A Dynamic Model of Rational "Panic Buying"
Online Appendix (Not for Publication)

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C Mobility at Grocery Stores

In Appendix C, we provide evidence that suggests the public health measures placed during the COVID-19 pandemic made grocery shopping more costly. We use Google Community Mobility Reports (available at https://www.google.com/covid19/mobility/?hl=en) to document how people in the United States changed grocery shopping activity around the stay-at-home order. In Figure C.1, each gray line represents how visits and length of stay at grocery stores changed in the 30 days before and after the stay-at-home order at each state (see Table C.1 for the day when the stay-at-home order was implemented around the first wave of COVID-19 spread). The black line represents its country-wide average, weighted by the state population as of April 2020. They are reported in the percent change compared to the median value for that day of the week during the 5-week period January 3-February 6, 2020.

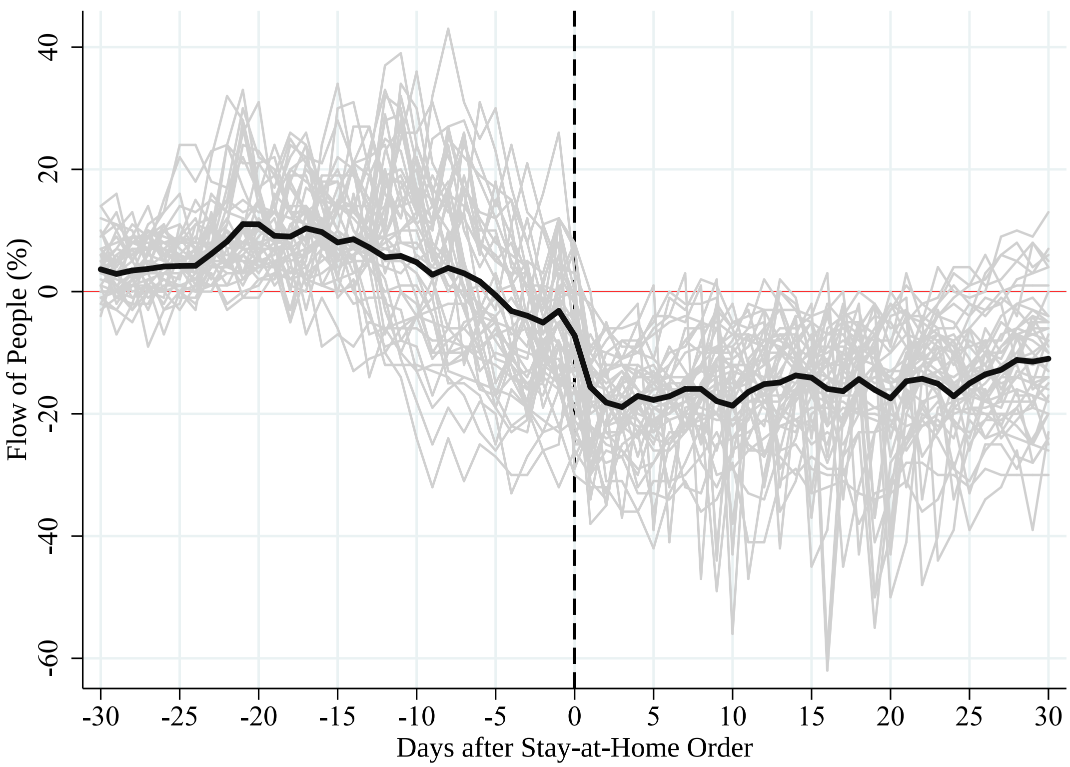


Figure C.1: Visits and Length of Stay at Grocery Stores Around Stay-at-Home Orders

Table C.1: Start Date of Stay-at-Home Orders

State	Start Date	State	Start Date
Alabama	April 4	Montana	March 28
Alaska	March 28	Nebraska	None
Arizona	March 31	Nevada	March 31
Arkansas	None	New Hampshire	March 27
California	March 19	New Jersey	March 21
Colorado	March 26	New Mexico	March 24
Connecticut	March 23	New York	March 20
Delaware	March 24	North Carolina	March 30
District of Columbia	April 1	North Dakota	None
Florida	April 3	Ohio	March 23
Georgia	April 3	Oklahoma	April 2
Hawaii	March 25	Oregon	March 23
Idaho	March 25	Pennsylvania	April 1
Illinois	March 21	Rhode Island	March 28
Indiana	March 24	South Carolina	April 7
Iowa	None	South Dakota	None
Kansas	March 30	Tennessee	April 2
Kentucky	March 26	Texas	April 2
Louisiana	March 23	Utah	None
Maine	April 2	Vermont	March 24
Maryland	March 30	Virginia	March 30
Massachusetts	March 24	Washington	March 23
Michigan	March 24	West Virginia	March 24
Minnesota	March 27	Wisconsin	March 25
Mississippi	April 3	Wyoming	None
Missouri	April 6		

D Interpretation of the Rationing Rule

In this section, we provide a microfoundation for the rationing rule of our model by demonstrating that it can be derived as a large-market limit of a finite economy.

We begin with a discrete-period model, where each period has a physical time length of $dt > 0$. Let $\mu > 0$ be a scaling parameter that characterizes the size of the market. We take the limit of $\mu \to \infty$ later while we always choose μ so that μdt becomes an integer. At each period, μdt consumers arrive at the market. In addition, $s\mu dt$ units of product are supplied to the store. Note that the ratio between the inflow of consumers (μ) and supplied product ($s\mu$) is fixed for any μ and dt.

In each period, all consumers, μdt of them, are randomly sorted. Each consumer i demands $Q_i(t) = A_i(t)dN_i(t)q_i(t)$ units of the product, where $A_i(t)$ represents whether the consumer searches or not, $dN_i(t)$ represents whether the consumer arrives at the store or not, and $q_i(t)$ represents the quantity of the product demanded. It is worth noting that we consider the "potential" demand, i.e., the demand of all consumers who search and arrive at the store, rather than only the demand of those who actually make a purchase. These two specifications are equivalent because $A_i(t)dN_i(t) = 0$ for all consumers who do not arrive at the store in period t.

The dynamics of the aggregate stock are primarily determined by $Q_i(t)$: all consumers, except for at most one person, face either (i) sufficient stock, where inventory levels are not a constraint, or (ii) no stock, where purchasing is not an option. As we focus on the large market limit, the presence of the marginal consumer diminishes. To simplify the analysis, we assume that if the stock does not deplete before consumer i's turn, she will purchase Q_i, and will be unable to purchase the product otherwise.

Without loss of generality, we can reorder consumers so that consumers arrive at the store in ascending order of their indices, $i = 1, 2, \ldots, \mu dt$. Consider an arbitrary rational number $\gamma \in (0, 1)$. We derive the condition under which the consumer who arrived at the store in the γth percentile can purchase the product in the limit of $\mu \to \infty$. To this end, we take a sequence μ in which $\gamma\mu dt$ becomes an integer. Note that whenever γ is rational, for any $\bar{\mu} > 0$, there exists μ such that both μdt and $\gamma\mu dt$ are integers. Let $i = \gamma\mu dt$ be the consumer who is positioned at the γth percentile.

Consumer i can meet her unconstrained demand $Q_i(t)$ if and only if

$$\sum_{j=1}^{\gamma\mu dt} Q_j(t) < s\mu dt,$$

or equivalently,

$$\gamma \cdot \frac{\sum_{j=1}^{\gamma\mu dt} Q_j(t)}{\gamma\mu dt} < s. \tag{D.1}$$

We consider a large market limit of this economy, i.e., take a limit of $\mu \to \infty$. Indeed, the main model assumes that infinitely many consumers arrive for any time interval $[t, t + dt]$. Since we assume that consumers are ordered uniformly at random, we can apply the law of large numbers. In the limit of $\mu \to \infty$, (D.1) becomes

$$\gamma \mathbb{E}[Q_i(t)] < s. \tag{D.2}$$

Rearranging the terms in (D.2) yields

$$\gamma < \frac{s}{\mathbb{E}[Q_i(t)]}.$$

Since this conclusion holds for all rational $\gamma \in (0, 1)$, the same conclusion holds for all real $\gamma \in (0, 1)$.

From now on, let us assume that there is a unit mass of (infinitely many) recurring consumers. The total amount of the product demanded in this length-dt period is given by

$$dD(t) \coloneqq \int_{i\in[0,1]} A_i(t) \cdot dN_i(t) \cdot q_i(t)di.$$

Meanwhile, the mass of consumers in this period is given by dt. Accordingly, we have

$$\mathbb{E}[Q_i(t)] = \frac{dD(t)}{dt} =: d(t).$$

Thus, the proportion of consumers who can buy the product without any constraints in each period can be expressed as $s/d(t)$ for any dt, which is consistent with the main model. Similarly, all other consumers are unable to purchase anything. Since this proportion remains constant regardless of the duration of each period, by taking the limit as $dt \to 0$, we derive the same rationing rule for the main continuous model.

E Supplementary Materials

E.1 Supplementary Materials for Section 5

E.1.1 Iteration and Cognitive Hierarchy

In the iterative scheme to find the equilibrium path of $R(t)$, we use the following algorithm. Let $\tilde{R}^j(t)$ represent the consumer's belief for $R(t)$ at round $j = 0, 1, 2, \ldots$, and $\hat{R}^j(t)$ represent the path of $R(t)$ that would be achieved if consumers acted on the belief $\{\tilde{R}^j(t)\}_{t \geq 0}$. Starting with an initial guess $\tilde{R}^0(t) = 1$ for all $t \geq 0$, we update the consumer's belief according to

$$\tilde{R}^{j+1} = (1 - \lambda)\tilde{R}^j + \lambda \hat{R}^j \quad \text{with} \quad \lambda \in (0, 1), \tag{E.1}$$

and repeat this until the differences between $\tilde{R}^j$ and $\hat{R}^j$ become sufficiently small.

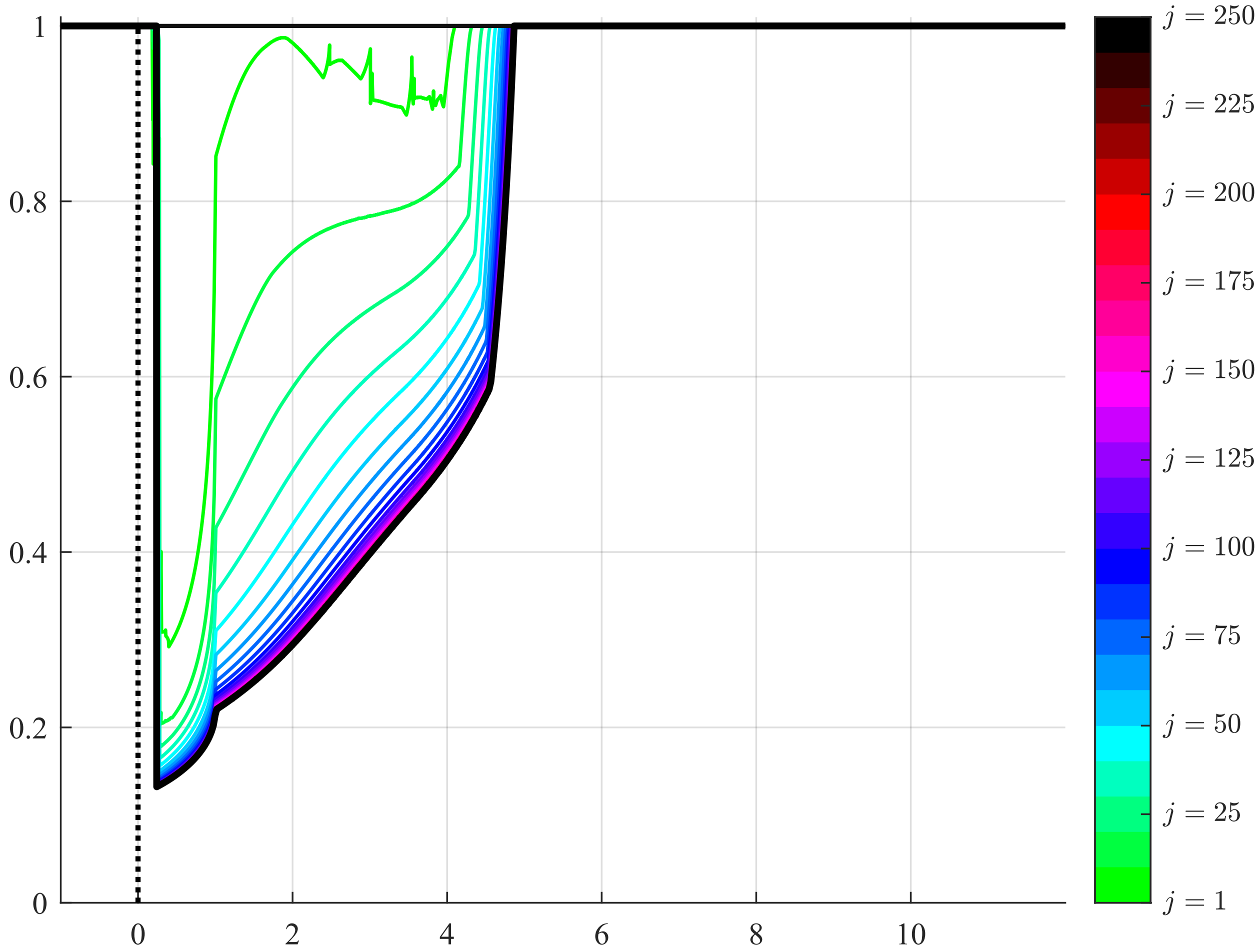
Figure E.1 depicts the updated beliefs of consumers $\tilde{R}^1, \tilde{R}^2, \ldots$, showing the convergence process of consumer's belief for the path of $R(t)$ to the equilibrium in the benchmark simulation. There are three noteworthy observations to make.

