Appendix to Financial Frictions, Trends, and the Great Recession (Not for Publication)

Pablo A. Guerron-Quintana Ryo Jinnai*

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In this appendix, we provide a comprehensive discussion of the financial crunch, data, estimation, derivations of the model as well as additional results not reported in the main text.

1 Some empirical evidence

1.1 On the financial crunch

A measure of liquidity usually used in the finance literature comes from margins for S&P 500 futures (Brunnermeier and Pedersen (2009)). The higher the margin is, the larger the amount of money an investor must maintain in a future contract. According to these authors, margins tend to increase during periods of liquidity crises. Indeed, they show that margins moved up in previous periods of illiquidity, as in 1987 (Black Monday) or 1998 (Asian and LTCM crises). Figure 1 shows these margins for the last decade.[1] As one can see, the most recent crisis led to a spike in the margins. At the peak in 2009, financiers required investors to keep 12 percent of the value of a future contract as a capital requirement. Note, however, that this measure of liquidity indicates that financial conditions started to improve by 2011, and seem to be back to more normal levels by the end of 2012.

Becker and Ivashina (2014) report the time series of the aggregate stock of bank credit in the last six decades. While bank credit is generally procyclical, the speed of the bank-debt shrinkage during the Great Recession was clearly exceptional. According to the survey of senior loan officers on bank

*Guerron-Quintana: Boston College and Espol, email: pguerron@gmail.com. Jinnai: Hitotsubashi University, email: rjinnai@ier.hit-u.ac.jp.

[1] The margins are computed as the dollar margin divided by the product of the underlying S&P 500 index and the size of the contract ($250 in this case). Data for margins are taken from Chicago Mercantile Exchange's website (http://www.cmegroup.com/clearing/risk-management/historical-margins.html). We thank Ronel Elul for helping with the computation.

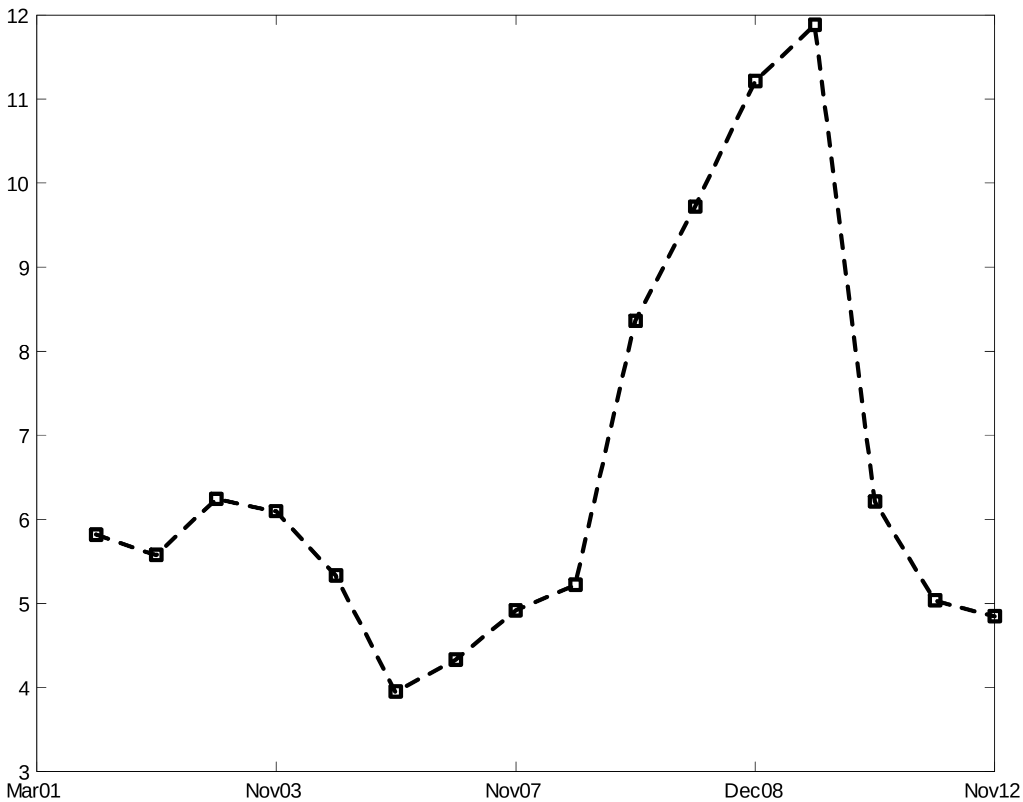

Figure 1: Margins for S&P 500 Futures

lending practices,[2] about 80 percent of loan officers reported in the aftermath of Lehman's collapse that standards for commercial and industrial loans have tightened over the past three months in their banks. This is the highest score in the last three recessions. Gap between loan rates and the bank's cost of funding spiked in spreads in 2008, suggesting that businesses (commercial and industrial; large and small) faced adverse financing conditions during the last recession.

On the cause of the dramatic shrinkage in lending activities, a number of studies report evidence suggesting that it was largely driven by an exogenous reduction in credit. Almeida, Campello, Laranjeira, and Weisbenner (2009) compare firms that needed to refinance a substantial fraction of their long-term debt over the year following August 2007 with firms that do not have a large refinancing in the period following the start of the financial crisis. After controlling other firm characteristics using a matching estimator, they find that investment of firms in the first group fell by one-third, while investment in the second group showed no investment reduction. Duchin, Ozbas, and Sensoy (2010) find a similar result by comparing firms that were carrying more cash prior to the onset of the crisis with firms that were carrying less cash. Campello, Graham, and Harvey (2010) surveyed 1,050 chief financial officers (CFOs) in 39 countries in the middle of the crisis and found that, after controlling other firm characteristics using a matching estimator, financially constrained firms planned to cut more investment, technology, marketing, and employment relative to financially unconstrained firms; to restrict their pursuit of attractive projects; and to cancel valuable investments.

Giroud and Mueller (2015) advocate the firm balance sheet channel. They first document that while the leverage of non-financial firms remained essentially flat during the Great Recession in aggregate, there is substantial variation in leverage changes at the firm level in the years prior to the Great Recession. They also show that firms with high leverage not only look like financially constrained firms based on popular measures but also act like financially constrained firms; specifically, these firms reduce employment, close down establishments, and cut back on investment. While the authors do not report a separate result for R&D, it is likely that the same firms reduce investment to intellectual properties as well.

A related and important channel in our model is that entrepreneurs/firms need funding to innovate. In reality, this funding often comes from Private Equity firms (PE), particularly those specialized in venture (growth) capital. We find informative that global investment by PEs collapsed in 2008, reached a cyclical low in 2009 and remained flat between 2010 and 2012 (Bain and Company (2014)).[3] A similarly dare picture arises from data on exits by PEs or funds raised by PEs. These observations should be cautiously interpreted because information on different components of private equity investment, particularly that by venture capital firms, is rather scant.

[2] It is published by the Federal Reserve Board, asking senior loan officers about "changes in the standards and terms on bank loans to businesses and households over the past three months." The most recent survey at the time of this writing (July 2013) included responses from officers at 73 domestic banks and 22 U.S. branches and agencies of foreign banking institutions.

[3] We thank an anonymous referee for mentioning the Bain's report on private equity.

However, data from funds raised by venture-capital PE firms show a substantial decline post-2008. Indeed, fund-raising remained flat between 2010 and 2012 and below its cyclical peak. Overall, we view this industry-based data as suggestive of funding headwinds faced by entrepreneurs during and after the Great Recession.

1.2 On firm creation

Evidence on firm entries is interesting to look at because the class of models we employ is often interpreted as capturing the process of firm creation and destruction (Bilbiie, Ghironi, and Melitz (2012)). Slowdown in entrepreneurial activities during the Great Recession is reported by the following authors. Using the Business Dynamics Statistics, Siemer (2014) reports that the number of firms in the U.S. contracted by more than 5 percent during the period 2007-2010. Furthermore, the number of startups, defined as firms one year old and younger, went down by 25 percent for the period 2006-2010. A critical factor driving the slowdown in firm creation was the lack of external financing. Sedlacek and Sterk (2014) report that firms created during recession episodes are on average smaller than their counterteparts started during expansions. Moreover, the size of the former firms tends to remain smaller even after the economy has recovered. In recent years, firms born in 2008 are around 13 percent smaller at age 5 than the equivalent firms created in 2006.

1.3 On innovative activities and financial shocks

Regarding the connection between innovative activies and financial shocks, we find that measures of financial restraint, such as tightening of lending standards published by the Federal Reserve Board, lead de-trended R&D expenditures for the period 1991-2013. The correlation between these measures is negative and depending on the number of leads and measure, the correlation fluctuates between -0.01 and -0.39. Important for our argument, if we condition the data to the period 2005-2013, the relevant period for the Great Recession, the negative correlation between financial conditions and R&D rises significantly. This connection is consistent with the mechanism in our model. That is, adverse financial conditions reduce R&D, which in turn reduces economic activity, delivering the break in the trend level.

2 Derivations of Benchmark Model

In this section, we provide details on the derivations behind the models in the main text.

2.1 Liquidity constraints

We assume that $1 < p_{n,t}\vartheta_t$ always hold. We show that entrepreneur's liquidity constraints

$$n_{t+1}^e \geq (1-\theta)\,\vartheta_t s_t + (1-\phi_t)\,(1-\delta_n)\,n_t \tag{1}$$

and

$$k_{t+1}^e \geq (1-\phi_t)\,(1-\delta\,(u_t))\,k_t \tag{2}$$

must bind at the optimum under this assumption. Suppose (1) is slack. In this case, the household can simultaneously increase product developments s_t by $\Delta > 0$ and an entrepreneur's consumption c_t^e by $(p_{n,t}\vartheta_t - 1)\,\Delta$ as long as Δ is sufficiently small. But because the entrepreneur's consumption increases while the household's asset portfolio at the beginning of the investment stage is constant, the assumption that (1) is slack contradicts the optimality. Suppose (2) is slack. In this case, the household can simultaneously decrease an entrepreneur's capital holding k_{t+1}^e by Δ, increase an entrepreneur's equity holding n_{t+1}^e by $(p_{k,t}/p_{n,t})\,\Delta$, increase a worker's capital holding k_{t+1}^w by $(\sigma_e/\sigma_w)\,\Delta$, and decrease a worker's equity holding n_{t+1}^w by $(\sigma_e/\sigma_w)\,(p_{k,t}/p_{n,t})\,\Delta$ as long as Δ is sufficiently small. These changes make (1) slack while they do not change the household's asset portfolio at the beginning of the investment stage. But since the household can increase the utility if (1) is slack, the assumption that (2) is slack contradicts the optimality.

