population filing BC	0.59	0.66	0.68

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