First, we cannot observe a rational expectations equilibrium (REE) with $R(t) = 1$ for all t after the shock hits the market. Initially, we set $\tilde{R}^0(t) = 1$ for all $t \geq 0$. In each iteration, $\tilde{R}^j$ is a weighted average of $\bar{R}^{j-1}$ and the aggregate dynamic that emerges as the optimal response against $\tilde{R}^{j-1}$. The existence of a time period t where $\tilde{R}^1(t) \neq 1$ indicates that, after the shock arrives, and consumers act rationally, there is a shortage.

Second, we find no evidence of an equilibrium in which the shortage is less severe. To search for a fixed point, we begin with the most optimistic initial guess, $\tilde{R}^0(t) = 1$ for all t, and iteratively compute best responses. Figure E.1 displays the monotonic convergence of $\tilde{R}^j$, suggesting that there is no fixed point (i.e., equilibrium) where the availability $R(t)$ is higher than the one we have presented, although this is not conclusive evidence of the uniqueness of the equilibrium transition dynamics.

Third, we can interpret the hoarding-demand spiral using cognitive hierarchy theory (Camerer, Ho, and Chong, 2004). In our economy, the level-0 agents are consumers who adhere to the stationary-equilibrium shopping strategy. The level-k agents are consumers who optimize their shopping strategy, taking into account the fundamental shock, while assuming that all other consumers are level $k-1$. As such, $\tilde{R}^j$ roughly represents the "level-j" consumer's belief about product availability. Figure E.1 illustrates this cognitive hierarchy by showing how each level of consumer's belief converges towards the equilibrium.[1]

[1] Two key differences exist between our updating rule and the standard cognitive hierarchy theory. First, we update consumers' beliefs about product availability $\tilde{R}^j$, rather than their shopping strategy. However, this difference is not significant as we are analyzing a mean-field game, in which a consumer's problem is



Note: This figure illustrates the process where consumers update their belief for the path of $R(t)$ using the updating rule (E.1) with $\lambda = 1/6$, each colored solid line representing $\tilde{R}^j(t)$ for $j = 0, 10, 20, 30, \ldots$. The horizontal axis represents the number of weeks after the announcement.

Figure E.1: Illustration of the Process of Searching for the Rational Belief for $R(t)$.

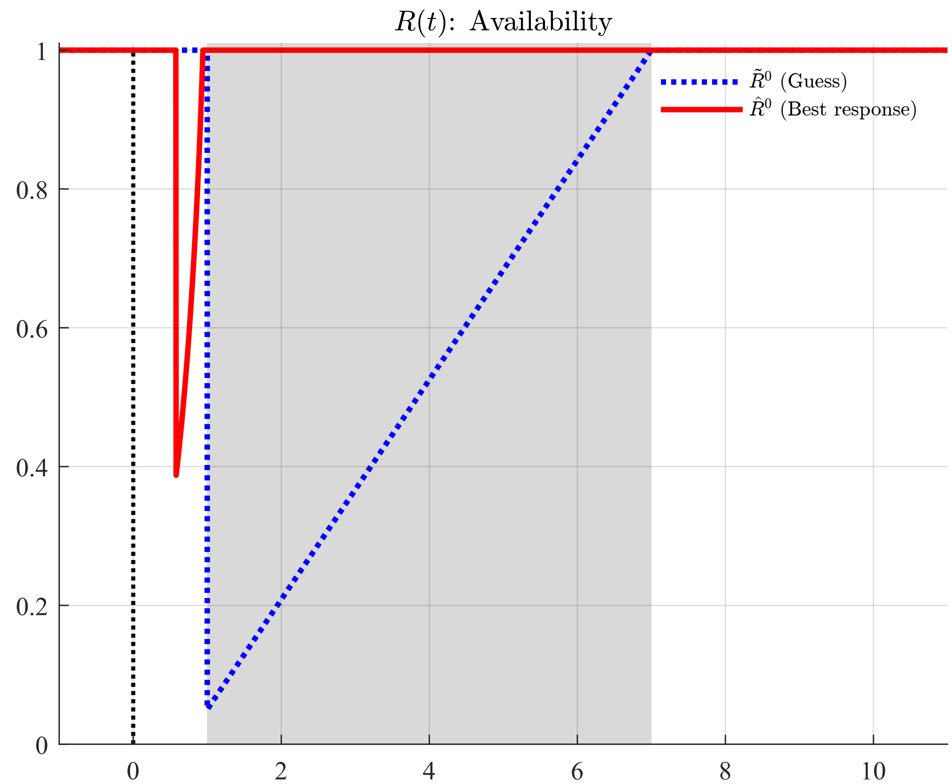
The impact of the shopping-cost shock on hoarding demand in the first round is relatively small. However, as consumers realize that other consumers will respond to the shock and there will be a slight shortage of products, they begin to stockpile more. This leads to an increase in hoarding demand driven by the fear of lower product availability, which in turn increases hoarding demand further in subsequent rounds. This iterative process continues until an equilibrium point is reached, capturing how the shortage grows through the spiral of defensive hoarding.

influenced by other consumers' strategies only through product availability. Second, we define the level-$j+1$ consumers' belief $\tilde{R}^{j+1}$ as a convex combination of (i) the availability in level j, $\tilde{R}^j$, and (ii) the availability obtained as the best response against $\tilde{R}^j$. While the standard cognitive hierarchy theory puts the entire weight on (ii), we only assign a 90% weight on (i) for computational stability. This is because assigning too much weight on (ii) would cause abrupt belief changes, making the calculation errors more significant.

E.1.2 Self-fulfilling Panics

In this section, we briefly investigate the potential for self-fulfilling panics. To be specific, we explore whether our model can rationalize $R(t) < 1$ without any exogenous changes in the model parameters. For that purpose, holding all the parameter values fixed, we consider an exogenous shift in consumer belief regarding $R(t)$, i.e., $\tilde{R}^0(t) < 1$ for a certain period (see Online Appendix E.1.1 for the definition of $\tilde{R}^0$).

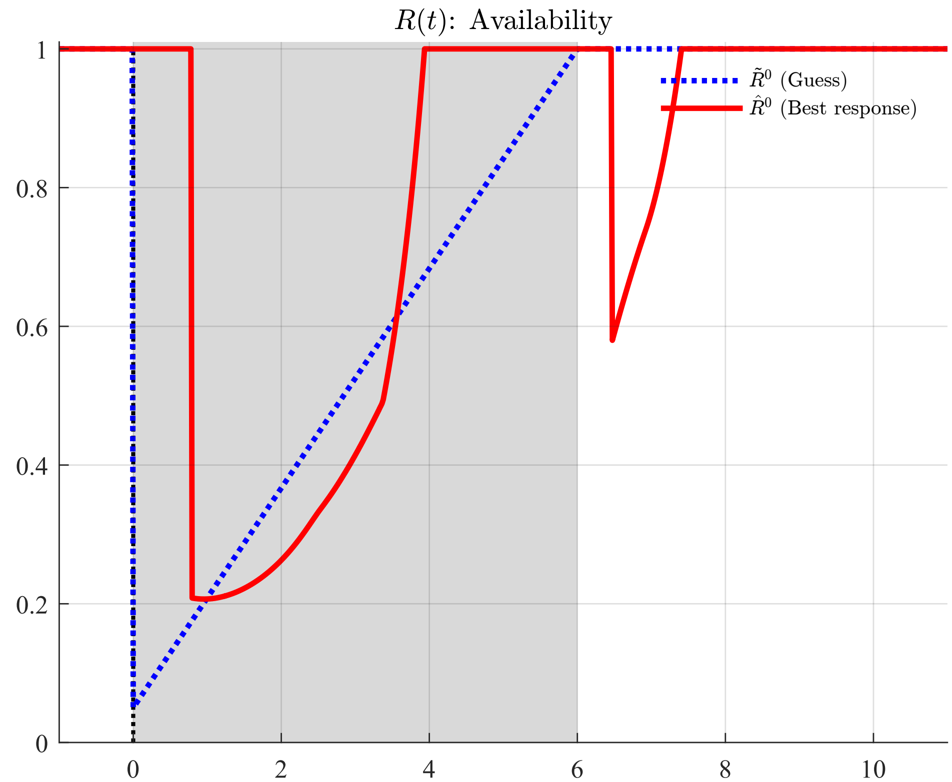
Here, instead of demonstrating a global absence of belief changes leading to self-fulfilling panics, we explore specific belief changes and provide intuitive explanations for why they do not result in (self-) fulfillment. In Figure E.2, we illustrate consumer belief $\tilde{R}^0(t)$ with a dotted line and the best response to this belief, $\hat{R}^0(t)$, with a solid line. It is evident that when a future shortage is anticipated without any changes in fundamentals, consumers make their purchases in advance of the expected shortage, resulting in the realized shortage occurring earlier than initially expected. Thus, the initial expectation of a future shortage cannot be fulfilled.



Note: All the parameter values are fixed.

Figure E.2: Response of $R(t)$ to an Exogenous Shift in $\tilde{R}(t)$.

In Figure E.3, we illustrate the scenario where consumers anticipate an immediate shortage. It also reveals that such an expectation does not lead to fulfillment, as consumers delay their purchases until the shortage is expected to ease, causing the realized shortage to occur later than initially anticipated.



Note: All the parameter values are fixed.

Figure E.3: Response of $R(t)$ to an Exogenous Shift in $\tilde{R}(t)$.

In summary, in the absence of any changes in fundamentals, consumers refrain from making purchases when severe shortages are anticipated. Consequently, expectations of shortages do not become self-fulfilling.

E.1.3 Additional Policy Simulations

E.1.3.1 Sale-Tax Increases We consider a month-long tax increase, comparing its effectiveness with different tax rates and implementation lags in Table E.1.

Policy Simulation (Short-Term Sales-Tax Hike)**.** The government imposes a special sales tax of τ percent for a month (4.3 weeks) from d weeks after recognizing the news of a shopping-cost increase. The after-tax price is given by

$$\hat{p}(t) = \begin{cases} \left(1 + \dfrac{\tau}{100}\right) \cdot p & \text{if } t \in [d, 4.3 + d]; \\ p & \text{otherwise.} \end{cases}$$

Table E.1: A Shopping-Cost Shock with Short-Term Sales-Tax Hike

Tax rate (τ)	**Implementation lags** (d)								
	$d = 0$			$d = 1/2$ (week)			$d = 1$ (week)		
	$R(t) < 0.33$	Ω_{tax}	GR	$R(t) < 0.33$	Ω_{tax}	GR	$R(t) < 0.33$	Ω_{tax}	GR
$\tau = 0$ (Benchmark)	2.12 (weeks)	5.05	0						
$\tau = 3.0$	1.91	4.60	2.04	2.00	4.80	1.29	2.04	4.88	1.24
$\tau = 6.0$	1.68	4.15	4.08	1.88	4.55	2.58	1.96	4.72	2.46
$\tau = 9.0$	1.41	3.70	6.12	1.76	4.31	3.86	1.88	4.56	3.68

Note: GR is the present value of increased government revenues ($GR = \int_0^\infty \mathrm{e}^{-rt}[\hat{p}(t) - p(t)]R(t)d(t)dt$).

Table E.1 shows that the sales-tax increase can be useful in reducing the welfare cost of a shopping-cost shock if it is implemented immediately, and that, if its implementation is delayed even a few days, the effect is limited since it encourages consumers to shop before the (after-tax) price will increase.

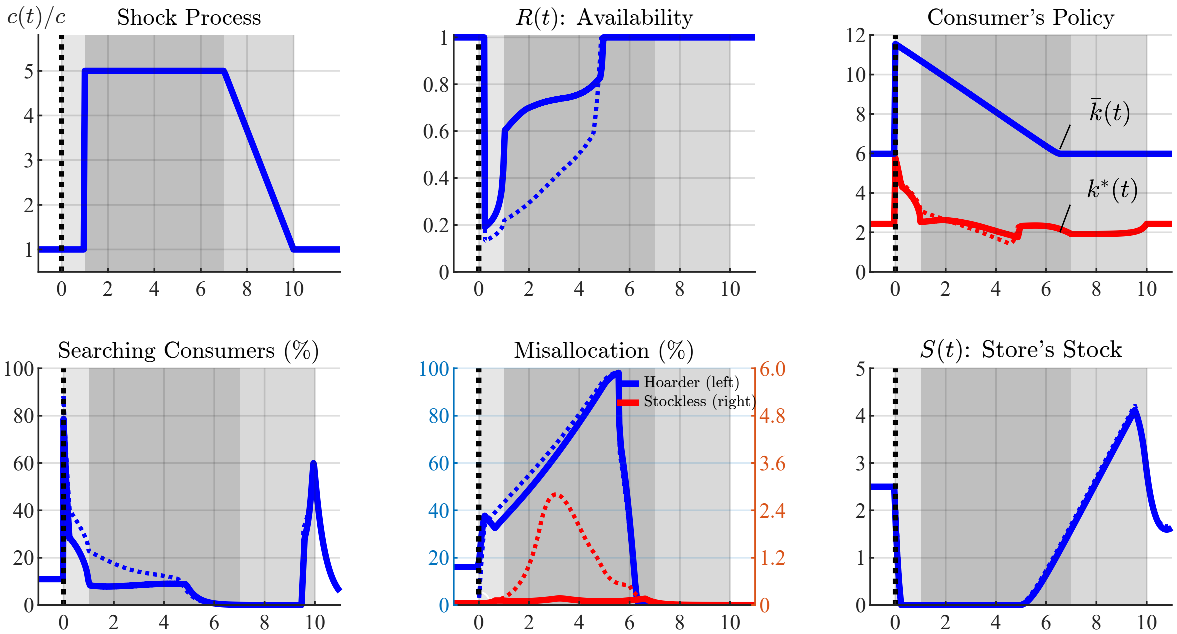
E.1.3.2 Non-Market Distribution We first consider a case in which the government distributes the product equally to all consumers. Since fairness is an important policy concern, the government often wants to accommodate the whole population when specific consumers' needs are not observed.