We will also restrict our attention to the case in which $p_{n,t}\vartheta_t < 1/\theta$ always holds, because otherwise, the household's problem is not properly formulated. More specifically, the household can simultaneously increase the product developments s_t by Δ, increase an entrepreneur's asset holdings n_{t+1}^e by $(1-\theta)\,\vartheta_t\Delta$, increase an entrepreneur's consumption c_t^e by $(\theta p_{n,t}\vartheta_t - 1)\,\Delta$, decrease a worker's asset holdings n_{t+1}^w by $(\sigma_e/\sigma_w)\,(1-\theta)\,\vartheta_t\Delta$, and increase a worker's consumption c_t^w by $p_{n,t}\,(\sigma_e/\sigma_w)\,(1-\theta)\,\vartheta_t\Delta$. Because these changes do not influence the household's portfolio at the beginning of the investment stage but can increase the worker's consumption arbitrarily large, the problem does not have a solution.

2.2 Solving the household's problem

Given that the liquidity constraints (1) and (2) are binding, the household problem can be rewritten as a maximization problem of

$$v\,(q_t;\Gamma_t,\Theta_t) = \max\left\{\sigma_e\frac{\left(c_t^e\right)^{1-\frac{1}{\psi}}}{1-\frac{1}{\psi}} + \sigma_w\frac{\left[c_t^w + \frac{\varphi_t}{\omega}\,(\Psi_t)\,(1-l_t)^{\omega}\right]^{1-\frac{1}{\psi}}}{1-\frac{1}{\psi}} + \beta\mathbb{E}_t\left[v\,(q_{t+1};\Gamma_{t+1},\Theta_{t+1})\right]\right\} \tag{3}$$

subject to

$$\begin{aligned} & c_t^w + \frac{i_t}{\chi_t} + p_{n,t} n_{t+1}^w + p_{k,t} k_{t+1}^w \\ = \quad & (1-\tau_p)(\Pi_t n_t + u_t R_t k_t) + p_{n,t}(1-\delta_n) n_t + p_{k,t}(1-\delta(u_t)) k_t + (1-\tau_l) W_t l_t + \tau_{tr,t}, \end{aligned} \tag{4}$$

$$c_t^e + s_t(1-\theta p_{n,t}\vartheta_t) = (1-\tau_p)(\Pi_t n_t + u_t R_t k_t) + \phi_t [p_{n,t}(1-\delta_n) n_t + p_{k,t}(1-\delta(u_t)) k_t] + \tau_{tr,t}, \tag{5}$$

$$n_{t+1} = \sigma_e [(1-\theta)\vartheta_t s_t + (1-\phi_t)(1-\delta_n) n_t] + \sigma_w n_{t+1}^w,$$

and

$$k_{t+1} = \sigma_e (1-\phi_t)(1-\delta(u_t)) k_t + \sigma_w k_{t+1}^w + \left(1-\Lambda\left(\frac{i_t}{i_{t-1}}\right)\right)\sigma_w i_t.$$

First order optimality conditions are

$$\beta \mathbb{E}_t \left[\frac{\partial v_{t+1}}{\partial k_{t+1}}\right] \sigma_e (1-\phi_t)(-\delta'(u_t)) + \sigma_e \mu_t^e [(1-\tau_p) R_t - \phi_t p_{k,t} \delta'(u_t)] + \sigma_w \mu_t^w [(1-\tau_p) R_t - p_{k,t}\delta'(u_t)] = 0, \tag{6}$$

$$(c_t^e)^{-\frac{1}{\psi}} = \mu_t^e, \tag{7}$$

$$\beta \mathbb{E}_t \left[\frac{\partial v_{t+1}}{\partial n_{t+1}}\right] (1-\theta)\vartheta_t - \mu_t^e (1-\theta p_{n,t}\vartheta_t) = 0, \tag{8}$$

$$\left[c_t^w + \frac{\varphi_t}{\omega}(\Psi_t)(1-l_t)^\omega\right]^{-\frac{1}{\psi}} = \mu_t^w, \tag{9}$$

$$\left[c_t^w + \frac{\varphi_t}{\omega}(\Psi_t)(1-l_t)^\omega\right]^{-\frac{1}{\psi}} \left(-\varphi_t(\Psi_t)(1-l_t)^{\omega-1}\right) + \mu_t^w (1-\tau_l) W_t = 0, \tag{10}$$

$$\beta \mathbb{E}_t \left[\frac{\partial v_{t+1}}{\partial n_{t+1}}\right] = \mu_t^w p_{n,t}, \tag{11}$$

$$\beta \mathbb{E}_t \left[\frac{\partial v_{t+1}}{\partial k_{t+1}}\right] = \mu_t^w p_{k,t}, \tag{12}$$

and

$$\beta \mathbb{E}_t \left[\frac{\partial v_{t+1}}{\partial k_{t+1}}\right] \sigma_w \left(1-\Lambda\left(\frac{i_t}{i_{t-1}}\right) - \Lambda'\left(\frac{i_t}{i_{t-1}}\right)\frac{i_t}{i_{t-1}}\right) + \beta \mathbb{E}_t \left[\frac{\partial v_{t+1}}{\partial i_t}\right] = \sigma_w \mu_t^w \frac{1}{\chi_t}. \tag{13}$$

where μ_t^w and μ_t^e are Lagrangian multipliers associated with (4) and (5), respectively, and $\partial v_t / \partial n_t$ and $\partial v_t / \partial k_t$ are defined as $\partial v_t / \partial n_t = \partial v(q_t; \Gamma_t, \Theta_t) / \partial n_t$ and $\partial v_t / \partial k_t = \partial v(q_t; \Gamma_t, \Theta_t) / \partial k_t$, respectively. Envelope conditions are

$$\begin{aligned} \frac{\partial v_t}{\partial n_t} = \quad & \beta \mathbb{E}_t \left[\frac{\partial v_{t+1}}{\partial n_{t+1}}\right] \sigma_e (1-\phi_t)(1-\delta_n) \\ & + \sigma_e \mu_t^e [(1-\tau_p)\Pi_t + \phi_t p_{n,t}(1-\delta_n)] + \sigma_w \mu_t^w [(1-\tau_p)\Pi_t + p_{n,t}(1-\delta_n)], \end{aligned} \tag{14}$$

$$\begin{aligned}\frac{\partial v_t}{\partial k_t} &= \beta \mathbb{E}_t \left[\frac{\partial v_{t+1}}{\partial k_{t+1}}\right] \sigma_e (1-\phi_t)(1-\delta(u_t)) \\ &\quad +\sigma_e \mu_t^e \left[(1-\tau_p) u_t R_t + \phi_t p_{k,t}(1-\delta(u_t))\right] + \sigma_w \mu_t^w \left[(1-\tau_p) u_t R_t + p_{k,t}(1-\delta(u_t))\right],\end{aligned} \tag{15}$$

and

$$\frac{\partial v_t}{\partial i_{t-1}} = \beta \mathbb{E}_t \left[\frac{\partial v_{t+1}}{\partial k_{t+1}}\right] \sigma_w \Lambda' \left(\frac{i_t}{i_{t-1}}\right)\left(\frac{i_t}{i_{t-1}}\right)^2. \tag{16}$$

Combining (9) and (10), we find the optimality condition for labor supply

$$\varphi_t(\Psi_t)(1-l_t)^{\omega-1} = (1-\tau_l) W_t.$$

The optimality condition for investment

$$\frac{1}{\chi_t} = p_{k,t}\left(1-\Lambda\left(\frac{i_t}{i_{t-1}}\right) - \Lambda'\left(\frac{i_t}{i_{t-1}}\right)\frac{i_t}{i_{t-1}}\right) + \mathbb{E}_t\left[\beta\left(\frac{\mu_{t+1}^w}{\mu_t^w}\right) p_{k,t+1}\Lambda'\left(\frac{i_{t+1}}{i_t}\right)\left(\frac{i_{t+1}}{i_t}\right)^2\right] \tag{17}$$

is derived as follows. Combining (12) and (16), we find

$$\frac{\partial v_t}{\partial i_{t-1}} = \mu_t^w p_{k,t}\sigma_w \Lambda'\left(\frac{i_t}{i_{t-1}}\right)\left(\frac{i_t}{i_{t-1}}\right)^2. \tag{18}$$

Substituting (12) and (18) into (13), we find (17). The optimality condition for product developments

$$(c_t^e)^{-\frac{1}{\psi}} = \left(\frac{\vartheta_t(1-\theta)}{1-\theta p_{n,t}\vartheta_t}\right)\beta \mathbb{E}_t\left[\frac{\partial v(q_{t+1};\Gamma_{t+1},\Theta_{t+1})}{\partial n_{t+1}}\right] \tag{19}$$

is derived from (7) and (8). Pricing equation for equity

$$p_{n,t} = \mathbb{E}_t\left[\beta\left(\frac{\mu_{t+1}^w}{\mu_t^w}\right)\left((1-\tau_p)\Pi_{t+1} + p_{n,t+1}(1-\delta_n) + \sigma_e \lambda_{t+1}\left[(1-\tau_p)\Pi_{t+1} + \phi_{t+1}p_{n,t+1}(1-\delta_n)\right]\right)\right] \tag{20}$$

is derived as follows. Substituting (11) into (14), we find

$$\frac{\partial v_t}{\partial n_t} = \mu_t^w p_{n,t}\sigma_e(1-\phi_t)(1-\delta_n) + \sigma_e\mu_t^e\left[(1-\tau_p)\Pi_t + \phi_t p_{n,t}(1-\delta_n)\right] + \sigma_w\mu_t^w\left[(1-\tau_p)\Pi_t + p_{n,t}(1-\delta_n)\right]. \tag{21}$$

(11) and (19) imply

$$(c_t^e)^{-\frac{1}{\psi}} = (1+\lambda_t)\mu_t^w \tag{22}$$

where

$$\lambda_t = \frac{p_{n,t}\vartheta_t - 1}{1-\theta p_{n,t}\vartheta_t}.$$

Combining (21) and (22), we find

$$\frac{\partial v_t}{\partial n_t} = \mu_t^w \left((1-\tau_p)\,\Pi_t + p_{n,t}(1-\delta_n) + \sigma_e \lambda_t \left[(1-\tau_p)\,\Pi_t + \phi_t p_{n,t}(1-\delta_n) \right] \right). \tag{23}$$

Combining (11) and (23), we find (20). Following analogous steps, we find

$$p_{k,t} = \mathbb{E}_t \left[\beta \left(\frac{\mu_{t+1}^w}{\mu_t^w} \right) \left(\begin{array}{c} (1-\tau_p)\, u_{t+1} R_{t+1} + p_{k,t+1}\left(1-\delta\left(u_{t+1}\right)\right) \\ +\sigma_e \lambda_{t+1} \left[(1-\tau_p)\, u_{t+1} R_{t+1} + \phi_{t+1} p_{k,t+1}\left(1-\delta\left(u_{t+1}\right)\right) \right] \end{array} \right) \right].$$

Optimality condition for capacity utilization rate

$$(1-\tau_p)\, R_t - p_{k,t}\delta'(u_t) + \sigma_e \lambda_t \left[(1-\tau_p)\, R_t - \phi_t p_{k,t} \delta'(u_t) \right] = 0 \tag{24}$$

is derived as follows. First, substituting (12) into (6), we find

$$\mu_t^w p_{k,t} \sigma_e (1-\phi_t)(-\delta'(u_t)) + \sigma_e \mu_t^e \left[(1-\tau_p)\, R_t - \phi_t p_{k,t}\delta'(u_t) \right] + \sigma_w \mu_t^w \left[(1-\tau_p)\, R_t - p_{k,t}\delta'(u_t) \right] = 0.$$

Then dividing both sides by μ_t^w and substituting (22), we find (24).