Simulation E.1 (Governmental Distribution to All Consumers)**.** The government distributes 4.5 days worth (4.5/7 unit) of the product to *all* consumers at $t = 0$: The initial condition is set to $S(0) = S_o - 4.5/7$ and $G(0, k) = G_o(k - 4.5/7)$ for all k.

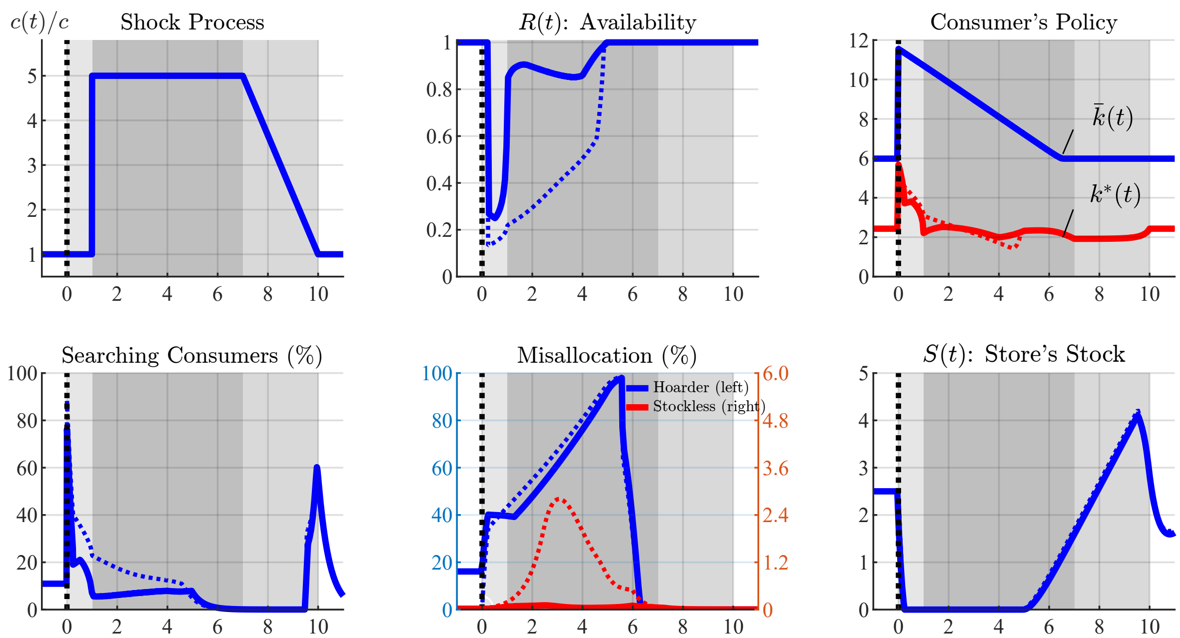
Figure E.4a presents the simulation results. The adoption of the distribution policy successfully curbs the consumer's tendency to rush for panic buying, which can be observed in the lower-left chart. This, in turn, reduces market congestion and minimizes product shortages. It is worth noting that increasing the product allocation per consumer can further alleviate panic buying. However, distributing the product equally to the entire population, as simulated in Simulation E.1, can be both costly and time-consuming. Therefore, alternative distribution rules are being investigated:

Simulation E.2 (Governmental Distribution to One Half of Consumers)**.** The government distributes *nine* days' worth (9/7 unit) of the product to *one half* of consumers at $t = 0$: the initial condition is set to $S(0) = S_o - 4.5/7$ and $G(0, k) = 1/2G_o(k) + 1/2G_o(k - 9/7)$ for all k.

The distribution policy is only targeted toward a portion of the population. Surprisingly, as shown in Figure E.4b, allocating the product to only half of the population yields similar



(a) Simulation E.1: The government distributes 4.5/7 unit to all the consumers at time 0: $S(0) = S_o - 4.5/7$ and $G(0,k) = G_o(k - 4.5/7)$ for all k.



(b) Simulation E.2: The government distributes 9/7 unit to 1/2 of consumers at time 0: $S(0) = S_o - 4.5/7$ and $G(0,k) = 1/2G_o(k) + 1/2G_o(k - 9/7)$ for all k.

Note: The horizontal axis represents the number of weeks after the announcement. The dotted lines show the results for the benchmark case.

Figure E.4: Governmental Distribution

results to distributing it to the entire population. This suggests that the government can improve the welfare of the *entire* population by catering to only *a fraction* of it.

In practice, it is also challenging to implement the distribution policy immediately. Therefore, we consider a scenario in which the government distributes the products one week after purchase.

There are various information settings that can be adopted for product distribution. For instance, consumers may be aware or unaware of whether they will receive the product from the government in the future. Such information plays a crucial role in shaping consumer behavior as those who are aware of receiving the product soon have no incentive to rush to the store.

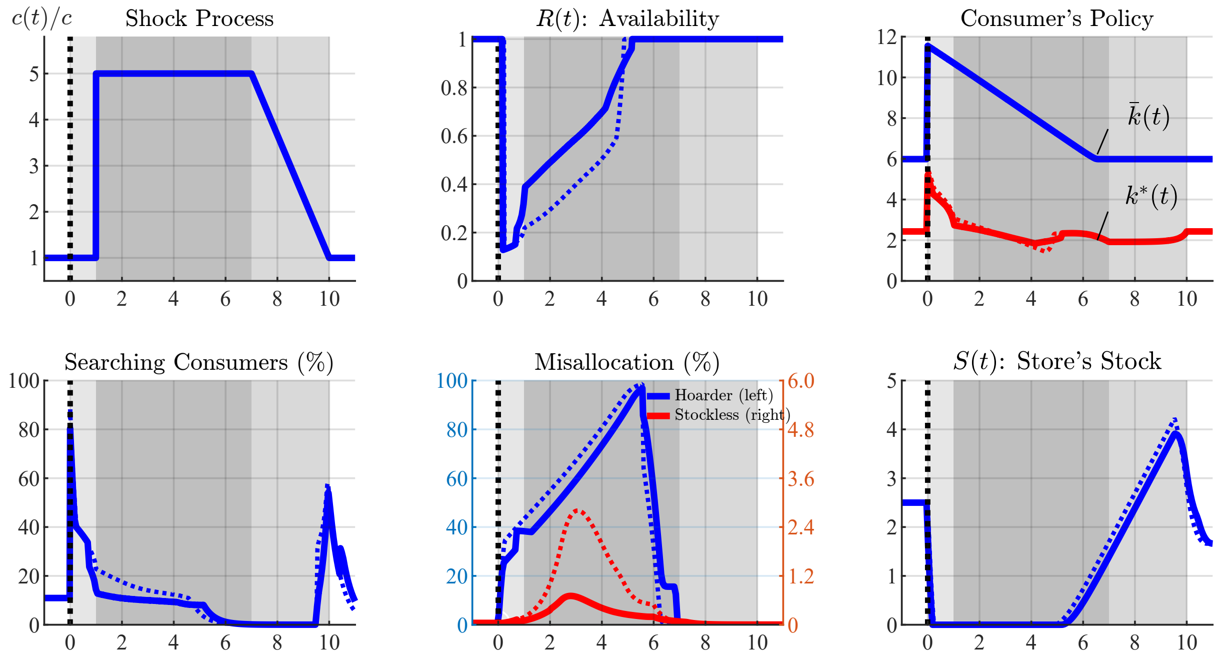
In this study, we adopt a conservative information setting where consumers have rational expectations of the evolution of product availability, denoted by $R(t)$. However, we assume that no consumer expects to receive the product at $t = 1$, which is an irrational expectation. This conservative assumption underestimates the effectiveness of the government's distribution policy since consumers' hoarding behavior can only decrease when they are aware of the chance of receiving the product. Conversely, if the conservative setting can suppress panic buying, the same results should be applicable to any information setting.

Simulations E.3 and E.4 investigate the scenario where the distribution is delayed by one week. In these simulations, the government purchases the product at $t = 0$ but distributes it at $t = 1$. As in Simulation E.2, the policy only covers half of the population.

Simulation E.3 (Delayed Governmental Distribution)**.** The government collects 4.5/7 unit of the product at time $t = 0$: $S(0) = S_o - 4.5/7$. The government distributes *nine* days worth (9/7 unit) of the product to one-half of consumers at $t = 1$: $G(1, k) = 1/2G(1^-, k) + 1/2G_o(1^-, k - 9/7)$ for all k. Before time t, each consumer behaves as if there is no chance to receive the governmental distribution, but they correctly anticipate the evolution of the availability, R.

Figure E.5 displays the simulation results, which reveal a marginal improvement due to the policy. However, the effect is not significant, at least with our conservative information setting. To explore whether the efficacy of the policy increases with an increase in the quantity of distributed products, we perform another simulation in which the government distributes two weeks' stock instead of nine day's stock.

Simulation E.4 (Delayed Governmental Distribution 2)**.** The government collects one unit of the product at time $t = 0$: $S(0) = S_o - 1$. The government distributes two units of the product to one-half of consumers at $t = 1$: $G(1, k) = 1/2G(1^-, k) + 1/2G_o(1^-, k - 2)$ for all



Note: The horizontal axis represents the number of weeks after the announcement. The dotted lines show the results for the benchmark case.

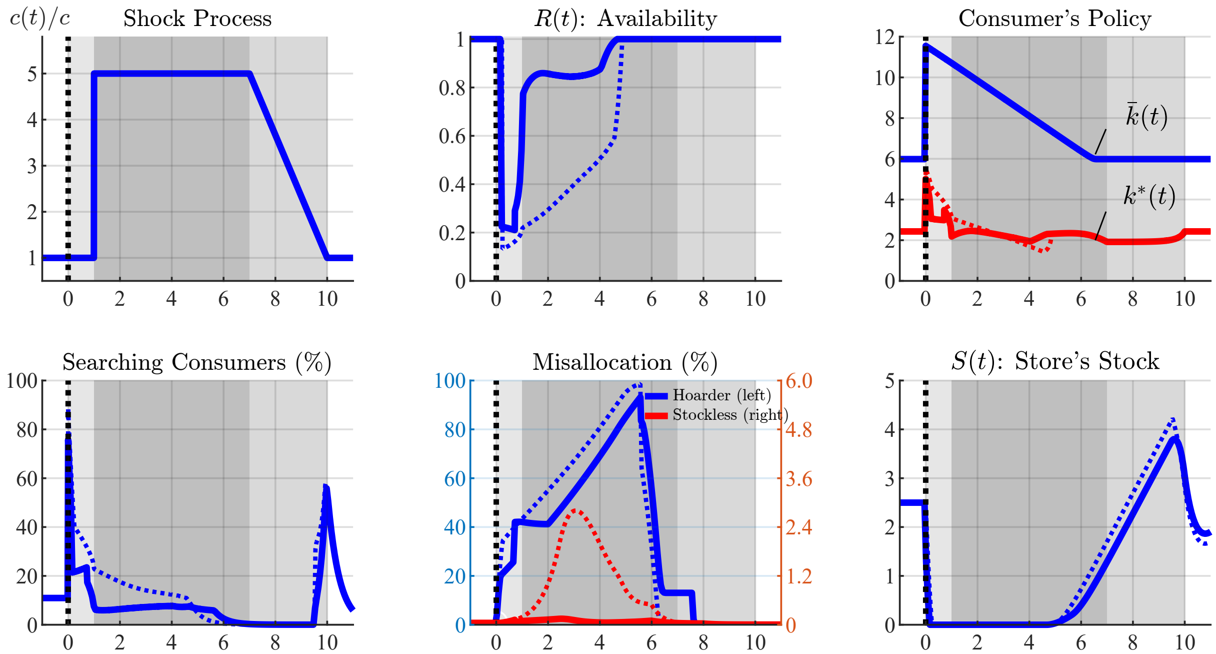
Figure E.5: Delayed Governmental Distribution (Simulation E.3)

k. Before time t, each consumer behaves as if there is no chance to receive the governmental distribution, but they correctly anticipate the evolution of the availability, R.

The result of this simulation is presented in Figure E.6. As shown in the top-middle chart, the availability of the product substantially improves with the distribution of two units. It is worth noting that in this scenario, the government only purchases one out of 2.5 units of the product at $t = 0$, implying that the policy can be scaled up further to suppress panic buying more effectively.

These simulation results suggest that the effectiveness of the government's distribution policy is not significantly affected by implementation delays. Even if there are delays in product delivery, consumers anticipate that market congestion will ease quickly, leading to fewer consumers rushing to the market upon the policy announcement.

Figure E.6 illustrates that despite delays in the delivery of consumer products, consumers anticipate the market congestion to ease quickly, leading to fewer consumers rushing to the market upon the announcement. In reality, consumers are aware that their stock increases at $t = 1$, which further improves product availability. This result contrasts with the tax policy, which may have an adverse effect if the government cannot implement it immediately after the announcement.



Note: The horizontal axis represents the number of weeks after the announcement. The dotted lines show the results for the benchmark case.

Figure E.6: Delayed Governmental Distribution (Simulation E.4)

E.1.3.3 Efficiency of Policy Interventions We further discuss the efficiency of policy interventions. Our welfare measure focuses on the impact of the average value. Here we examine who gets better off through the policy interventions discussed above. To this end, the difference in the flow value ($rV(0,k)$) to consumers with stock k at time 0 with and without policy interventions is displayed in Figures E.7-E.9. That is, these figures display $r(\tilde{V}(0,k) - V(0,k))$, where $\tilde{V}(t,k)$ and $V(t,k)$ be consumers' value given stock k at time t, with and without policy interventions, respectively.

Figure E.7 displays the distributional welfare impacts of the purchase-quota policy. It shows that (i) all consumers would be better off by implementing the policy, and (ii) it would especially benefit consumers with small stock. This is because such consumers urgently need to shop and would benefit greatly from having difficulties in shopping relieved by the policy.

Figure E.8 displays the distributional welfare impacts of the future sales-tax reduction (Section 5.6.2), showing that the tax policy makes all consumers better off.

Figure E.9 displays the distributional welfare impacts of the governmental distribution policies (Online Appendix E.1.3.2). We find that government rationing increases value for all consumers under any scenario and that consumers with low inventories benefit greatly, similar to purchase quotas and tax reductions.

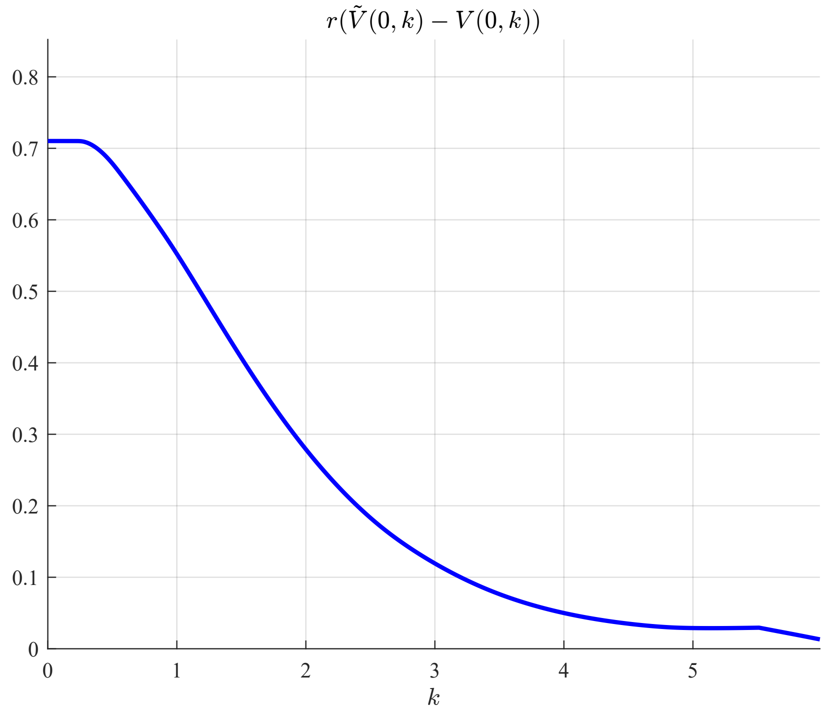


Figure E.7: The Increase in Consumers' Value at Time 0 due to the Purchase Quota

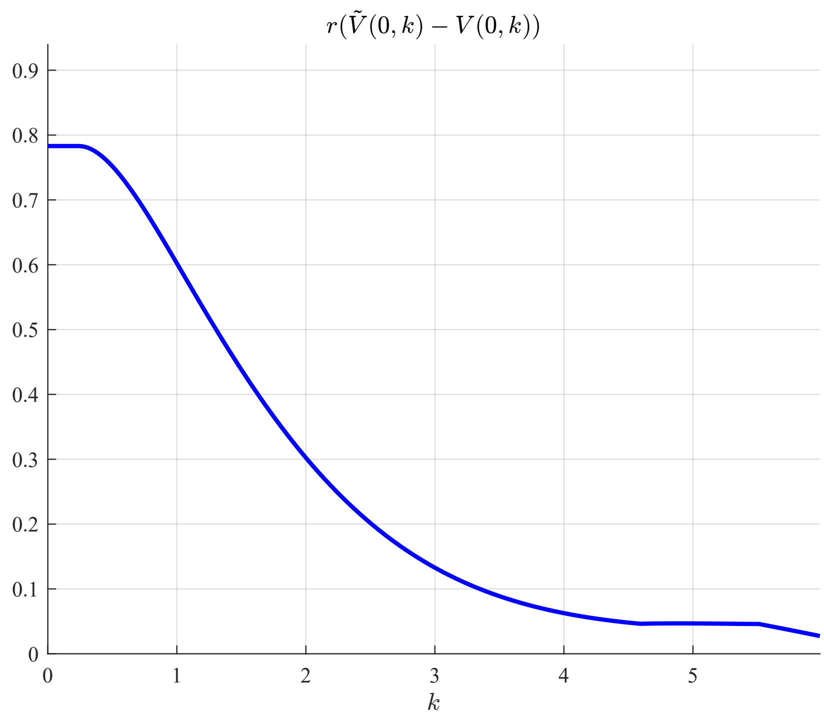
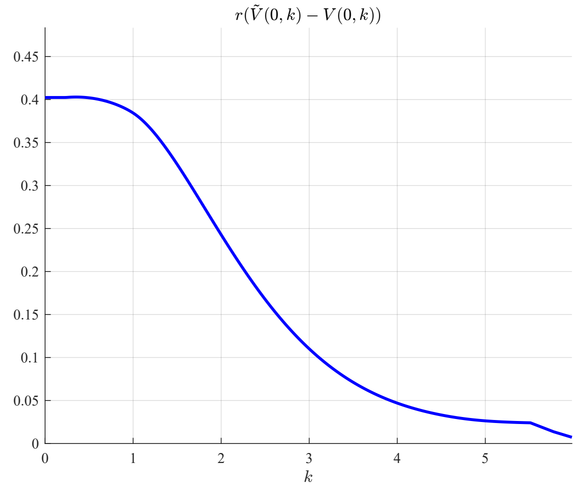
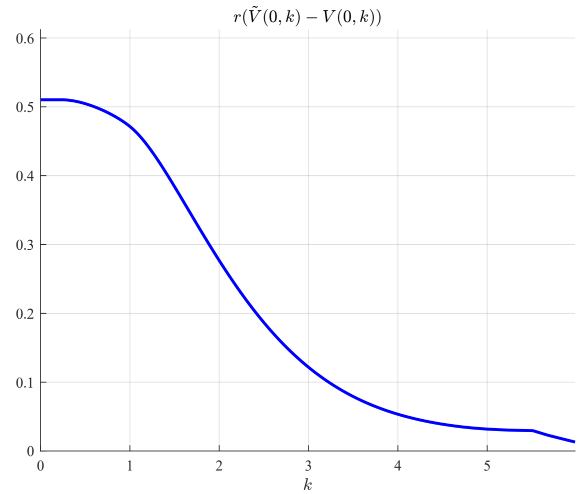


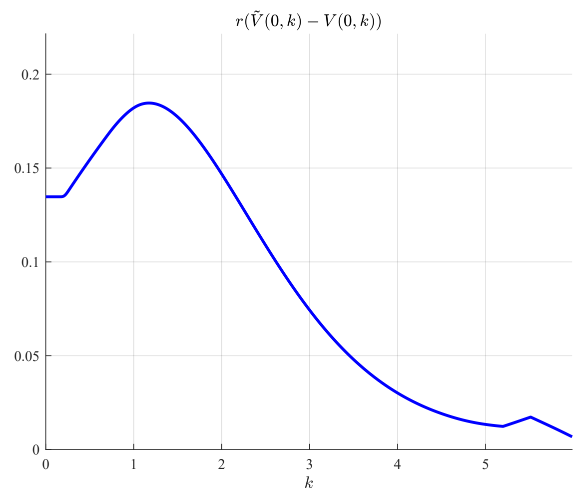
Figure E.8: The Increase in Consumers' Value at Time 0 due to the Future Tax Reduction of 20%



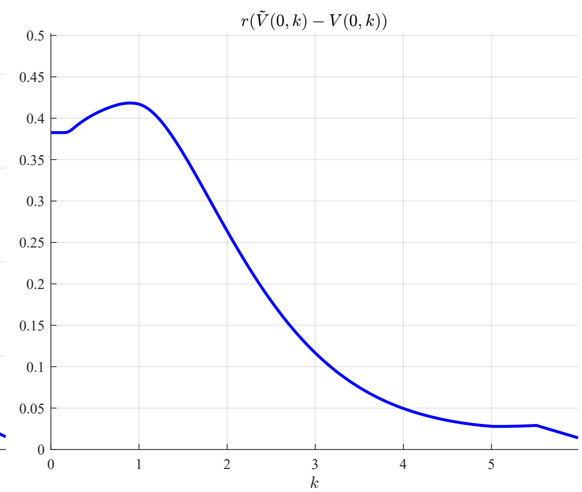
(a) All to 4.5/7 unit



(b) Half to 9/7 unit



(c) Half to 9/7 unit (5 days delay)



(d) Half to 2 units (5 days delay)

Note: Panel (a), (b), (c), and (d) are for Simulation E.1, E.2, E.3, and E.4, respectively.

Figure E.9: The Increase in Consumers' Value at Time 0 due to the Governmental Distribution

E.2 Supplementary Materials for Section 6

Here, we introduce *consumption shock* by considering the time variation in the consumption rate as follows:

$$u(t, x_i(t)) = \begin{cases} 0, & x_i(t) \geq \mu(t); \\ -a, & x_i(t) < \mu(t), \end{cases} \tag{E.2}$$

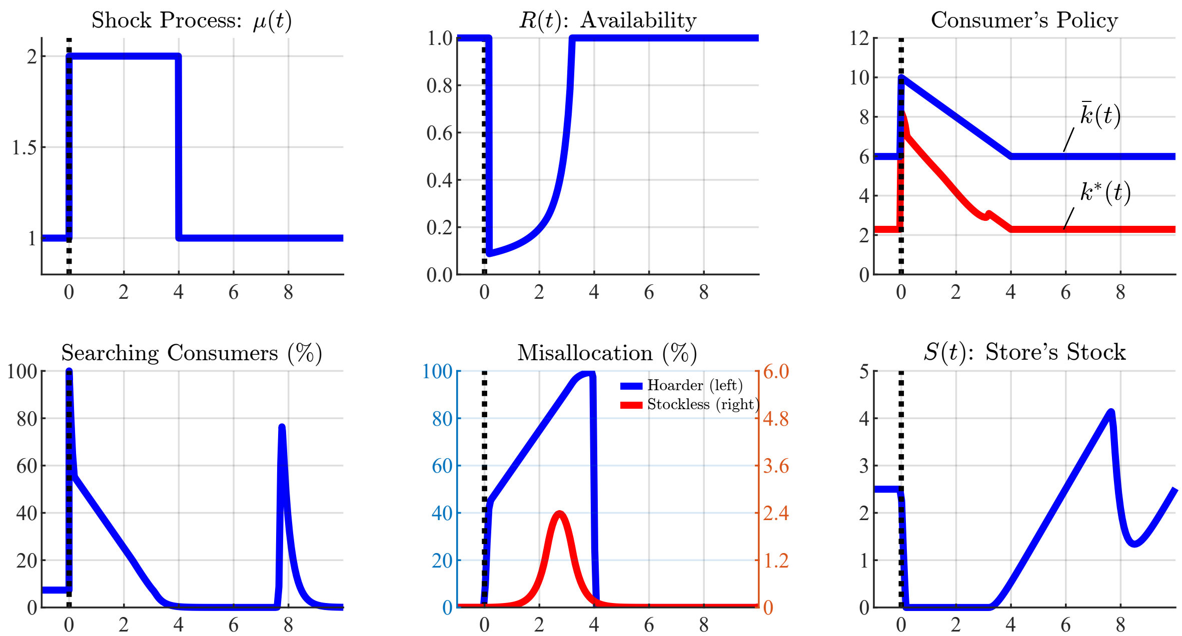
where $\mu(t)$ is a time-varying positive parameter with $\lim_{t\to\infty} \mu(t) = 1$. Given (E.2), the stock is consumed at rate of $\mu(t)$, i.e., $x(t, k_i(t)) = \mu(t)$ if $k_i(t) > 0$ and $x(t, k_i(t)) = 0$ otherwise. Namely, the good is consumed $\mu(t)$ units per unit of time (as long as stock is available).

In the following, we consider a consumption shock that increases the rate of consumption by 100 percent over a four-week period and compare the impact when it is unanticipated and when it is anticipated.

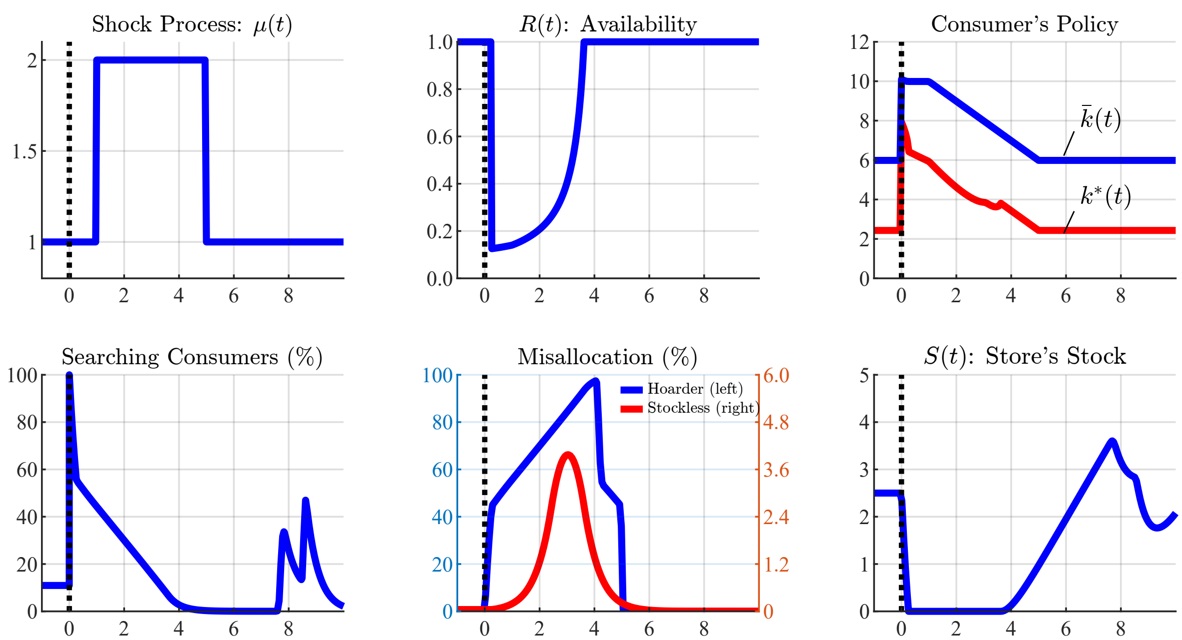
Simulation E.5 (Unanticipated Consumption Shock)**.** The rate of instantaneous consumption is $\mu(t) = 2.0$ for $t \in [0, 4]$ and $\mu(t) = 1.0$ otherwise.