2.3 Model summary

The following equations summarizes the model.

$$Y_t = (u_t K_t)^{\alpha} (Z_t \sigma_w l_t)^{1-\alpha},$$

$$Z_t = (\bar{A})(A_t)(N_t),$$

$$(c_t^e)^{-\frac{1}{\psi}} = (1+\lambda_t)\left[c_t^w + \frac{\varphi_t}{\omega}(\Psi_t)(1-l_t)^{\omega} \right]^{-\frac{1}{\psi}},$$

$$\varphi_t(\Psi_t)(1-l_t)^{\omega-1} = (1-\tau_l)\, W_t,$$

$$W_t = (1-\xi)(1-\alpha)\frac{Y_t}{\sigma_w l_t},$$

$$\lambda_t = \frac{p_{n,t}\vartheta_t - 1}{1-\theta p_{n,t}\vartheta_t},$$

$$p_{n,t} = \mathbb{E}_t \left[\begin{array}{c} \beta \left(\frac{c_{t+1}^w + \frac{\varphi_{t+1}}{\omega}(\Psi_{t+1})(1-l_{t+1})^{\omega}}{c_t^w + \frac{\varphi_t}{\omega}(\Psi_t)(1-l_t)^{\omega}} \right)^{-\frac{1}{\psi}} \\ \left((1-\tau_p)\,\Pi_{t+1} + p_{n,t+1}(1-\delta_n) + \sigma_e \lambda_{t+1} \left[(1-\tau_p)\,\Pi_{t+1} + \phi_{t+1} p_{n,t+1}(1-\delta_n) \right] \right) \end{array} \right],$$

$$\Pi_t = \left(\frac{\nu-1}{\nu} \right) \xi \frac{Y_t}{N_t},$$

$$R_t = (1-\xi)\,\alpha \frac{Y_t}{u_t K_t},$$

$$p_{k,t}=\mathbb{E}_t\left[\begin{array}{c}\beta\left(\frac{c_{t+1}^w+\frac{\varphi_{t+1}}{\omega}(\Psi_{t+1})(1-l_{t+1})^\omega}{c_t^w+\frac{\varphi_t}{\omega}(\Psi_t)(1-l_t)^\omega}\right)^{-\frac{1}{\psi}}\\ \left(\begin{array}{c}(1-\tau_p)\,u_{t+1}R_{t+1}+p_{k,t+1}\left(1-\delta\left(u_{t+1}\right)\right)\\ +\sigma_e\lambda_{t+1}\left[(1-\tau_p)\,u_{t+1}R_{t+1}+\phi_{t+1}p_{k,t+1}\left(1-\delta\left(u_{t+1}\right)\right)\right]\end{array}\right)\end{array}\right],$$

$$(1-\tau_p)\,R_t-p_{k,t}\delta'\left(u_t\right)+\sigma_e\lambda_t\left[(1-\tau_p)\,R_t-\phi_t p_{k,t}\delta'\left(u_t\right)\right]=0,$$

$$\begin{aligned}\frac{1}{\chi_t} &= p_{k,t}\left(1-\Lambda\left(\frac{i_t}{i_{t-1}}\right)-\Lambda'\left(\frac{i_t}{i_{t-1}}\right)\frac{i_t}{i_{t-1}}\right)\\ &\quad+\mathbb{E}_t\left[\beta\left(\frac{c_{t+1}^w+\frac{\varphi_{t+1}}{\omega}\left(\Psi_{t+1}\right)\left(1-l_{t+1}\right)^\omega}{c_t^w+\frac{\varphi_t}{\omega}\left(\Psi_t\right)\left(1-l_t\right)^\omega}\right)^{-\frac{1}{\psi}}p_{k,t+1}\Lambda'\left(\frac{i_{t+1}}{i_t}\right)\left(\frac{i_{t+1}}{i_t}\right)^2\right],\end{aligned}$$

$$c_t^e+s_t\left(1-\theta p_{n,t}\vartheta_t\right)=\Pi_tN_t+u_tR_tK_t+\phi_t\left[p_{n,t}\left(1-\delta_n\right)N_t+p_{k,t}\left(1-\delta\left(u_t\right)\right)K_t\right]+\tau_lW_t\sigma_wl_t-Gov_t,$$

$$\frac{Gov_t}{N_t}=g_t,$$

$$N_{t+1}=\left(1-\delta_n\right)N_t+\vartheta_t\left(\sigma_es_t\right),$$

$$K_{t+1}=\left(1-\delta\left(u_t\right)\right)K_t+\left(1-\Lambda\left(\frac{i_t}{i_{t-1}}\right)\right)\sigma_wi_t,$$

$$\vartheta_t=\frac{\zeta N_t}{\left(\sigma_es_t\right)^{1-\eta}\left(N_t\right)^\eta},$$

$$\left(1-\frac{\xi}{\nu}\right)Y_t=\sigma_ec_t^e+\sigma_wc_t^w+\sigma_w\frac{i_t}{\chi_t}+\sigma_es_t+Gov_t,$$

$$\Psi_t=p_{k,t}K_t,$$

$$Stock_t=p_{k,t}\left(1-\delta\left(u_t\right)\right)K_t+p_{n,t}N_{t+1},$$

$$\log\left(\frac{TFP_t}{TFP_{t-1}}\right)=\alpha\log\left(\frac{u_t}{u_{t-1}}\right)+(1-\alpha)\log\left(\frac{A_t}{A_{t-1}}\right)+(1-\alpha)\log\left(\frac{N_t}{N_{t-1}}\right).$$

Detrending them by N_t, we obtain a new system of equations with stationary variables. It is summarized by the following equations:

$$\hat{Y}_t=\left(u_t\hat{K}_t\right)^\alpha\left(\left(\bar{A}\right)\left(A_t\right)\left[\sigma_wl_t\right]\right)^{1-\alpha},$$

$$\left(\hat{c}_t^e\right)^{-\frac{1}{\psi}}=\left(1+\lambda_t\right)\left[\hat{c}_t^w+\frac{\varphi_t}{\omega}\left(\hat{\Psi}_t\right)\left(1-l_t\right)^\omega\right]^{-\frac{1}{\psi}},$$

$$\varphi_t\left(\hat{\Psi}_t\right)\left(1-l_t\right)^{\omega-1}=\left(1-\tau_l\right)\hat{W}_t,$$

$$\hat{W}_t=(1-\xi)\left(1-\alpha\right)\frac{\hat{Y}_t}{\sigma_wl_t},$$

$$\lambda_t=\frac{p_{n,t}\vartheta_t-1}{1-\theta p_{n,t}\vartheta_t},$$

$$p_{n,t} = \mathbb{E}_t \left[\begin{array}{c} \beta \left(\gamma_{t+1} \frac{\hat{c}^w_{t+1} + \frac{\varphi_{t+1}}{\omega} \left(\hat{\Psi}_{t+1} \right) (1-l_{t+1})^{\omega}}{\hat{c}^w_t + \frac{\varphi_t}{\omega} \left(\hat{\Psi}_t \right) (1-l_t)^{\omega}} \right)^{-\frac{1}{\psi}} \\ \left((1-\tau_p) \Pi_{t+1} + p_{n,t+1} (1-\delta_n) + \sigma_e \lambda_{t+1} \left[(1-\tau_p) \Pi_{t+1} + \phi_{t+1} p_{n,t+1} (1-\delta_n) \right] \right) \end{array} \right],$$

$$\Pi_t = \left(\frac{\nu - 1}{\nu} \right) \xi \hat{Y}_t,$$

$$R_t = (1-\xi) \alpha \frac{\hat{Y}_t}{u_t \hat{K}_t},$$

$$p_{k,t} = \mathbb{E}_t \left[\begin{array}{c} \beta \left(\gamma_{t+1} \frac{\hat{c}^w_{t+1} + \frac{\varphi_{t+1}}{\omega} \left(\hat{\Psi}_{t+1} \right) (1-l_{t+1})^{\omega}}{\hat{c}^w_t + \frac{\varphi_t}{\omega} \left(\hat{\Psi}_t \right) (1-l_t)^{\omega}} \right)^{-\frac{1}{\psi}} \\ \left(\begin{array}{c} (1-\tau_p) u_{t+1} R_{t+1} + p_{k,t+1} (1-\delta(u_{t+1})) \\ +\sigma_e \lambda_{t+1} \left[(1-\tau_p) u_{t+1} R_{t+1} + \phi_{t+1} p_{k,t+1} (1-\delta(u_{t+1})) \right] \end{array} \right) \end{array} \right],$$

$$(1-\tau_p) R_t - p_{k,t} \delta'(u_t) + \sigma_e \lambda_t \left[(1-\tau_p) R_t - \phi_t p_{k,t} \delta'(u_t) \right] = 0,$$