Simulation E.6 (Anticipated Consumption Shock)**.** The rate of instantaneous consumption is $\mu(t) = 2.0$ for $t \in [1, 5]$ and $\mu(t) = 1.0$ otherwise. Agents become aware of the rise in consumption at time $t = 0$.

The simulation results in Figure E.10 suggest that the anticipated consumption shock has a similar impact on shortages as the anticipated shopping-cost shock. However, in contrast to panic buying caused by shopping costs, unanticipated consumption shocks (as in Simulation E.5) also result in severe shortages. This is because a consumption shock makes the goods absolutely scarce, and consumers need to compete for the limited resources, regardless of whether the shock is anticipated or not. Therefore, the timing of information has little effect on the severity of panic buying caused by consumption shocks.



(a) Simulation E.5: Unanticipated Consumption Shock



(b) Simulation E.6: Anticipated Consumption Shock

Note: The horizontal axis represents the number of weeks after the news.

Figure E.10: An Unanticipated and Anticipated Consumption Shock

E.3 Supplementary Materials for Section 7

E.3.1 The Magnitude of the Shopping-Cost Shock

In the benchmark case, we considered the case in which the flow shopping costs are increased by 500 percent. In Figure E.11, by varying the parameter $\bar{c}$, we investigate how the size of the shock affects social welfare.

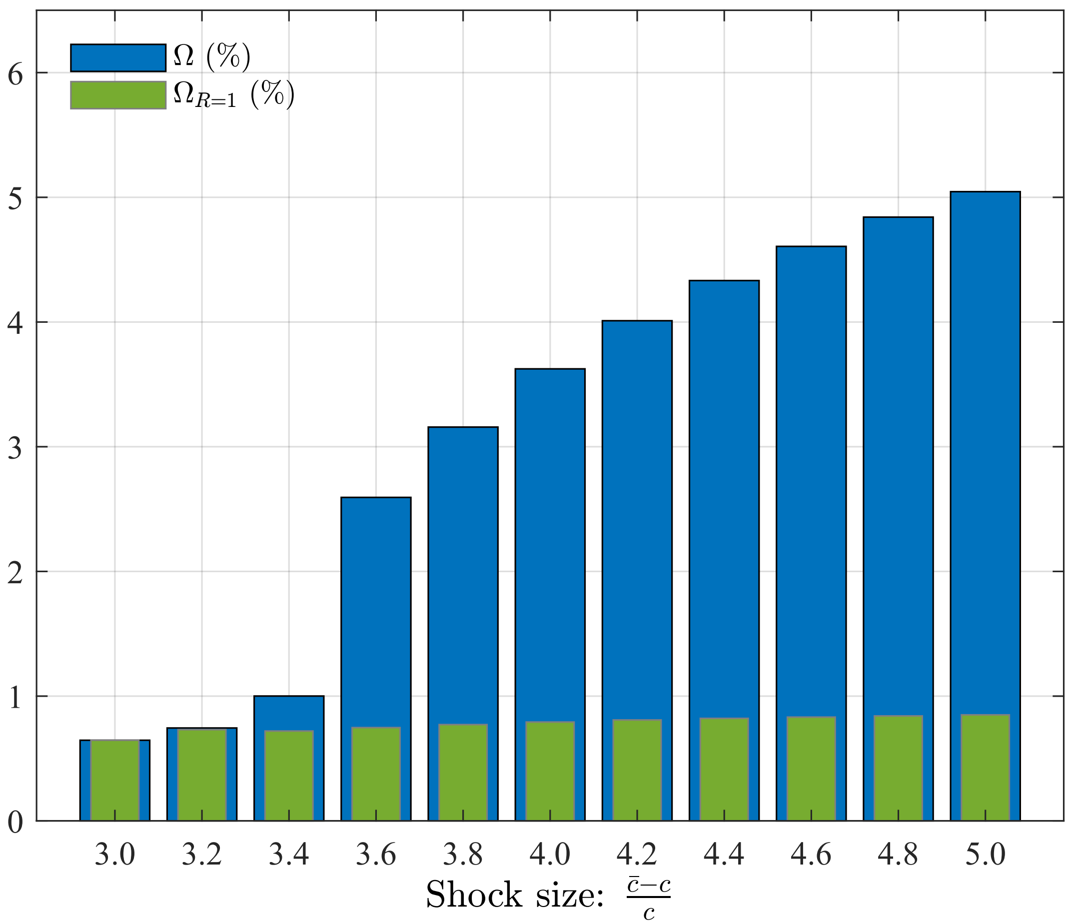


Figure E.11: Welfare Cost of a Shopping-Cost shock with Different Magnitudes

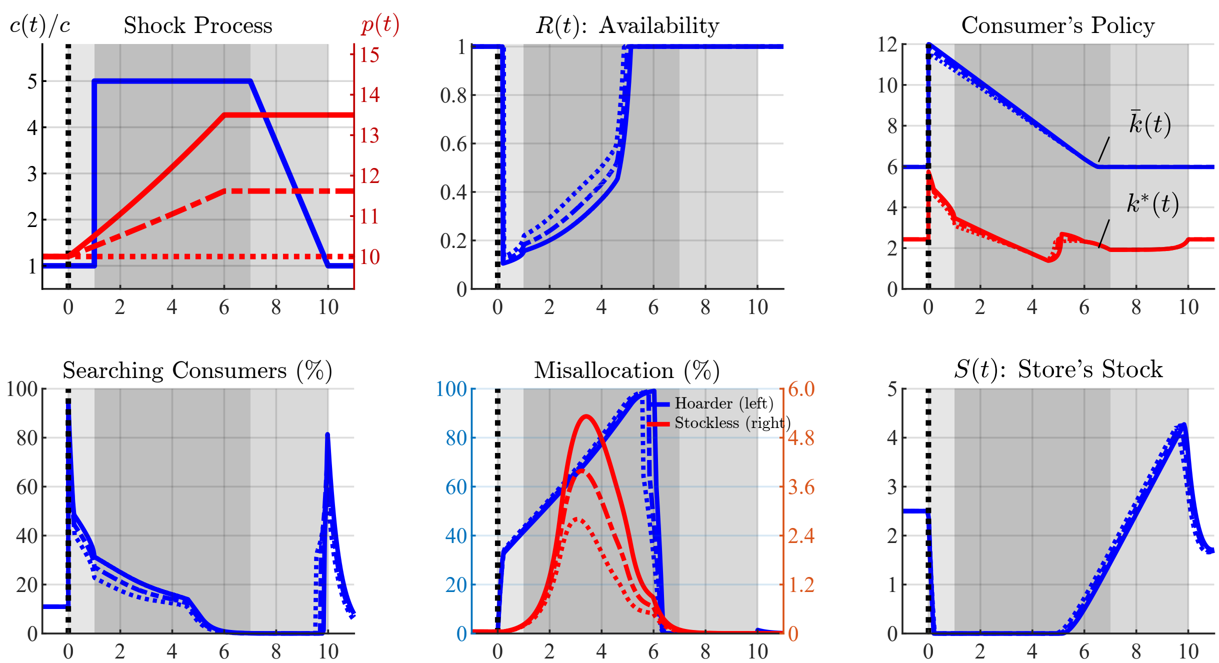
The figure shows that the welfare cost is very small when the shock size is about 300 percent but drastically severe when the size of the shock is greater than 360 percent. This implies that there is a nonlinear S-shaped relationship between the gross welfare cost of a shopping-cost shock and the size of the shock $(\bar{c} - c)/c$: If the increase in shopping costs exceeds a certain level, there is a serious shortage of products, resulting in substantial costs to consumers.

E.3.2 Price Dynamics

In the benchmark simulation, we assumed that the market price is kept constant at the stationary-equilibrium level ($p(t) = p$). Here, we allow the market price to change in response to the increase in demand. In light of extensive empirical evidence suggesting that stores are reluctant to increase the price in emergency situations in order to maintain their reputation, we assume that the market price is rigid and gradually rises in response to the spike in demand. Specifically, we consider the following scenarios:

Simulation E.7 (Inflation)**.** During the first six weeks after time 0, the market price increases at a monthly rate of 10 percent.

Simulation E.8 (High Inflation)**.** During the first six weeks after time 0, the market price increases at a monthly rate of 20 percent.



Note: The horizontal axis represents the number of weeks after the announcement. The dotted, dash-dotted, and solid lines show the results for the benchmark case (no inflation), Simulation E.7 (10 percent inflation), Simulation E.8 (20 percent inflation), respectively.

Figure E.12: Response to a Shopping-Cost Shock with Inflation

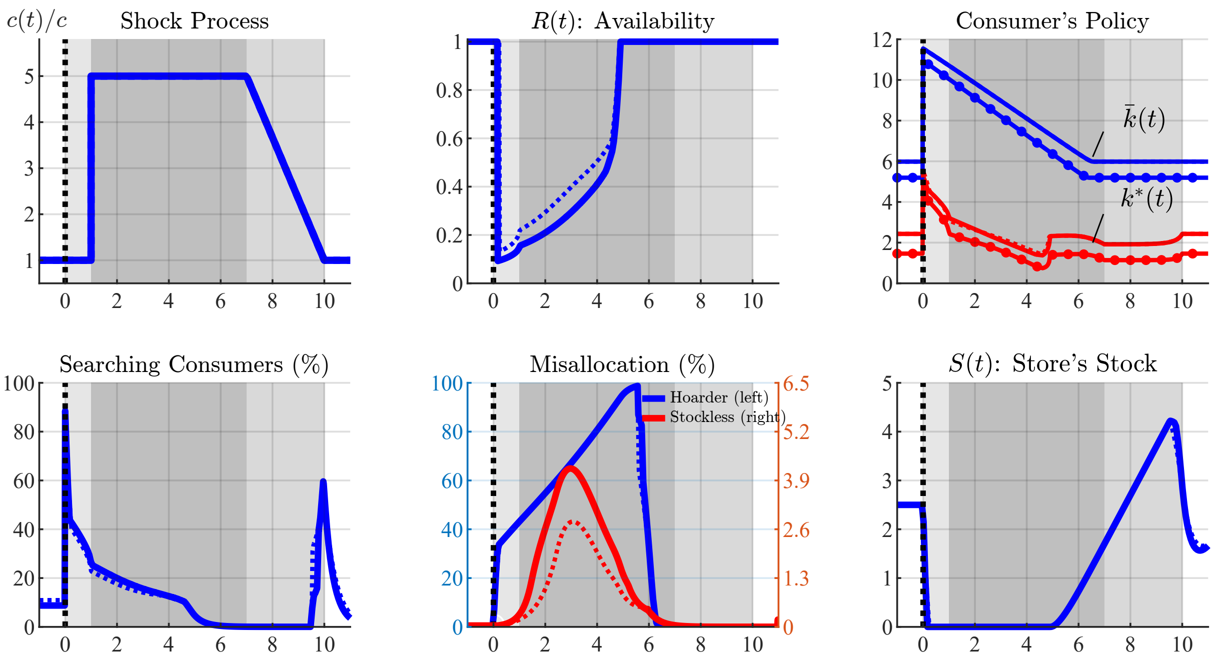
Figure E.12 illustrates how inflation exacerbates the extent of shortages. While the changes in individual consumer policies in response to inflation might seem modest, the anticipated inflation encourages consumers to shop earlier, thereby contributing to an aggregate exacerbation of shortages. Consequently, the likelihood of consumers becoming stockless increases. The figure indicates that this effect becomes more substantial as consumers encounter even higher inflation rates.[2]

E.3.3 Model Extension

We extend the model by introducing heterogeneity in the degree of product market frictions faced by consumers. Kano (2018) documents a large dispersion in the purchase cycle of toilet paper. In particular, there is a marked difference in inventory at the time of purchase; on average, households purchase when they reach a two-week stock, but there are many households that purchase with about half that amount in stock. Here, we examine how the heterogeneity in the purchase cycle affects the response to the shopping-cost shock by incorporating consumers who are heterogeneous in the degree of product market frictions captured by parameters (α, c). Specifically, we consider two types of household: (i) average shoppers with making purchases once every 4 weeks and having 2 weeks' stock remaining on average at the time of purchase; (ii) accessible shoppers with making purchases once every 4 weeks and having 1.2 weeks' stock remaining on average at the time of purchase. Following the calibration strategy in Section 4, average shoppers face product market frictions with $(\alpha, c) = (2.29, 14.63)$, while accessible shoppers face $(\alpha, c) = (3.82, 26.89)$.

Figure E.13 illustrates the response to a shopping cost shock, similar to the benchmark, in an economy where the population is composed of an equal number of average and accessible shoppers. The results reveal that the presence of accessible shoppers intensifies the impact of the shopping cost increase. Specifically, the shortage becomes more severe and persistent, and the risk of becoming stockless is higher than in the absence of accessible shoppers. This is because accessible shoppers have less inventory when they receive the news of rising shopping costs, leading them to hoard products more intensely than the average shoppers. Consequently, the intensive hoarding by accessible shoppers hastens the onset of shortages, which further accelerates hoarding by the average consumers. The findings imply that to avoid panic buying, it is crucial to prevent highly accessible consumers from rushing to stores and purchasing products.