$$\begin{aligned} \frac{1}{\chi_t} &= p_{k,t} \left(1 - \Lambda \left(\gamma_t \frac{\hat{\imath}_t}{\hat{\imath}_{t-1}} \right) - \Lambda' \left(\gamma_t \frac{\hat{\imath}_t}{\hat{\imath}_{t-1}} \right) \left(\gamma_t \frac{\hat{\imath}_t}{\hat{\imath}_{t-1}} \right) \right) \\ &+ \mathbb{E}_t \left[\beta \left(\gamma_{t+1} \frac{\hat{c}^w_{t+1} + \frac{\varphi_{t+1}}{\omega} \left(\hat{\Psi}_{t+1} \right) (1-l_{t+1})^{\omega}}{\hat{c}^w_t + \frac{\varphi_t}{\omega} \left(\hat{\Psi}_t \right) (1-l_t)^{\omega}} \right)^{-\frac{1}{\psi}} p_{k,t+1} \Lambda' \left(\gamma_{t+1} \frac{\hat{\imath}_{t+1}}{\hat{\imath}_t} \right) \left(\gamma_{t+1} \frac{\hat{\imath}_{t+1}}{\hat{\imath}_t} \right)^2 \right], \end{aligned}$$

$$\hat{c}^e_t + \hat{s}_t (1 - \theta p_{n,t} \vartheta_t) = \Pi_t + u_t R_t \hat{K}_t + \phi_t \left[p_{n,t} (1-\delta_n) + p_{k,t} (1-\delta(u_t)) \hat{K}_t \right] + \tau_l \hat{W}_t \sigma_w l_t - g_t,$$

$$\gamma_{t+1} = 1 - \delta_n + \vartheta_t (\sigma_e \hat{s}_t)$$

$$\gamma_{t+1} \hat{K}_{t+1} = (1-\delta(u_t)) \hat{K}_t + \left(1 - \Lambda \left(\gamma_t \frac{\hat{\imath}_t}{\hat{\imath}_{t-1}} \right) \right) \sigma_w \hat{\imath}_t.$$

$$\vartheta_t = \zeta (\sigma_e \hat{s}_t)^{\eta - 1},$$

$$\left(1 - \frac{\xi}{\nu} \right) \hat{Y}_t = \sigma_e \hat{c}^e_t + \sigma_w \hat{c}^w_t + \sigma_w \frac{\hat{\imath}_t}{\chi_t} + \sigma_e \hat{s}_t + g_t,$$

$$\hat{\Psi}_t = p_{k,t} \hat{K}_t,$$

$$\widehat{Stock}_t = p_{k,t} (1-\delta(u_t)) \hat{K}_t + p_{n,t} \gamma_{t+1},$$

$$\log \left(\frac{TFP_t}{TFP_{t-1}} \right) = \alpha \log \left(\frac{u_t}{u_{t-1}} \right) + (1-\alpha) \log \left(\frac{A_t}{A_{t-1}} \right) + (1-\alpha) \log (\gamma_t).$$

Hat variables denote the original variables divided by N_t, i.e., $\hat{Y}_t = Y_t / N_t$, and so on. γ_{t+1} is defined as $\gamma_{t+1} = N_{t+1}/N_t$.

2.4 Calibration

We present our calibration strategy. We calibrate A and φ so that the steady state detrended gross output $\hat{Y}$ is set at $\hat{Y} = 1$, and the steady state labor supply l is set at $l = 1/3$. We also assume that the home production function has the same elasticity of labor as the aggregate production function in the market sector, i.e., $\omega = 1 - \alpha$.

GDP is written as a function of ξ and ν,

$$\widehat{\mathcal{Y}} = 1 - \frac{\xi}{\nu}.$$

The aggregate wage income is written as

$$\hat{W}(\sigma_w l) = (1-\xi)(1-\alpha).$$

The labor share is a function of α, ν, and ξ,

$$\frac{\hat{W}(\sigma_w l)}{\widehat{\mathcal{Y}}} = \frac{(1-\xi)(1-\alpha)}{1-\xi/\nu}. \tag{25}$$

where ξ is a function of α and ν because we impose a parameter restriction

$$\xi = \frac{1-\alpha}{\nu-\alpha}.$$

Pricing equation for capital in the steady state is

$$1 = \beta(\gamma)^{-\frac{1}{\psi}}\left(uR(1-\tau_p) + 1 - \delta_k + \sigma_e\lambda\left(uR(1-\tau_p) + \phi(1-\delta_k)\right)\right), \tag{26}$$

where

$$uR = (1-\xi)\,\alpha\frac{1}{\hat{K}},$$

$$\hat{K} = \frac{\sigma_w \hat{\imath}}{\gamma - 1 + \delta_k},$$

$$\sigma_w \hat{\imath} = \left(1 - \frac{\xi}{\nu}\right)\left(\frac{\sigma_w \hat{\imath}}{\widehat{\mathcal{Y}}}\right),$$

and

$$\lambda = \frac{p_n\vartheta - 1}{1 - \theta p_n\vartheta}.$$

Pricing equation for a product in the steady state is

$$\frac{p_n\vartheta}{\vartheta} = \beta(\gamma)^{-\frac{1}{\psi}}\left(\Pi(1-\tau_p) + \frac{p_n\vartheta}{\vartheta}(1-\delta_n) + \sigma_e\lambda\left(\Pi(1-\tau_p) + \phi\frac{p_n\vartheta}{\vartheta}(1-\delta_n)\right)\right) \tag{27}$$

where

$$\Pi = \left(\frac{\nu - 1}{\nu}\right)\xi,$$

$$\vartheta = \frac{\gamma - 1 + \delta_n}{\sigma_e \hat{s}},$$

and

$$\sigma_e \hat{s} = \left(1 - \frac{\xi}{\nu}\right)\left(\frac{\sigma_e \hat{s}}{\widehat{\mathcal{Y}}}\right).$$

Entrepreneur's budget constraint in the steady state is

$$\hat{c}^e + \frac{\sigma_e \hat{s}}{\sigma_e}\left(1 - \theta p_n \vartheta\right) = \Pi + (1 - \xi)\,\alpha + \phi\left[\frac{p_n \vartheta}{\vartheta}\left(1 - \delta_n\right) + \left(1 - \delta_k\right)\hat{K}\right] + \tau_l\,(1 - \xi)\,(1 - \alpha) - g, \quad (28)$$

where g is the steady state value of the government consumption shock, which is calibrated to match the government consumption share in GDP, i.e.,

$$g = \underbrace{\left(\frac{Gov}{\mathcal{Y}}\right)}_{\text{government consumption share in GDP}} \left(1 - \frac{\xi}{\nu}\right).$$

Optimal intratemporal resource allocation in the steady state is

$$\left(\hat{c}^e\right)^{-\frac{1}{\psi}} = (1 + \lambda)\left[\hat{c}^w + \frac{1 - \tau_l}{\omega}\frac{(1 - \xi)\,(1 - \alpha)}{1 - \sigma_e}\frac{1 - l}{l}\right]^{-\frac{1}{\psi}} \quad (29)$$

where

$$\omega = 1 - \alpha.$$

We assume that the steady state value of the M.E.I. shock χ is $\chi = 1$. The resource constraint in the steady state is

$$1 = \left(1 - \frac{\xi}{\nu}\right)^{-1}\left(\sigma_e \hat{c}^e + (1 - \sigma_e)\,\hat{c}^w\right) + \frac{\sigma_w \hat{\imath}}{\widehat{\mathcal{Y}}} + \frac{\sigma_e \hat{s}}{\widehat{\mathcal{Y}}} + \frac{g}{\widehat{\mathcal{Y}}}. \quad (30)$$

Note that equations (25) to (30) are a six-equation, six-unknown system if we specify values of β, γ, δ_k, δ_n, τ_p, τ_l, $Gov/\mathcal{Y}$, $\hat{W}\left(\sigma_w l\right)/\widehat{\mathcal{Y}}$, $\sigma_w \hat{\imath}/\widehat{\mathcal{Y}}$, and $\sigma_e \hat{s}/\widehat{\mathcal{Y}}$. Solving them, we calibrate paramteters α, ν, and σ_e, and find unknown steady state values $p_n\vartheta$, $\hat{c}^e$, and $\hat{c}_w$.

Other steady state values are backed out as follows.

$$g = \underbrace{\left(\frac{Gov}{\mathcal{Y}}\right)\left(1 - \frac{\xi}{\nu}\right)}_{\text{known}}$$

$$\sigma_w \hat{\imath} = \underbrace{\left(1 - \frac{\xi}{\nu}\right)\left(\frac{\sigma_w \hat{\imath}}{\widehat{\mathcal{Y}}}\right)}_{\text{known}}$$

$$\hat{\imath} = \underbrace{\frac{\sigma_w \hat{\imath}}{\sigma_w}}_{\text{known}}$$

$$\hat{K} = \underbrace{\frac{\sigma_w \hat{\imath}}{\gamma - 1 + \delta_k}}_{\text{known}}$$

$$\lambda = \underbrace{\frac{p_n \vartheta - 1}{1 - \theta p_n \vartheta}}_{\text{known}}$$

$$\sigma_e \hat{s} = \underbrace{\left(1 - \frac{\xi}{\nu}\right)\left(\frac{\sigma_e \hat{s}}{\widehat{\mathcal{Y}}}\right)}_{\text{known}}$$

$$\hat{s} = \underbrace{\frac{\sigma_e \hat{s}}{\sigma_e}}_{\text{known}}$$

$$\vartheta = \underbrace{\frac{\gamma - 1 + \delta_n}{\sigma_e \hat{s}}}_{\text{known}}$$

$$\Pi = \underbrace{\left(\frac{\nu - 1}{\nu}\right)\xi}_{\text{known}}$$

$$\hat{W} = \underbrace{(1 - \xi)(1 - \alpha)\frac{1}{\sigma_w l}}_{\text{known}}$$

$$\hat{\Psi} = \underbrace{p_k \hat{K}}_{\text{known}}$$

$$\varphi = \underbrace{\frac{(1 - \tau_l)\hat{W}}{\hat{\Psi}(1 - l)^{\omega - 1}}}_{\text{known}}$$

We assume $u = 1$.

$$R = \underbrace{(1 - \xi)\alpha\frac{1}{\hat{K}}}_{\text{known}}$$

$$\delta'(u) = \underbrace{\frac{1 + \sigma_e \lambda}{1 + \phi \sigma_e \lambda}(1 - \tau_p) R}_{\text{known}}$$

$$A = \underbrace{\left(\hat{K}\right)^{\frac{-\alpha}{1-\alpha}} \left(\bar{A}\sigma_w l\right)^{-1}}_{\text{known}}$$

$$\zeta = \underbrace{\vartheta \left(\sigma_e \hat{s}\right)^{1-\eta}}_{\text{known}}$$