[2] Awaya and Krishna (2021) provide a two-period model framework in which the price of a storable good is endogenously determined by the market-clearing price. In their model, when consumers purchase a large amount in the first period, the second-period price increases and becomes even higher than the first-period price. This situation resembles Simulations E.7 and E.8 in the sense that the price does not increase instantaneously at the beginning of the game. Awaya and Krishna (2021) also show that price controls mitigate panic buying and enhance social welfare.



Note: The horizontal axis represents the number of weeks after the announcement. The dotted lines show the results for the benchmark case. In the upper-right panel, the lines with marks represent the response of accessible shoppers.

Figure E.13: Response to a Shopping-Cost Shock with Heterogeneous Households

F Proofs

F.1 Proofs of Proposition 1 (i)-(iii)

Proof. We prove the proposition in six steps. In steps 1-4, we show that there exists a unique inaction region. Then, in step 5, we show the uniqueness of optimal stopping time, k^*, and $\bar{k}$. In step 6, we drive the expressions for the value functions and $\bar{k}$.

Step 1. We first prove $0 \in \mathcal{A}$ and

$$\alpha \left(V^A(0) - V_o^*(0)\right) - c > 0 \tag{F.1}$$

by contradiction. Suppose $0 \notin \mathcal{A}$, we must have

$$V_o(0) = -\frac{a}{r} = V^N(0) > V_o^*(0), \tag{F.2}$$

where

$$V^N(k) := \int_0^\infty \mathrm{e}^{-rs} h\left(\max\{k-s,0\}\right) ds = \frac{1}{r}\left[1-(1+a)\mathrm{e}^{-rk} - \bar{b}\left[\mathrm{e}^{-rk}\left(\frac{1}{r}+k\right)-\frac{1}{r}\right]\right].$$

By definition of V_o^*,

$$\begin{aligned}
V_o^*(0) &= -\frac{a+c}{r} + \alpha\frac{V^A(0) - V_o^*(0)}{r} \\
&> -\frac{a+c}{r} + \alpha\frac{\sup_{q\geq 0}\ V^N(q) - pq - V(0)}{r} \\
&= -\frac{a+c}{r} + \alpha\frac{\sup_{q\geq 0}\ V^N(q) - pq + a/r}{r},
\end{aligned}$$

where the second line used the fact $V^A(k) = \sup_{q\geq 0}\ V_o(k+q) - pq \geq \sup_{q\geq 0}\ V^N(k+q) - pq$ and the third line used $V^N(0) = -a/r$. Then, using (F.2), we have

$$-\frac{a}{r} > -\frac{a+c}{r} + \alpha\frac{\sup_{q\geq 0}\ V^N(q) - pq + a/r}{r},$$

or

$$c > \alpha\left[\sup_{q\geq 0}\ V^N(q) - pq + \frac{a}{r}\right].$$

This clearly contradicts Assumption 1. Then, we must have $0 \in \mathcal{A}$, which implies

$$V_o(0) = V_o^*(0) = -\frac{a+c}{r} + \alpha\frac{V^A(0) - V_o^*(0)}{r} > -\frac{a}{r}.$$

This immediately implies (F.1).

Step 2. We next prove $[0, \varepsilon] \in \mathcal{A}$ for sufficiently small $\varepsilon > 0$ by contradiction. Suppose that $\mathcal{A} = \{0\}$, that is $V_o^*(k) < V_o(k)$ for all $k > 0$.

By construction of V_o and V_o^*, we have $V_o(\varepsilon) = \max\{\tilde{V}_o(\varepsilon), V_o^*(\varepsilon)\}$, where

$$\tilde{V}_o(\varepsilon) = h(\varepsilon)dt + (1 - rdt)V_o(\varepsilon - dt) \tag{F.3}$$

and

$$V_o^*(\varepsilon) = \left[h(\varepsilon) - c + \alpha V^A(\varepsilon)\right] dt + (1 - (\alpha + r)dt)V_o^*(\varepsilon - dt) \tag{F.4}$$

for any $\varepsilon > 0$. Take $\varepsilon = dt > 0$. Then, taking difference (F.3) from (F.4), we have

$$V_o^*(\varepsilon) - \tilde{V}_o(\varepsilon) = \left[\alpha \left(V^A(\varepsilon) - V_o^*(0)\right) - c\right] \varepsilon. \tag{F.5}$$

Since

$$V^A(\varepsilon) = \sup_{q \geq 0} V_o(\varepsilon + q) - pq = \sup_{q' \geq \varepsilon} V_o(q') - p(q' - \varepsilon) = \left(\sup_{q' \geq \varepsilon} V_o(q') - pq'\right) + p\varepsilon,$$

we have, for a sufficiently small ε,

$$V^A(\varepsilon) = V^A(0) + p\varepsilon. \tag{F.6}$$

Substituting (F.6) into (F.5), we have

$$\frac{V_o^*(\varepsilon) - \tilde{V}_o(\varepsilon)}{\varepsilon} = \alpha \left(V^A(0) - V_o^*(0)\right) - c + p\varepsilon.$$

Rearranging the terms yields

$$\alpha \left(V^A(0) - V_o^*(0)\right) - c = -\frac{\tilde{V}_o(\varepsilon) - V_o^*(\varepsilon)}{\varepsilon} - p\varepsilon < 0. \tag{F.7}$$

where the last inequality comes from the assumption $V_o(\varepsilon) = \tilde{V}_o(\varepsilon) > V_o^*(\varepsilon)$. Here, (F.7) contradicts to (F.1).

Step 3. Using the same arguments as Step 2, we can show that if $[0, \hat{k}] \in \mathcal{A}$ such that $\alpha \left(V^A(\hat{k}) - V_o^*(\hat{k})\right) - c > 0$, then $[0, \hat{k} + \varepsilon'] \in \mathcal{A}$ for a small $\varepsilon' > 0$. Then, continuity of V_o^* and the instantaneous payoff function $h(k)$ show that $[0, k^*] \in \mathcal{A}$ with $\alpha \left(V^A(k^*) - V_o^*(k^*)\right) = c$.

Step 4. We show that the interval $\mathcal{A}$ is connected. That is, $\mathcal{A} = [0, k^*]$. This is almost obvious. Because $h(k)$ is strictly decreasing for $k \geq k^*$, there is no reason to increase k at the cost of shopping search.

Step 5. We show the uniqueness of optimal stopping-time policies, or k^* and $\bar{k}$. Since we have shown the problem has a unique inaction region and have assumed $V^o(k)$ is continuous, applying the uniqueness theorem for optimal stopping (Øksendal, 2003, Theorem 10.1.12) to this problem derives a unique stopping time. This implies a unique k^*.

The uniqueness of $\bar{k}$ is clear since for $k > k^*$, $V_o(k) - pk$ is continuous and strictly concave with $V_o'(k^*) - p > 0$. (See step 6 for the explicit expression).

Step 6. Finally, given that optimal policy, we derive V and V^* satisfying:

$$V_o(k) = \mathbb{1}_{\{k \geq k^*\}} \left[\int_0^{k-k^*} \mathrm{e}^{-rs'} h(k-s')ds' + \mathrm{e}^{-r(k-k^*)} V_o^*(k^*) \right] + \mathbb{1}_{\{k<k^*\}} V_o^*(k), \tag{F.8}$$

and

$$V_o^*(k) = \int_0^\infty \mathrm{e}^{-(\alpha+r)s'} \left[h(\max\{k-s', 0\}) + \alpha V^A(\max\{k-s', 0\}) - c \right] ds',$$

where $V^A(k) = \max_{q \geq 0} \; V_o(k+q) - pq$. It is therefore confirmed that

$$rV_o(k) = \mathbb{1}_{\{k \geq k^*\}} \left[h(k) - \frac{\partial V_o(k)}{\partial k} x(k) \right] + \mathbb{1}_{\{k<k^*\}} V_o^*(k) \tag{F.9}$$

and

$$rV^*(k) = h(k) - c - \frac{\partial V^*(k)}{\partial k} x(k) + \alpha \left[V^A(k) - V^*(k) \right]. \tag{F.10}$$

Lemma F.1. *$V_o(k)$ and $V_o^*(k)$ are respectively expressed as follows:*

$$V_o(k) = \mathbb{1}_{\{k \geq k^*\}} \left[\frac{1}{r} \mathrm{e}^{-r(k-k^*)} \left(b(k^*) - \frac{\bar{b}}{r} + rV_o^*(k^*) \right) + \frac{1}{r} \left(\frac{\bar{b}}{r} - b(k) \right) \right] + \mathbb{1}_{\{k<k^*\}} V_o^*(k), \tag{F.11}$$

and

$$V_o^*(k) = \alpha \Lambda(k) + \frac{1}{\alpha+r} \left[\left(1 - \mathrm{e}^{-(\alpha+r)k} \right) \frac{\bar{b}}{\alpha+r} - b(k) - \mathrm{e}^{-(\alpha+r)k} a - c \right], \tag{F.12}$$

where

$$\Lambda(k) \coloneqq \int_0^k \mathrm{e}^{-(\alpha+r)(k-s)} V^A(s) ds + \mathrm{e}^{-(\alpha+r)k} \frac{V^A(0)}{\alpha+r}.$$

They satisfy the value-matching condition

$$V_o(k^*) = V_o^*(k^*) = \lim_{k \uparrow k^*} V_o^*(k) = \lim_{k \uparrow k^*} V_o(k), \tag{F.13}$$

and the smooth pasting condition

$$V_o'(k^*) = V_o^{*'}(k^*) = \lim_{k \uparrow k^*} V_o^{*'}(k) = \lim_{k \uparrow k^*} V_o'(k). \tag{F.14}$$

Proof of Lemma F.1. First, we derive (F.11). The first term of the right-hand side of (F.8) is

$$\begin{aligned}\int_0^{k-k^*} \mathrm{e}^{-rs'} h(k-s')ds' + \mathrm{e}^{-r(k-k^*)} V_o^*(k^*) &= -\int_k^{k^*} \mathrm{e}^{-r(k-s)} h(s)ds + \mathrm{e}^{-r(k-k^*)} V_o^*(k^*) \\ &= \int_k^{k^*} \mathrm{e}^{-r(k-s)} b(s)ds + \mathrm{e}^{-r(k-k^*)} V_o^*(k^*).\end{aligned}$$

Then, use $b(k) = \bar{b}k$ and then apply integration by part to obtain

$$\begin{aligned}\int_k^{k^*} \mathrm{e}^{-r(k-s)} b(s)ds =& \bar{b} \int_k^{k^*} \mathrm{e}^{-r(k-s)} sds \\ =& \bar{b} \left[\frac{1}{r} \left[\mathrm{e}^{-r(k-s)} s \right]_k^{k^*} - \frac{1}{r} \int_k^{k^*} \mathrm{e}^{-r(k-s)} ds \right] \\ =& \frac{\bar{b}}{r} \left[\mathrm{e}^{-r(k-s)} \left(s - \frac{1}{r} \right) \right]_k^{k^*} \\ =& \frac{1}{r} \left[\mathrm{e}^{-r(k-k^*)} \left(b(k^*) - \frac{\bar{b}}{r} \right) + \frac{\bar{b}}{r} - b(k) \right].\end{aligned}$$

Then, we derive (F.13) and (F.14). Given (F.8), it is immediate to derive the value matching condition (F.13). Then, (F.10) and the fact $\alpha \left(V^A(k^*) - V_o^*(k^*) \right) = c$ implies

$$rV^*(k^*) = -b(k^*) - V^{*'}(k^*). \tag{F.15}$$

Then, the value matching condition and (F.9) yield the smooth pasting condition (F.14).

Finally, we derive (F.12).

$$\begin{aligned}V^*(k) &= \int_0^{\infty} \mathrm{e}^{-(\alpha+r)s'} \left[h(\max\{k-s',0\}) + \alpha V^A((\max\{k-s',0\}) - c \right] ds' \\ &= \int_0^k \mathrm{e}^{-(\alpha+r)(k-s)} \left[h(s) + \alpha V^A(s) \right] ds + \frac{1}{\alpha+r} \left[\mathrm{e}^{-(\alpha+r)k} \left(h(0) + \alpha V^A(0) \right) - c \right] \\ &= \alpha\Lambda(k) + \frac{1}{\alpha+r} \left[\left(1 - \mathrm{e}^{-(\alpha+r)k} \right) \frac{\bar{b}}{\alpha+r} - b(k) \right] - \frac{1}{\alpha+r} \left(\mathrm{e}^{-(\alpha+r)k} a + c \right) \\ &= \alpha\Lambda(k) + \frac{1}{\alpha+r} \left[\left(1 - \mathrm{e}^{-(\alpha+r)k} \right) \frac{\bar{b}}{\alpha+r} - b(k) - \mathrm{e}^{-(\alpha+r)k} a - c \right],\end{aligned}$$

where

$$\Lambda(k) = \int_0^k \mathrm{e}^{-(\alpha+r)(k-s)} V^A(s) ds + \mathrm{e}^{-(\alpha+r)k} \frac{V^A(0)}{\alpha+r}.$$

□

Note that Lemma F.1 implies that, for $k \geq k^*$,

$$V_o'(k) = -\mathrm{e}^{-r(k-k^*)} \left[b(k^*) - \frac{\bar{b}}{r} + r V_o^*(k^*) \right] - \frac{\bar{b}}{r},$$

and

$$\begin{aligned} V_o''(k) =& r\mathrm{e}^{-r(k-k^*)} \left[b(k^*) - \frac{\bar{b}}{r} + r V^*(k^*) \right] \\ =& - r\mathrm{e}^{-r(k-k^*)} \left[\frac{\bar{b}}{r} + V_o^{*\prime}(k^*) \right], \end{aligned}$$

where the second line used (F.15) and (F.14).