3 Growth accounting and TFP

We first review standard growth accounting. See Fernald (2014) for further discussion. Let's assume a constant returns aggregate production function

$$Y_t = \tilde{A}_t F\left(W_t K_t, E_t L_t\right), \tag{31}$$

where $\tilde{A}_t$ is technology, W_t is the workweek of capital, and E_t is effort. First order Taylor approximation of the right-hand side is

$$\begin{aligned} \tilde{A}_t F\left(W_t K_t, E_t L_t\right) &= \tilde{A}_{t-1} F\left(W_{t-1} K_{t-1}, E_{t-1} L_{t-1}\right) \\ &\quad + F\left(W_{t-1} K_{t-1}, E_{t-1} L_{t-1}\right)\left(\tilde{A}_t - \tilde{A}_{t-1}\right) \\ &\quad + \tilde{A}_{t-1} F_1\left(W_{t-1} K_{t-1}, E_{t-1} L_{t-1}\right)\left(K_{t-1}\left(W_t - W_{t-1}\right) + W_{t-1}\left(K_t - K_{t-1}\right)\right) \\ &\quad + \tilde{A}_{t-1} F_2\left(W_{t-1} K_{t-1}, E_{t-1} L_{t-1}\right)\left(L_{t-1}\left(E_t - E_{t-1}\right) + E_{t-1}\left(L_t - L_{t-1}\right)\right) \end{aligned} \tag{32}$$

where F_1 and F_2 are first order derivative of $F(\cdot,\cdot)$ with respect to first and second arguments, respectively. Combining (31) and (32), we obtain

$$\begin{aligned} Y_t - Y_{t-1} &= F\left(W_{t-1} K_{t-1}, E_{t-1} L_{t-1}\right)\left(\tilde{A}_t - \tilde{A}_{t-1}\right) \\ &\quad + \tilde{A}_{t-1} F_1\left(W_{t-1} K_{t-1}, E_{t-1} L_{t-1}\right)\left(K_{t-1}\left(W_t - W_{t-1}\right) + W_{t-1}\left(K_t - K_{t-1}\right)\right) \\ &\quad + \tilde{A}_{t-1} F_2\left(W_{t-1} K_{t-1}, E_{t-1} L_{t-1}\right)\left(L_{t-1}\left(E_t - E_{t-1}\right) + E_{t-1}\left(L_t - L_{t-1}\right)\right) \end{aligned}$$

Applying the approximation $\left(Y_t - Y_{t-1}\right)/Y_{t-1} = \log\left(Y_t/Y_{t-1}\right)$, we obtain

$$\begin{aligned} \log\left(\frac{Y_t}{Y_{t-1}}\right) &= \log\left(\frac{\tilde{A}_t}{\tilde{A}_{t-1}}\right) \\ &\quad + \underbrace{\tilde{A}_{t-1} F_1\left(W_{t-1} K_{t-1}, E_{t-1} L_{t-1}\right)\frac{W_{t-1} K_{t-1}}{Y_{t-1}}}_{\text{output elasticity for capital}}\left(\log\left(\frac{W_t}{W_{t-1}}\right) + \log\left(\frac{K_t}{K_{t-1}}\right)\right) \\ &\quad + \underbrace{\tilde{A}_{t-1} F_2\left(W_{t-1} K_{t-1}, E_{t-1} L_{t-1}\right)\frac{E_{t-1} L_{t-1}}{Y_{t-1}}}_{\text{output elasticity for labor}}\left(\log\left(\frac{E_t}{E_{t-1}}\right) + \log\left(\frac{L_t}{L_{t-1}}\right)\right). \end{aligned}$$

Because the production function is constant returns to scale, cost minimization implies that output elasticity for capital is equal to capital share in costs;

$$
\begin{aligned}
\log\left(\frac{Y_t}{Y_{t-1}}\right) &= \log\left(\frac{\tilde{A}_t}{\tilde{A}_{t-1}}\right) \\
&+ \underbrace{\alpha_{t-1}}_{\text{capital share in costs}} \left(\log\left(\frac{W_t}{W_{t-1}}\right) + \log\left(\frac{K_t}{K_{t-1}}\right)\right) \\
&+ \underbrace{(1-\alpha_{t-1})}_{\text{labor share in costs}} \left(\log\left(\frac{E_t}{E_{t-1}}\right) + \log\left(\frac{L_t}{L_{t-1}}\right)\right).
\end{aligned}
$$

where α_{t-1} is total payments to capital as a share in the total costs of capital and labor.

Fernald (2014) defines growth in TFP by

$$
\log\left(\frac{TFP_t}{TFP_{t-1}}\right) \equiv \log\left(\frac{Y_t}{Y_{t-1}}\right) - \alpha_{t-1}\log\left(\frac{K_t}{K_{t-1}}\right) - (1-\alpha_{t-1})\log\left(\frac{L_t}{L_{t-1}}\right).
$$

It is easy to see that TFP defined as such is a noisy measure of technology

$$
\log\left(\frac{TFP_t}{TFP_{t-1}}\right) = \log\left(\frac{\tilde{A}_t}{\tilde{A}_{t-1}}\right) + \alpha_{t-1}\log\left(\frac{W_t}{W_{t-1}}\right) + (1-\alpha_{t-1})\log\left(\frac{E_t}{E_{t-1}}\right).
$$

We define growth in TFP in an analogous way in our model,

$$
\log\left(\frac{TFP_t}{TFP_{t-1}}\right) = \log\left(\frac{\mathcal{Y}_t}{\mathcal{Y}_{t-1}}\right) - \alpha\log\left(\frac{K_t}{K_{t-1}}\right) - (1-\alpha)\log\left(\frac{L_t}{L_{t-1}}\right).
$$

Growth in TFP has a clean decomposition,

$$
\log\left(\frac{TFP_t}{TFP_{t-1}}\right) = (1-\alpha)\left[\log\left(\frac{A_t}{A_{t-1}}\right) + \log\left(\frac{N_t}{N_{t-1}}\right)\right] + \alpha\log\left(\frac{u_t}{u_{t-1}}\right).
$$

Notice that the use of the constant α in our model is consistent with the aforementioned growth accounting theory, because the capital share in total costs in our model economy is constant at α:

$$
\frac{R_t\left(u_t K_t\right)}{R_t\left(u_t K_t\right) + W_t\left(\sigma_w l_t\right)} = \frac{(1-\xi)\,\alpha}{(1-\xi)(1-\alpha) + (1-\xi)\,\alpha} = \alpha.
$$

4 Data

Here, we describe in detail the data used in the estimation of our model. We take nominal GDP and personal consumption expenditures from the BEA. The investment series corresponds to gross private domestic investment. As explained in the main text, we use Nakamura's (Nakamura (2003))

R&D series in lieu of NIPA's intellectual property series. The main text contains additional details' on Nakamura's measure. We adjust both the GDP and investment series to reflect this change. Although it affects the level of these series, it has a negligible impact on the growth rates which we use in the estimation. Our measure of labor is hours of all persons in the nonfarm business sector divided by civilian population 16 and older (Altig, Christiano, Eichenbaum, and Linde (2011)); the measure is HP filtered. Total factor productivity is taken from Fernald (Fernald (2014)).

The value of the stock market corresponds to corporate equity in the nonfinancial corporate business sector. The data come from the Federal Reserve Board (Flow of Funds Table L.213) and are seasonally adjusted. We do not include corporate debt and the valuation of the financial sector in our measure because our model does not have them explicitly. Our measure of financial conditions is based on the liquidity/financial risk shocks considered by Stock and Watson (2012). Specifically, we take the 4-quarter moving average of their first principal component of the TED spread, excess bond premium (Gilchrist and Zakrajsek (2012)), and bank loan supply shock (Bassett, Chosak, Driscoll, and Zakrajsek (2012)). We use the implicit GDP deflator to convert nominal series into real variables. Except for labor and liquidity, all other variables are expressed in growth rates. The sample covers the period 1970.Q1 - 2011.Q4.

5 Estimation

We use a random-walk Metropolis Hasting simulator to characterize the posterior distributions of the parameters of interest. The acceptance rate of the simulator is set to approximately 30% (Robert and Casella (2004)). After an extensive search for the mode and a burn-in period, the posteriors' statistics were computed with $600,000$ draws. We ensure convergence of the chains to their ergodic distributions by checking different objects. Figure 2 shows the raw chains for each of the parameters estimated in our model. As one can see, there is substantial variation and no obvious persistence within each chain. Figure 3 in turn presents the histograms for each estimated parameter. Clearly, the posterior distributions are unimodal and concentrated away from the prior distributions. Following Robert and Casella (2004), we report cumulative means and cumulative sums (CUSUM) plots in Figures 4 and 5, respectively. The cumulative means, which are computed in steps of 1,000, show that the mean of the estimated parameters stabilizes after 300,000 iterations. Although there is some variation, note the size of the scale. Therefore, for practical purposes, we can be relatively confident that the means are stable. The CUSUM plots in turn reveal irregular patterns around zero, which suggest adequate mixing of the chains.

6 Additional Results to the Benchmark Model

We provide supplementary materials to and additional results from the benchmark model.

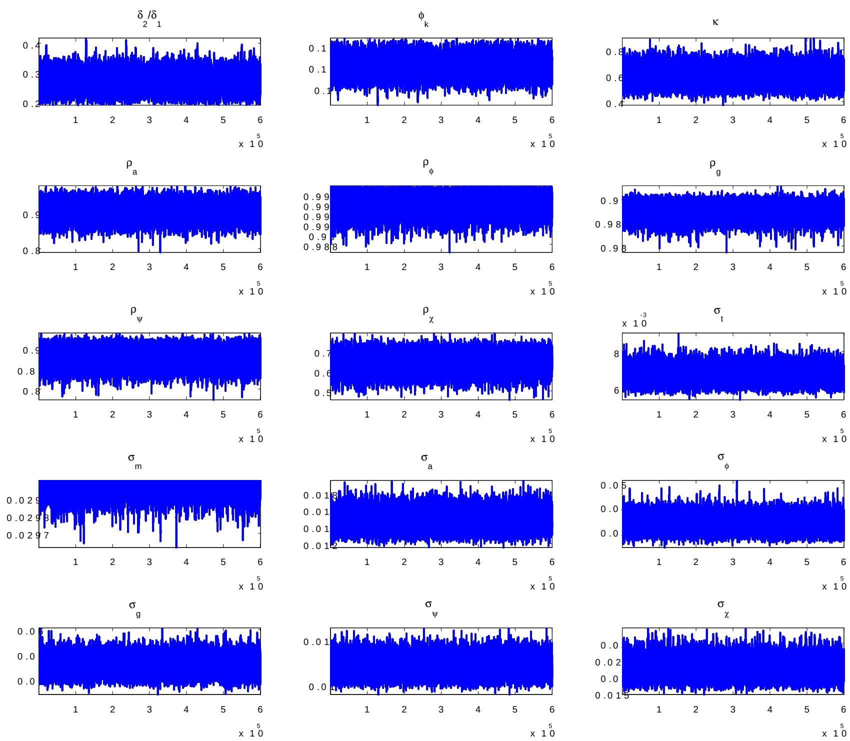

Figure 2: Raw Chains from Metropolis-Hasting Simulator

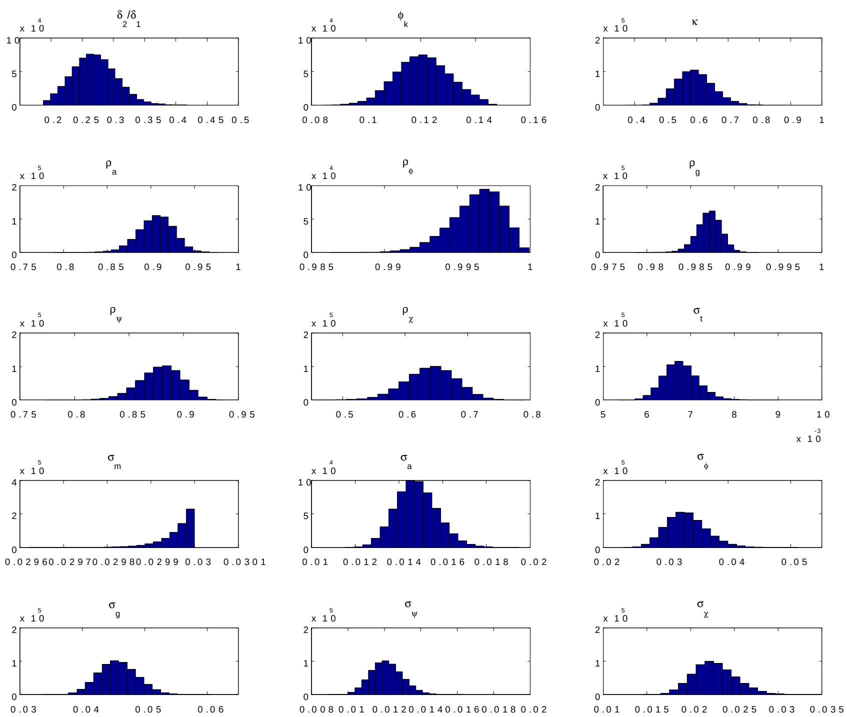

Figure 3: Estimated Parameters Histograms

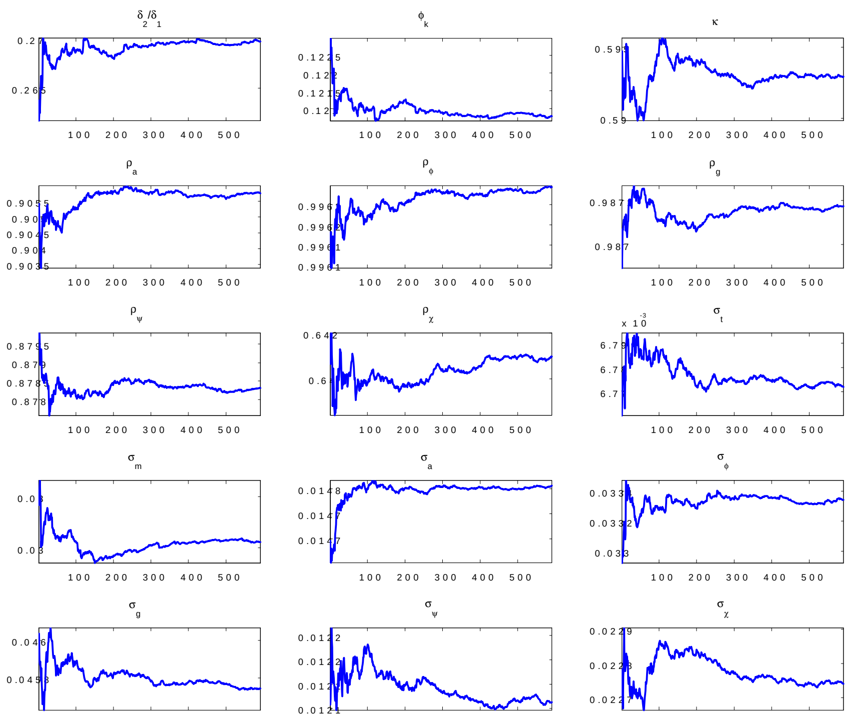

Figure 4: Cumulative Means

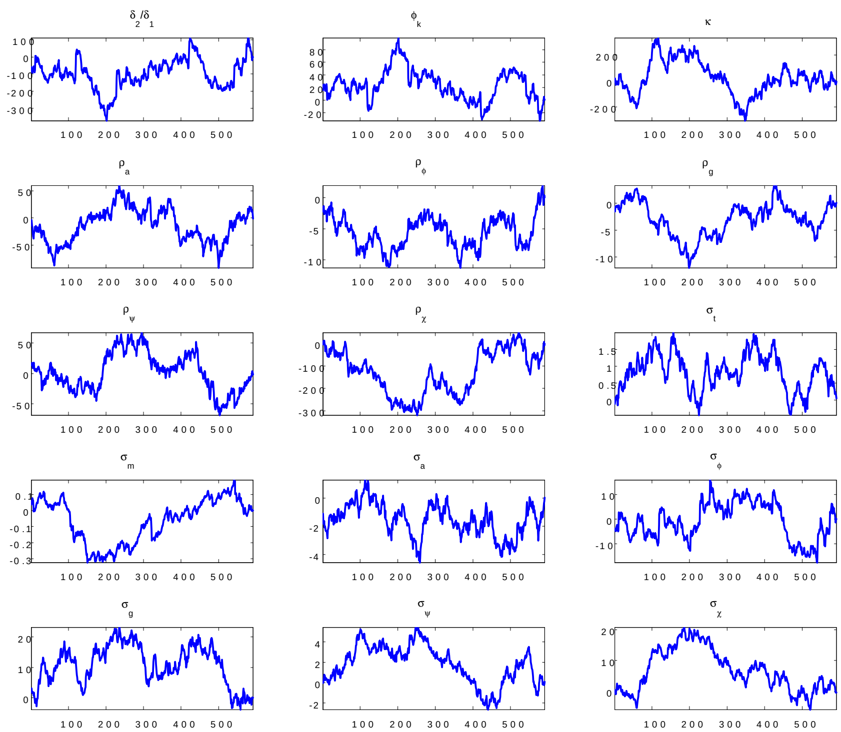

Figure 5: Cusum Statistics Benchmark Model

6.1 Shock Contribution during the Great Recession

Figure 6 plots the paths of different observables if only one shock is the driver of the economy at a time. The paths for each variable given each shock are depicted in the figure. As stressed in the main text, both the liquidity (solid black line with cross) and the government consumption (solid green line with dot) dragged the economy into the recession, pulling down production, consumption, hours worked, and TFP. Contrary, the preference shock (red dashed line) and the technology shock (red dotted line) counteracted to these forces. The M.E.I. shock (solid red line) is an important contributor to the collapse of investment and an initial drag to the economy. Both the liquidity shock and the government consumption shock are main culprits in the collapse in the stock market. The figure also highlights the importance of liquidity during the recovery phase, in particular for output, consumption, and investment. The shock to the efficiency of investment is also a contributor to the post-crisis period, particularly so for labor and investment.

6.2 Back to Trend: Other Shocks

In the main text, we report the counterfactual paths of liquidity and government consumption shock that would bring the economy back to its trend. We do the same exercise for the other structural shocks. Results are plotted in Figure 7. Notice wild movements in M.E.I. (first panel), technology (second panel), and preference shock (third panel). Indeed, the size of the fluctuations of these shocks that are necessary to avert the crisis is about an order of magnitude bigger than the shocks to liquidity and government consumption reported in the main text. Productivity had to be persistently above its average starting in mid 2008 and well into 2012; a similar story can be told for preference shock. These results indicate that the main causes of the Great Recession are unusual movements in liquidity and government consumption shocks—recall that restoring them to normal conditions was enough to avoid the Great Recession—but not unusual movements in other structural shocks. Quite the opposite, we need unusual movements in these shocks to avoid the Great Recession.

6.3 Learning-by-doing model

We first examine asset pricing implications in a simple endogenous growth model to highlight the basic mechanism and to examine its robustness. The household's problem is summarized as follows. The head of the household chooses instructions to its members to maximize the value function defined as

$$v\left(k_t; \Gamma_t, \Theta_t\right) = \max\left\{\sigma_e \frac{\left(c_t^e\right)^{1-\frac{1}{\psi}}}{1-\frac{1}{\psi}} + \sigma_w \frac{\left[c_t^w \left(1-l_t\right)^{\varrho}\right]^{1-\frac{1}{\psi}}}{1-\frac{1}{\psi}} + \beta \mathbb{E}_t\left[v\left(k_{t+1}; \Gamma_{t+1}, \Theta_{t+1}\right)\right]\right\}$$