Here, we postulate $V_o^{*\prime}(k^*) > 0$, implying that $V_o''(k) < 0$ for $k \geq k^*$ and therefore $V_o(k)$ is strictly concave for $k \geq k^*$. In this case, $V^A(0) = \max_{q \geq 0} \; V_o(q) - pq$ has a unique solution. Let $\bar{k}$ be the solution. It must be true that (i) $\bar{k} = k^*$ if $V_o'(k^*) \leq p$ or (ii) $\bar{k} > k^*$ if $V_o'(k^*) > p$. But it is clear that the case (i) contradicts to the fact that $k^* \in \mathcal{A}$. Hence, the case (ii) must be held and $\bar{k}$ satisfies

$$V_o'(\bar{k}) = -\mathrm{e}^{-r(\bar{k}-k^*)} \left(b(k^*) - \frac{\bar{b}}{r} + r V^*(k^*) \right) - \frac{\bar{b}}{r} = p,$$

or

$$\bar{k} = k^* - \frac{1}{r} \log \left(- \frac{\bar{b}/r + p}{b(k^*) - \bar{b}/r + r V_o^*(k^*)} \right) = k^* + \frac{1}{r} \log \underbrace{\left(1 + \frac{V_o'(k^*) - p}{\bar{b}/r + p} \right)}_{>1}.$$

As a consequence, (when postulating $V_o^{*\prime}(k^*) > 0$), we must have

$$V^A(k) = \max_{q \geq 0} \; V_o(k+q) - pq = \begin{cases} V_o(\bar{k}) - p(\bar{k} - k), & \text{for } k \in [0, \bar{k}], \\ V_o(k), & \text{for } k \in (\bar{k}, \infty). \end{cases} \tag{F.16}$$

Furthermore, use (F.9) to derive the following

$$V_o(\bar{k}) = -\frac{b(\bar{k}) + V_o'(\bar{k})}{r} = -\frac{b(\bar{k}) + p}{r}.$$

Plugging this into (F.16) yields

$$V^A(k) = \begin{cases} -\dfrac{p + b(\bar{k})}{r} - p(\bar{k} - k) & \text{for} \quad k \in [0, \bar{k}], \\ \dfrac{1}{r}\left[\mathrm{e}^{-r(k-k^*)}\left(b(k^*) - \dfrac{\bar{b}}{r} + rV^*(k^*)\right) + \left(\dfrac{\bar{b}}{r} - b(k)\right)\right] & \text{for} \quad k \in (\bar{k}, \infty). \end{cases}$$

Finally, we verify that our postulation was true. Using (F.12), we have

$$V^{*'}(k) = \alpha\Lambda'(k) - \left(1 - \mathrm{e}^{-(\alpha+r)k}\right)\frac{\bar{b}}{\alpha + r} + \mathrm{e}^{-(\alpha+r)k}a.$$

We then show that, for $k \in [0, \bar{k}]$

$$\Lambda'(k) = -(\alpha + r)\Lambda(k) + V^A(k) = \left(1 - \mathrm{e}^{-(\alpha+r)k}\right)\frac{p}{\alpha + r}.$$

since

$$\begin{aligned} \Lambda'(k) &= -(\alpha + r)\left[\int_0^k \mathrm{e}^{-(\alpha+r)(k-s)}\left(V^A(0) + ps\right)ds + \mathrm{e}^{-(\alpha+r)k}\frac{V^A(0)}{\alpha + r}\right] + (V^A(0) + pk) \\ &= -\left(1 - \mathrm{e}^{-(\alpha+r)k}\right)V^A(0) - \mathrm{e}^{-(\alpha+r)k}V^A(0) + (V^A(0) + pk) - p(\alpha + r)\int_0^k \mathrm{e}^{-(\alpha+r)(k-s)}sds \\ &= pk - pk + \left(1 - \mathrm{e}^{-(\alpha+r)k}\right)\frac{p}{\alpha + r} \end{aligned}$$

for $k \in [0, \bar{k}]$. Hence, we have

$$V^{*'}(k) = \left(1 - \mathrm{e}^{-(\alpha+r)k}\right)\frac{\alpha p - \bar{b}}{\alpha + r} + \mathrm{e}^{-(\alpha+r)k}a.$$

for $k < \bar{k}$. Assumption 2 ensures $\alpha p > \bar{b}$ and thus $V^{*'}(k) > 0$ for all $k < \bar{k}$. Since $k^* < \bar{k}$, we have shown $V^{*'}(k^*) > 0$. □

In sum, the value functions in the stationary equilibrium is given by

$$rV_o(k) = \mathbb{1}_{\{k \geq k_o^*\}}\left[\mathrm{e}^{-r(k-k_o^*)}\left(\bar{b}k_o^* - \frac{\bar{b}}{r} + rV_o^*(k_o^*)\right) + \left(\frac{\bar{b}}{r} - \bar{b}k\right)\right] + \mathbb{1}_{\{k < k_o^*\}}rV_o^*(k),$$

where the value of exercising a control $V^*(k)$ satisfies

$$\begin{aligned} V_o^*(k) =& \alpha \left[\int_0^k \mathrm{e}^{-(\alpha+r)(k-s)} V_o^A(s) ds - \mathrm{e}^{-(\alpha+r)k} \frac{(p + \bar{b}\bar{k}_o)/r + p\bar{k}_o}{\alpha + r} \right] \\ &+ \frac{1}{\alpha+r} \left[\left(1 - \mathrm{e}^{-(\alpha+r)k}\right) \frac{\bar{b}}{\alpha+r} - \bar{b}k - \mathrm{e}^{-(\alpha+r)k} a - c \right], \end{aligned}$$

with

$$V^A(k) = \mathbb{1}_{\{k \le \bar{k}_o\}} \left[p(k - \bar{k}_o) - (p + \bar{b}\bar{k}_o)/r \right] + \mathbb{1}_{\{k > \bar{k}_o\}} V_o(k).$$

F.2 Proofs of Lemma 1 and Proposition 1 (iv) and (v)

Let $G(0,k)$ be the distribution function for the consumer's stock at the initial period $t = 0$. Because of the exogenous exit, the share of consumers who exist from the initial period decreases at the rate of θ, and such consumers disappear in the long run. Thus, the choice of $g(0,k)$ does not affect the long-run distribution. Thus, in computing the long-run distribution, without loss of generality, we can assume $G(0,k) = G_o(k)$.

In the stationary equilibrium, it must be held that

$$\frac{\partial g(t,k)}{\partial t} = \begin{cases} \dfrac{\partial g(t,k)}{\partial k} x(k) + \theta \left[g_{new}(k) - g(t,k) \right] - \alpha g(t,k) & \text{for } k \in [0, k_o^*], \\ \dfrac{\partial g(t,k)}{\partial k} x(k) + \theta \left[g_{new}(k) - g(t,k) \right] + \alpha G(t, k_o^*) \delta(k - \bar{k}_o) & \text{for } k \in [k_o^*, \bar{k}_o]. \end{cases}$$

Then we show that there exists a unique $g(t,k)$ that satisfies $\frac{\partial g(t,k)}{\partial t} = 0$. Namely, we show that the solution for the ordinary differential equation is unique.

$$0 = \begin{cases} -g_o'(k) - \alpha g_o(k) & \text{for } k \in (0, k_o^*), \\ -g_o'(k) + \alpha G_o(k_o^*) \delta(k - \bar{k}_o) & \text{for } k \in [k_o^*, \bar{k}_o], \end{cases}$$

with $\lim\limits_{k \uparrow k_o^*} g_o(k) = g_o(k_o^*)$ and $G_o(0) = G(k_o^*) \mathrm{e}^{\alpha k_o^*}$.

It is clear that

$$g_o(k) = \begin{cases} C & \text{for } k \in [k_o^*, \bar{k}_o], \\ C\mathrm{e}^{-\alpha(k_o^* - k)} & \text{for } k \in (0, k_o^*) \end{cases}$$

Hence,

$$
\begin{aligned}
G(k_o^*) =& G(0) + C\int_0^{k_o^*} \mathrm{e}^{-\alpha(k_o^*-k)}dk \\
=& G(k_o^*)\mathrm{e}^{\alpha k_o^*} + \frac{C}{\alpha}(1-\mathrm{e}^{\alpha k_o^*}).
\end{aligned}
$$

Therefore,

$$
G(k_o^*) = \frac{C}{\alpha}.
$$

Furthermore, it must be true that

$$
G(k_o^*) + \int_{k_o^*}^{\infty} g_o(k)dk = 1.
$$

This requires

$$
C\left(\frac{1}{\alpha} + \bar{k}_o - k_o^*\right) = 1.
$$

Namely,

$$
C = \frac{\alpha}{1+\alpha(\bar{k}_o - k_o^*)}.
$$

This shows that

$$
g_o(k) = \begin{cases} \frac{\alpha}{1+\alpha(\bar{k}_o-k_o^*)}\mathrm{e}^{-\alpha(k_o^*-k)} & \text{for } k \in (0, k_o^*), \\ \frac{\alpha}{1+\alpha(\bar{k}_o-k_o^*)} & \text{for } k \in [k_o^*, \bar{k}_o], \end{cases}
$$

and has a mass point at $k=0$ with $G_o(0) = \frac{\mathrm{e}^{-\alpha k_o^*}}{1+\alpha(\bar{k}_o-k_o^*)}$

Proposition 1 (v) is obvious from the average waiting time between two occurrences in a Poisson process.

G Algorithm Description

We define differential operators (or infinitesimal generators of the process) $\mathscr{K}$ and $\mathscr{T}$ as

$$(\mathscr{K}V)(t,k) = -\partial_k V(t,k)x(k)$$

and

$$(\mathscr{T}V)(t,k) = \partial_t V(t,k).$$

The value function $V(t,k)$ can be written as a solution of the Hamilton-Jacobi-Bellman variational inequality (HJBVI, henceforth):[3]

$$\min\left\{rV(t,k) - h(k) - (\mathscr{K}V)(t,k) - (\mathscr{T}V)(t,k), V(t,k) - V^*(t,k)\right\} = 0, \tag{G.1}$$

where $V^*(t,k)$ is the value function of exercising the option, which satisfies the following HJB equation:

$$(r + \alpha R(t))V^*(t,k) = h(k) - c(t) + (\mathscr{K}V^*)(t,k) + (\mathscr{T}V^*)(t,k) + \alpha R(t)V^A(t,k).$$

We will find an approximated solution of the HJBVI (G.1) in a discretized space. We begin with the description of our notations. Set an equidistant grid over the consumer's stock level, $k_1 = 0, k_2, \ldots k_L$ with $\Delta_k = k_\ell - k_{\ell-1}$ for all $\ell = 2, \ldots, L$. Throughout, we use bold letters to denote vectors and subscript ℓ to denote the ℓ-th element of a vector. For example, $\boldsymbol{k} = (k_1, \ldots, k_\ell, \ldots, k_L)'$ and $\boldsymbol{h} = (h_1, \ldots, h_\ell, \ldots, h_L)' = (h(k_1), \ldots, h(k_\ell), \ldots, h(k_L))'$. Let $\boldsymbol{v}(t)$ be $\boldsymbol{v}(t) = (V(t,k_1), \ldots, V(t,k_L))'$. Similarly, let $\boldsymbol{v}^*(t) = (V^*(t,k_1), \ldots, V^*(t,k_L))'$.

We then discretize the differential operator $\mathscr{K}$. Since a functional operator is the infinite-dimensional analogue of a matrix, the operator $\mathscr{K}$ can be discretized by a matrix $\boldsymbol{K}$. Specifically, we approximate the partial derivative based on the following finite difference scheme:

$$\partial_k V(t,k_\ell) = \frac{V(t,k_\ell) - V(t,k_{\ell-1})}{\Delta_k}.$$

[3] Note that solving the HJBVI (G.1) is equivalent to finding the function $V(t,k)$ that satisfies the complementary slackness conditions:

$$\begin{aligned} V(t,k) &\geq V^*(t,k) \quad \text{if} \quad rV(t,k) = h(k) + (\mathscr{K}V)(t,k) + (\mathscr{T}V)(t,k), \\ V(t,k) &= V^*(t,k) \quad \text{if} \quad rV(t,k) \geq h(k) + (\mathscr{K}V)(t,k) + (\mathscr{T}V)(t,k). \end{aligned}$$

Using the above scheme along with the boundary condition, we can write

$$-\partial_k V(t,k_\ell)x(k_\ell) = \begin{cases} 0, & \ell = 1 \\ -\dfrac{V(t,k_\ell) - V(t,k_{\ell-1})}{\Delta_k} = v_{\ell-1}(t)\omega_+ + v_\ell(t)\omega_-, & \ell = 2,\ldots,L \end{cases}$$

where $\omega_+ = 1/\Delta_k$ and $\omega_- = -1/\Delta_k$. Then, we can build a $L \times L$ sparse matrix $\boldsymbol{K}$ such that

$$\boldsymbol{K}\boldsymbol{v}(t) = \begin{pmatrix} 0 & 0 & 0 & \cdots & \cdots & \cdots & 0 \\ \omega_+ & \omega_- & 0 & 0 & \cdots & \cdots & 0 \\ 0 & \omega_+ & \omega_- & 0 & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & \cdots & 0 & \omega_+ & \omega_- & 0 \\ 0 & \cdots & \cdots & \cdots & 0 & \omega_+ & \omega_- \end{pmatrix} \begin{pmatrix} v_1(t) \\ v_2(t) \\ v_3(t) \\ \vdots \\ v_{L-1}(t) \\ v_L(t) \end{pmatrix} = \begin{pmatrix} 0 \\ v_1(t)\omega_+ + v_2(t)\omega_-, \\ v_2(t)\omega_+ + v_3(t)\omega_-, \\ \vdots \\ v_{L-2}(t)\omega_+ + v_{L-1}(t)\omega_- \\ v_{L-1}(t)\omega_+ + v_L(t)\omega_- \end{pmatrix}. \tag{G.2}$$

With notations introduced above, the approximation of (G.1) in the discretized space is given by

$$\min\left\{ r\boldsymbol{v}(t) - \boldsymbol{h} - \boldsymbol{K}\boldsymbol{v}(t) - \frac{\boldsymbol{v}(t+dt) - \boldsymbol{v}(t)}{dt}, \boldsymbol{v}(t) - \boldsymbol{v}^*(t) \right\} = 0,$$

In the similar way, we can find the expression for $\boldsymbol{v}^*(t)$ in the discretized space as follows:

$$(\alpha + rR(t))\boldsymbol{v}^*(t) = \boldsymbol{h} - c(t)\mathbf{1}_L + \boldsymbol{K}\boldsymbol{v}^*(t) + \left(\frac{\boldsymbol{v}^*(t+dt) - \boldsymbol{v}^*(t)}{dt} \right) + \alpha R(t)\boldsymbol{v}^A(t). \tag{G.3}$$

where $\boldsymbol{v}^A(t)$ is the approximation of $V^A(t,k)$ in the discretized space.