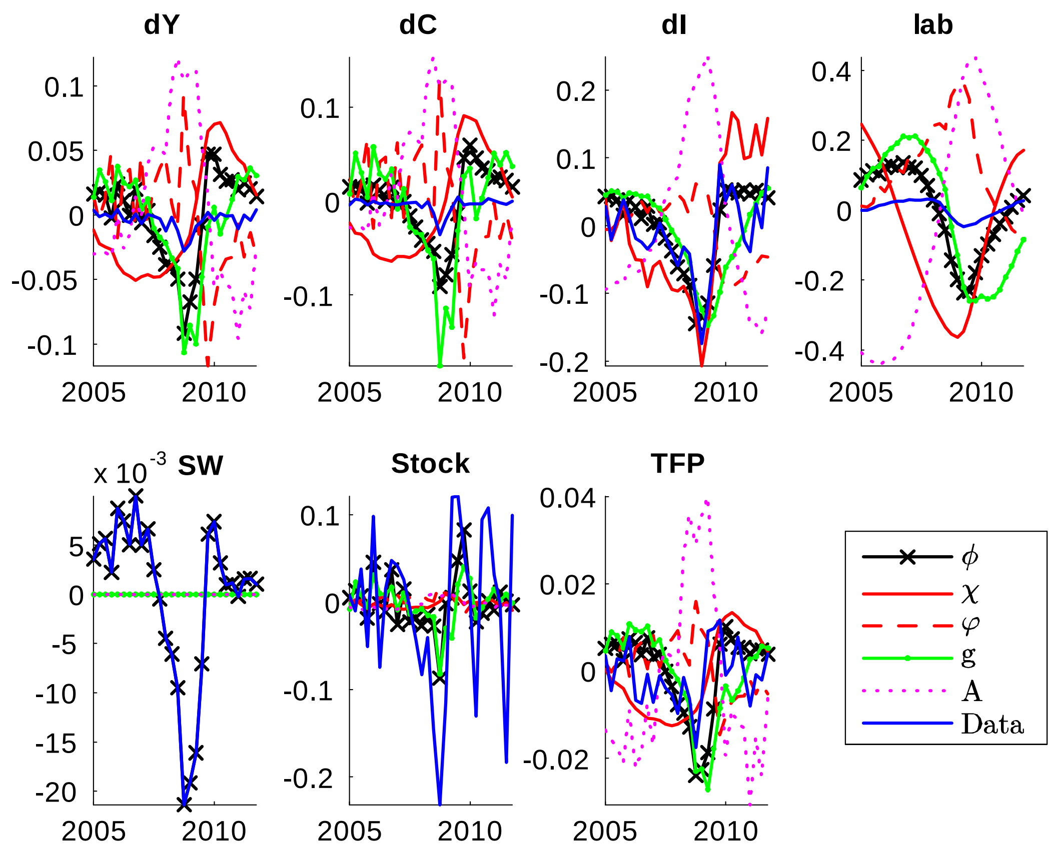

Figure 6: Shock Contribution: Benchmark Model

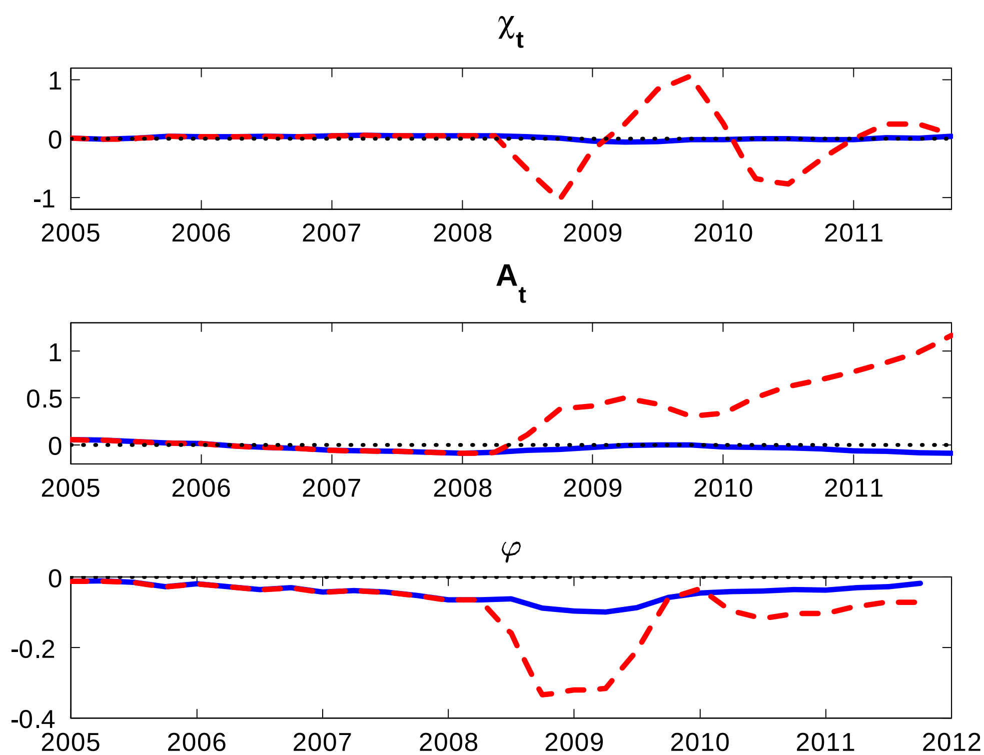

Figure 7: Back to Trend: M.E.I., Technology, and Preference Shock

subject to

$$
\begin{aligned}
\sigma_e x_t^e + \sigma_w x_t^w &= \sigma_e c_t^e + \sigma_w c_t^w, \\
x_t^e + i_t + p_{k,t} k_{t+1}^e &= R_t k_t + p_{k,t} (1-\delta) k_t + p_{k,t} i_t, \\
x_t^w + p_{k,t} k_{t+1}^w &= R_t k_t + p_{k,t} (1-\delta) k_t + W_t l_t, \\
k_{t+1}^e &\geq (1-\theta) i_t + (1-\phi_t)(1-\delta) k_t, \qquad (33) \\
k_{t+1} &= \sigma_e k_{t+1}^e + \sigma_w k_{t+1}^w,
\end{aligned}
$$

and

$$x_t^e \geq 0. \qquad (34)$$

Γ_t and Θ_t denote vectors of endogenous and exogenous state variables, respectively. To simplify our analysis, we allow resource sharing within a household at the consumption stage, with x_t^e and x_t^w denoting final consumption goods brought to the house by an entrepreneur and a worker, respectively. Other than that, interpretations of the variables, equations, and inequalities are similar to the benchmark model. We will restrict our attention to the case in which $1 < p_{k,t} < 1/\theta$ always hold. Under these conditions, both the entrepreneur's liquidity constraint (33) and her non-negativity constraint for intra-temporal resource transfer (34) must be binding at the optimum.

The price of capital is determined by the following equation:

$$p_{k,t} = \mathbb{E}_t \left[\beta \left(\frac{c_{t+1}^e}{c_t^e} \right)^{-\frac{1}{\psi}} \left(R_{t+1} + p_{k,t+1} (1-\delta) + \sigma_e \lambda_{t+1} \left[R_{t+1} + p_{k,t+1} \phi_{t+1} (1-\delta) \right] \right) \right] \qquad (35)$$

where λ_t is liquidity services defined as

$$\lambda_t = \frac{p_{k,t} - 1}{1 - \theta p_{k,t}}.$$

There is a representative firm using labor L_t and capital K_t to produce the final (consumption) goods according to the production technology

$$Y_t = \zeta (A_t L_t)^{1-\alpha} (K_t)^{\alpha}. \qquad (36)$$

α is the capital share, ζ is a scale parameter, and A_t is the index of knowledge available to the economy which both the households and the firm take as given. The firm maximizes profits defined as $Y_t - W_t L_t - R_t K_t$. The law of motion of the aggregate capital stock is given by $K_{t+1} = (1-\delta) K_t + \sigma_e i_t$.

The competitive equilibrium is defined in a standard way. We make two important assumptions about productivity growth, following Arrow (1962), Sheshinski (1967), and Romer (1986).[4] First,

[4] We closely follow chapter 4 of Barro and Sala-i-Martin (1999).

we assume that knowledge is a by-product of investment. Second, we assume that the knowledge is a public good that anyone can access at zero cost. Combining these assumptions, we replace A_t by K_t in equation (36) and write the production function as

$$Y_t = \zeta K_t \left(L_t\right)^{1-\alpha}.$$

Notice that the long-run tendency for capital to experience diminishing returns is eliminated. As such, the economy can grow indefinitely with an endogenous mechanism.

We measure the aggregate stock market value by the price times the stock of capital, i.e., $p_{k,t}K_{t+1}$. Its response to a positive liquidity shock is plotted in the top panel of figure 8 in a solid blue line.[5] Clearly a positive liquidity shock causes a stock market boom. In the middle panel, we show that the same shock causes a stock market bust in an otherwise identical exogenous growth model.[6] In the bottom panel, we show that the same shock causes a stock market bust in a version of the endogenous growth model in which we set the intertemporal elasticity of substitution to one (i.e., log utility).[7] These results underscore the importance of the endogenous growth mechanism and the intertemporal elasticity of substitution to obtain correlation between liquidity shocks and the stock market value.

To clarify the mechanism, we derive the following equation from (35),

$$p_{k,t}K_{t+1} = \mathbb{E}_t \left[\sum_{j=1}^{\infty} \beta^j \left(\frac{c^e_{t+j}}{c^e_t} \right)^{-\frac{1}{\psi}} \left(\alpha Y_{t+j} - \sigma_e i_{t+j} \right) \right]. \tag{37}$$

Not surprisingly, the aggregate stock market value is nothing but the present discounted value of future cash flows. Let us take a first-order Taylor approximation of equation (37). In the exogenous growth model, it is

$$\begin{aligned} \underbrace{\log \left(\frac{p_{k,t}K_{t+1}}{p_k \left(\gamma^t K_1\right)} \right)}_{\text{IRF}} \quad = \quad & \left(1 - \beta\gamma^{1-\frac{1}{\psi}}\right) \sum_{j=1}^{\infty} \left(\beta\gamma^{1-\frac{1}{\psi}}\right)^{j-1} \\ & \underbrace{\mathbb{E}_t \left[-\frac{1}{\psi} \log \left(\frac{\hat{c}^e_{t+j}}{\hat{c}^e_t} \right) + \frac{\alpha \hat{Y}}{\alpha \hat{Y} - \sigma_e \hat{\imath}} \log \left(\frac{\hat{Y}_{t+j}}{\hat{Y}} \right) - \frac{\sigma_e \hat{\imath}}{\alpha \hat{Y} - \sigma_e \hat{\imath}} \log \left(\frac{\hat{\imath}_{t+j}}{\hat{\imath}} \right) \right]}_{\text{cycle}}. \end{aligned} \tag{38}$$

[5] We set $\beta = 0.99$, $\alpha = 0.36$, $\eta = 0.8$, $\sigma_e = 0.06$, $\delta = 0.03$, $\phi = \theta = 0.2$, and the persistence of liquidity shock at 0.9. These are standard values that have been used in the literature (Shi (2015)). Our benchmark calibration of the intertemporal elasticity of substitution is $\psi = 1.85$, a standard value in the finance literature (Kung and Schmid (2015)). The scale parameter ζ is set at $\zeta = 0.32$ and the elasticity of leisure ϱ is set at $\varrho = 1.80$; they are calibrated to match both the growth rate of the economy and hours worked per person in the non-stochastic steady state to the empirical counterparts.