For later use, we define a $L \times L$ sparse matrix $\boldsymbol{M}(t)$ that captures the rate of transition of the consumer's stock associated with the purchase of the good.[4] Its (ℓ, n) element is given by

$$\boldsymbol{M}_{\ell,n}(t) = \begin{cases} -\alpha R(t), & \text{for} \quad n = \ell \quad \text{if} \quad k_\ell \in \mathcal{A}(t) \\ \alpha R(t), & \text{for} \quad n = \bar{k}(t) \quad \text{if} \quad k_\ell \in \mathcal{A}(t) \\ 0, & \text{otherwise} \end{cases},$$

where $\mathcal{A}(t)$ is the action region is the discretized space. The sum of each row of $\boldsymbol{K}$ and $\boldsymbol{M}(t)$ equals to zero. Furthermore, we define a $L \times L$ diagonal matrix $\boldsymbol{D}$ all of whose diagonal elements are $-\theta$, which captures the rate of transition of the consumer's stock associated with exit.

We turn to the time evolution of the cross-sectional distribution of the stock level. We

[4] The matrix $\boldsymbol{K}$, which is given by (G.2), can be interpreted as the rate of transition of the consumer's stock associated with consumption.

denote $\boldsymbol{g}(t) = [g(t,k_1),\dots,g(t,k_L)]'$ and $\boldsymbol{g}_{new} = [g_{new}(k_1),\dots,g_{new}(k_L)]'$. Since the KF operator is the adjoint operator of the HJB operator, the KF equation (8) in the discretized space, can be written as

$$\dot{\boldsymbol{g}}(t) = \left(\boldsymbol{K}^T + \boldsymbol{M}(t)^T + \boldsymbol{D}^T\right)\boldsymbol{g}(t) + \theta \boldsymbol{g}_{new}.$$

where $\dot{\boldsymbol{g}}(t) = [\partial g(t,k_1)/\partial t,\dots,\partial g(t,k_L)/\partial t]'$ and $\boldsymbol{A}^T$, $\boldsymbol{M}(t)^T$, and $\boldsymbol{D}^T$ are the transpose of the intensity matrices $\boldsymbol{A}$, $\boldsymbol{M}(t)$, and $\boldsymbol{D}$, respectively.

In the following, we describe the algorithm to obtain the stationary distribution in Section G.1 and the transitional dynamics in Section G.2.

G.1 Stationary Distribution

1. Set a concave function $\boldsymbol{v}^0$ as an initial guess for the value function. Specifically, we use $\boldsymbol{v}^0$ such that $r\boldsymbol{v}^0 = \boldsymbol{h} + \boldsymbol{K}\boldsymbol{v}^0$.

2. Given $\boldsymbol{v}^n$, find $\boldsymbol{v}^{n+1}$ by solving

$$\min\left\{\frac{\boldsymbol{v}^{n+1}-\boldsymbol{v}^n}{\Delta} + r\boldsymbol{v}^{n+1} - \boldsymbol{h} - \boldsymbol{K}\boldsymbol{v}^{n+1}, \boldsymbol{v}^{n+1} - \boldsymbol{v}^*(\boldsymbol{v}^n)\right\} = 0, \tag{G.4}$$

where

$$\boldsymbol{v}^*(\boldsymbol{v}^n) = \boldsymbol{B_a}^{-1}\left(\boldsymbol{h} - c\boldsymbol{1} + \alpha \boldsymbol{v}^{\boldsymbol{A}}(\boldsymbol{v}^n)\right)$$

with $\boldsymbol{B_a} = (\alpha + r)\boldsymbol{I}_L - \boldsymbol{K}$.

2-A. Define matrix $\boldsymbol{B}$ as

$$\boldsymbol{B} = \left(r + \frac{1}{\Delta}\right)\boldsymbol{I}_L - \boldsymbol{K}.$$

Then, rewrite (G.4) into

$$\min\left\{\boldsymbol{B}\boldsymbol{v}^{n+1} - \frac{1}{\Delta}\boldsymbol{v}^n - \boldsymbol{h}, \boldsymbol{v}^{n+1} - \boldsymbol{v}^*\left(\boldsymbol{v}^n\right)\right\} = 0. \tag{G.5}$$

Now, find that that solving (G.5) is equivalent to solving the following problem:

$$\begin{aligned}\left(\boldsymbol{v}^{n+1} - \boldsymbol{v}^*(\boldsymbol{v}^n)\right)'\left(\boldsymbol{B}\boldsymbol{v}^{n+1} - \frac{1}{\Delta}\boldsymbol{v}^n - \boldsymbol{h}\right) &= 0\\ \boldsymbol{v}^{n+1} - \boldsymbol{v}^*(\boldsymbol{v}^n) &\geq 0\\ \boldsymbol{B}\boldsymbol{v}^{n+1} - \frac{1}{\Delta}\boldsymbol{v}^n - \boldsymbol{h} &\geq 0\end{aligned} \tag{G.6}$$

2-B. Define

$$\boldsymbol{z}^{n+1} = \boldsymbol{v}^{n+1} - \boldsymbol{v}^*(\boldsymbol{v}^n) \quad \text{and} \quad \boldsymbol{y}^n = \boldsymbol{B}\boldsymbol{v}^*(\boldsymbol{v}^n) - \boldsymbol{v}^n/\Delta - \boldsymbol{h}.$$

Then, (G.6) is reduced to the following Linear Complementarity Problem (LCP):

$$\begin{aligned}(\boldsymbol{z}^{n+1})'(\boldsymbol{B}\boldsymbol{z}^{n+1} + \boldsymbol{y}^n) &= 0 \\ \boldsymbol{z}^{n+1} &\geq 0 \\ \boldsymbol{B}\boldsymbol{z}^{n+1} + \boldsymbol{y}^n &\geq 0\end{aligned}$$

Then, given $\boldsymbol{v}^n$ (equivalently $\boldsymbol{y}^n$), the above problem solves $\boldsymbol{z}^{n+1}$ and therefore $\boldsymbol{v}^{n+1}$.

3. Repeat the step 2 until $\boldsymbol{v}^{n+1}$ is sufficiently close to $\boldsymbol{v}^n$.

4. Find $\boldsymbol{g}$

4-A. Set $\boldsymbol{M}$. The (ℓ, n) elements are given by

$$\boldsymbol{M}_{\ell,n} = \begin{cases} -\alpha, & \text{for} \quad n = \ell \quad \text{if} \quad k_\ell \in \mathcal{A} \\ \alpha, & \text{for} \quad n = \bar{k} \quad \text{if} \quad k_\ell \in \mathcal{A} \\ 0, & \text{otherwise} \end{cases}$$

4-B. Find $\boldsymbol{g}$ such that

$$\boldsymbol{0} = \left(\boldsymbol{K}^{\mathrm{T}} + \boldsymbol{M}^{\mathrm{T}} + \boldsymbol{D}^{\mathrm{T}}\right)\boldsymbol{g} + \theta\boldsymbol{g}_{new},$$

or

$$\boldsymbol{g} = -\left(\boldsymbol{K}^{\mathrm{T}} + \boldsymbol{M}^{\mathrm{T}} + \boldsymbol{D}^{\mathrm{T}}\right)^{-1}\theta\boldsymbol{g}_{new}.$$

G.2 Transitional Dynamics

We describe the algorithm to find the transition of the equilibrium over time. First, we discretize the time horizon as $\boldsymbol{T} = (t_1, \ldots, t_\tau, \ldots, t_{T+1})'$ for $\tau \in \mathbb{Z}$ with $t_1 = 0$ and a large integer T, using the equidistant grid points with distance $\Delta_t = \Delta_k$ where $\Delta_t = t_\tau - t_{\tau-1}$ for all $\tau = 2, \ldots, T+1$. Let $\boldsymbol{v}$, $\boldsymbol{v}^*$ and $\boldsymbol{g}$ denote the vectors for the value functions and the density function for the consumer's stock in the stationary equilibrium, respectively. We keep the following notation: $x_\ell(\tau) = x(t_\tau, k_\ell)$ and $\boldsymbol{x}(\tau) = (x(t_\tau, k_1), \ldots, x(t_\tau, k_\ell), \ldots, x(t_\tau, k_L))'$.

1. Set $\boldsymbol{v}(T+1) = \boldsymbol{v}$, $\boldsymbol{v}^*(T+1) = \boldsymbol{v}^*$, $\boldsymbol{g}(1) = \boldsymbol{g}$, and initial store's stock $S(1) = S_o > 0$.

2. Set initial guess $\tilde{\boldsymbol{R}} = (\tilde{R}(1), \ldots, \tilde{R}(T+1))'$ for the consumer's brief $R(t)_{\{t\geq 0\}}$.

3. Given $\tilde{\boldsymbol{R}}$, find the paths $\{\boldsymbol{v}(\tau)\}_{\tau=1}^{T+1}$ and $\{\boldsymbol{M}(\tau)\}_{\tau=1}^{T+1}$ iteratively backward in time.

   3-A. Set $\tau = T$.

   3-B. Given $\boldsymbol{v}(\tau+1)$, find $\tilde{\boldsymbol{v}}(\tau)$ such that

$$r\tilde{\boldsymbol{v}}(\tau) = \boldsymbol{h} + \boldsymbol{K}\tilde{\boldsymbol{v}}(\tau) + \frac{\boldsymbol{v}(\tau+1) - \tilde{\boldsymbol{v}}(\tau)}{\Delta_t}.$$

   3-C. Given $\boldsymbol{v}^*(\tau+1)$ and $\tilde{\boldsymbol{v}}(\tau)$, find $\boldsymbol{v}^*(\tau)$ such that

$$(\alpha + r\tilde{R}(\tau))\boldsymbol{v}^*(\tau) = \boldsymbol{h} - c(\tau)\mathbf{1}_L + \boldsymbol{K}\boldsymbol{v}^*(\tau) + \frac{\boldsymbol{v}^*(\tau+1) - \boldsymbol{v}^*(\tau)}{\Delta_t} + \alpha\tilde{R}(\tau)\boldsymbol{v}^A(\tilde{\boldsymbol{v}}(\tau)).$$

   3-D. Find $\boldsymbol{v}(\tau)$ such that

$$\boldsymbol{v}(\tau) = \max\{\tilde{\boldsymbol{v}}(\tau), \boldsymbol{v}^*(\tau)\}$$

   that is, $\boldsymbol{v}(\tau)$ is the element-wise maximum of $\tilde{\boldsymbol{v}}(\tau)$ and $\boldsymbol{v}^*(\tau)$. At the same time, find the optimal policy, $k^*(\tau)$ and $\bar{k}(\tau)$.

   3-E. Set the transition intensity matrix $\boldsymbol{M}(\tau)$ as in (G.3)

   3-F. Repeat until $\tau = 1$

4. Given $\{\boldsymbol{M}(\tau)\}_{\tau=1}^{T+1}$, find the paths $\{\boldsymbol{g}(\tau)\}_{\tau=1}^{T+1}$ and $\{R(\tau)\}_{\tau=1}^{T+1}$ forwardly.

   4-A. Set $\tau = 1$

   4-B. Given $\boldsymbol{g}(\tau)$ and the optimal policy, find $D(\tau)$ as follows:

$$D(\tau) = \sum_{\ell=1,\ldots,L} \mathbb{1}_{\{k_\ell \le k^*(\tau)\}}(\bar{k}(\tau) - k_\ell)g_\ell(\tau).$$

   4-C. Given $\boldsymbol{g}(\tau)$ and $S(\tau)$, find $R(\tau)$ using the following rule:

$$R(\tau) = \min\left\{\frac{S(\tau) + s\cdot\Delta_t}{D(\tau)}, 1\right\}.$$

   4-D. Given $\boldsymbol{g}(\tau)$, find $\boldsymbol{g}(\tau+1)$ according to the rule: for some integer $n \ge 1$ and $i = 0, \ldots, n-1$

$$\frac{\boldsymbol{g}(\tau + (i+1)/n) - \boldsymbol{g}(\tau + i/n)}{\Delta_t/n} = (\boldsymbol{A} + \boldsymbol{M}(\tau) + \boldsymbol{D})^{\mathrm{T}}\boldsymbol{g}(\tau + i/n) + \theta\boldsymbol{g}_{new}.$$

4-E. Given $R(\tau)$ and $S(\tau)$, find $S(\tau+1)$ using the following rule:

$$S(\tau+1) = S(\tau) + (s \cdot \Delta_t - R(\tau)D(\tau)).$$

4-F. Repeat until $\tau = T$ and let $\boldsymbol{R} = (R(1), \ldots, R(T+1))'$.

5. If $\|\boldsymbol{R} - \tilde{\boldsymbol{R}}\| < \epsilon$, find the equilibrium transitional dynamics. Otherwise, update the guess $\tilde{\boldsymbol{R}}$ based on the following rule:

$$\tilde{\boldsymbol{R}} \leftarrow \lambda \tilde{\boldsymbol{R}} + (1-\lambda)\boldsymbol{R}.$$

with $\lambda \in (0,1)$, and repeat steps 3 and 4.

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