[6] We assume $A_t = \gamma^t$ in the exogenous growth model and calibrate γ to match the growth rate of the economy.

[7] Other parameters are either set at the same values or re-calibrated to hit the same empirical targets.

Variables with a hat indicate that they are divided by the deterministic trend; for example, $\hat{Y}_t$ is defined as $\hat{Y}_t = Y_t/\gamma^t$ where γ is the (gross) growth rate of the economy. Variables with no subscript denote non-stochastic steady state values of the corresponding variables. The left-hand side is the impulse response function of the aggregate stock market value. To see this point, suppose that all the stationary variables in the economy are at their corresponding non-stochastic steady state values in period 0. The denominator is period t stock market value when there is no disturbance hitting the economy from period 1 to period t. Now assume that the economy is actually disturbed by a favorable liquidity shock in period 1. The left-hand side is the log-deviation of the actual aggregate stock market value in period t from its counterfactual no-shock benchmark value, which is by definition the impulse response function. The right-hand side is the weighted average of stochastic discount factors and future cash flows. Note that it contains only cyclical terms; that is, both the stochastic discount factor $-\frac{1}{\psi}\log\left(\frac{\hat{c}^e_{t+j}}{\hat{c}^e_t}\right)$ and the future cash flows $\frac{\alpha\hat{Y}}{\alpha\hat{Y}-\sigma_e\hat{\imath}}\log\left(\frac{\hat{Y}_{t+j}}{\hat{Y}}\right) - \frac{\sigma_e\hat{\imath}}{\alpha\hat{Y}-\sigma_e\hat{\imath}}\log\left(\frac{\hat{\imath}_{t+j}}{\hat{\imath}}\right)$ are stationary, always returning to their steady states once the disturbances subside.

If we take a first-order Taylor approximation of the same equation in the endogenous growth model, we find

$$
\begin{aligned}
\underbrace{\log\left(\frac{p_{k,t}K_{t+1}}{p_k\left(\gamma^t K_1\right)}\right)}_{\text{IRF}} &= \underbrace{\log\left(\frac{K_t/K_1}{\gamma^{t-1}}\right)}_{\text{realized growth}} \\
&\quad \underbrace{+\left(1-\beta\gamma^{1-\frac{1}{\psi}}\right)\sum_{j=1}^{\infty}\left(\beta\gamma^{1-\frac{1}{\psi}}\right)^{j-1}\mathbb{E}_t\left[\left(1-\frac{1}{\psi}\right)\log\left(\frac{K_{t+j}/K_t}{\gamma^j}\right)\right]}_{\text{future growth}} \\
&\quad +\left(1-\beta\gamma^{1-\frac{1}{\psi}}\right)\sum_{j=1}^{\infty}\left(\beta\gamma^{1-\frac{1}{\psi}}\right)^{j-1} \\
&\quad \underbrace{\mathbb{E}_t\left[-\frac{1}{\psi}\log\left(\frac{\hat{c}^e_{t+j}}{\hat{c}^e_t}\right)+\frac{\alpha\hat{Y}}{\alpha\hat{Y}-\sigma_e\hat{\imath}}\log\left(\frac{\hat{Y}_{t+j}}{\hat{Y}}\right)-\frac{\sigma_e\hat{\imath}}{\alpha\hat{Y}-\sigma_e\hat{\imath}}\log\left(\frac{\hat{\imath}_{t+j}}{\hat{\imath}}\right)\right]}_{\text{cycle}}.
\end{aligned} \tag{39}
$$

Variables with a hat indicate that they are divided by the endogenous trend; for example, $\hat{Y}_t$ is defined as $\hat{Y}_t = Y_t/K_t$. Note both a similarity and a difference between (38) and (39). The similarity is that the cyclical component, the third term of equation (39), looks exactly the same as the right-hand side of equation (38), although variables are detrended by model-specific trends. The difference is that there are two additional terms in equation (39). One of them (the first term) measures the impact of actual growth in the endogenous trend relative to the non-stochastic steady state growth on the stock market. The other one (the second term) is the contribution from the weighted average of future growth in the endogenous trend relative to the non-stochastic steady state growth.

The decomposition is plotted in Figure 8, with a factor's contribution represented by the height of a bar. Clearly the most important factor raising the stock market value in the endogenous growth model is future growth. In contrast, the stock market value does not rise in the short run in the exogenous growth model because the impulse response function is solely driven by negative cyclical terms. In the bottom panel, we see that the contribution from the future growth term disappears when the intertemporal elasticity of substitution is one, and so does the intial stock market boom. This is the insight clarified by Bansal and Yaron (2004).

6.4 Government bonds

In our benchmark model, government consumption shocks are estimated to be as important as liquidity innovations during the crisis. This finding holds under our assumption of a balanced budget, but deficits do occur in practice. As a robustness check, this section introduces government bonds to finance fiscal expenditures. We modify the government's period budget constraint to

$$Gov_t + \tau_{tr,t} + B_t = \tau_p \left(\Pi_t N_t + u_t R_t K_t\right) + \tau_l W_t \sigma_w l_t + p_{b,t} B_{t+1}.$$

Here, B_{t+1} is the amount of bonds issued in period t whose duration is always one period. We assume that they are perfectly liquid following the literature (Del Negro, Eggertsson, Ferrero, and Kiyotaki (2017) and Shi (2015)). The bond price $p_{b,t}$ satisfies the Euler equation

$$p_{b,t} = \mathbb{E}_t \left[\beta \left(\frac{\mu^w_{t+1}}{\mu^w_t}\right)(1 + \sigma_e \lambda_{t+1})\right].$$

The term $\sigma_e \lambda_{t+1}$ illustrates the liquidity services provided by government bonds. Following Del Negro, Eggertsson, Ferrero, and Kiyotaki (2017), we assume that the government follows a fiscal rule given by

$$\tau_{tr,t} - \hat{\tau}_{tr} N_t = -\psi_\tau \left(B_t - \hat{B} N_t\right)$$

where $\psi_\tau > 0$, $\hat{\tau}_{tr}$, and $\hat{B}$ are constants. Note that the amount of transfer payments $\tau_{tr,t}$ is predetermined in this model, because both B_t and N_t are. The positive coefficient ψ_τ implies that government deficits require higher lump-sum taxes. This is important to ensure the dynamic stability of our model. The bond market clearing condition is

$$B_{t+1} = \sigma_e b^e_{t+1} + \sigma_w b^w_{t+1}$$

where b^e_{t+1} and b^w_{t+1} are the amounts of the government bonds purchased by entrepreneurs and workers in period t, respectively. The rest of the model is essentially the same as in the benchmark model.

The following parameters are fixed in the benchmark model: the discount factor β, the in-

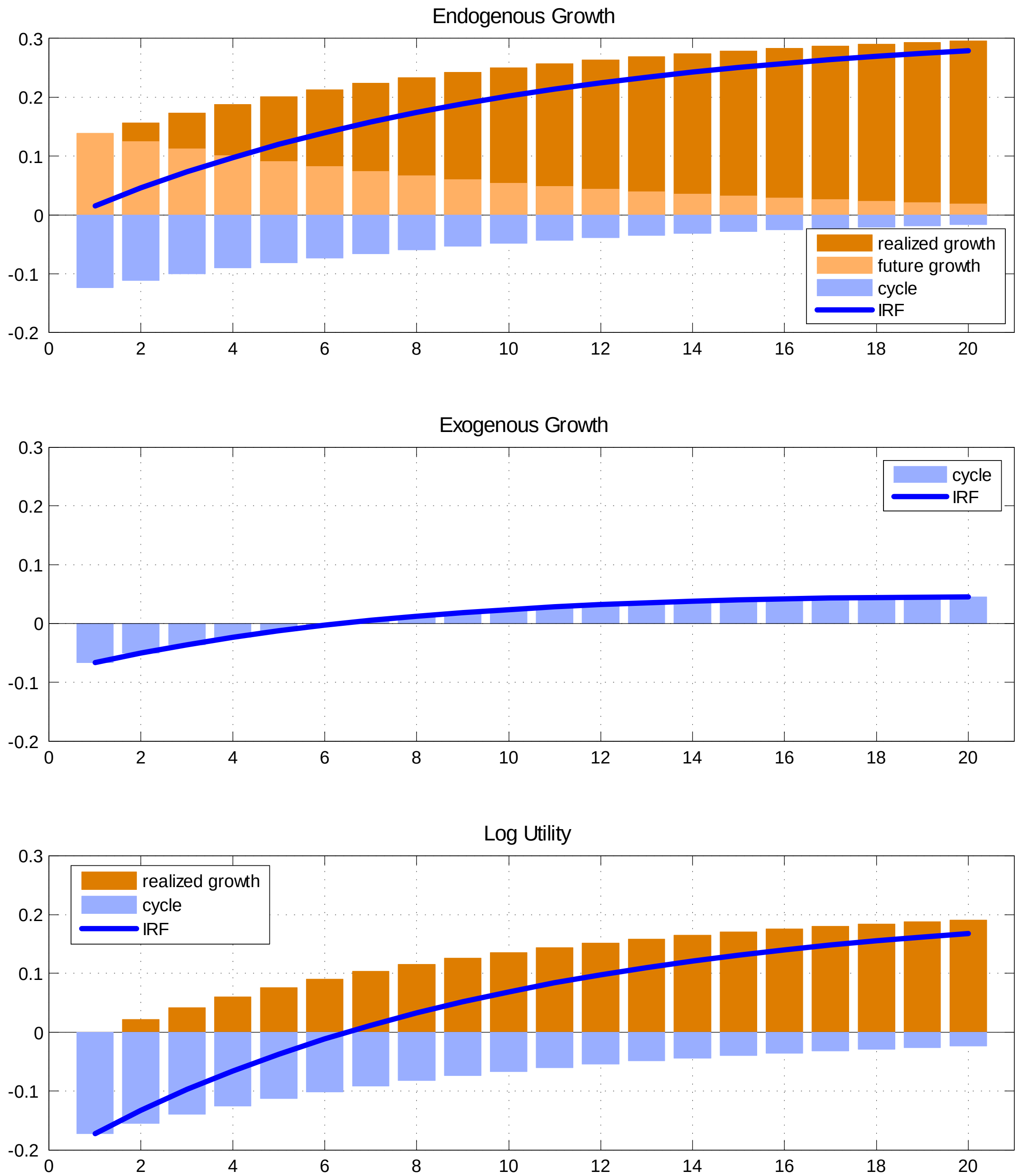

Figure 8: Stock Market Response and Factor Decomposition after Liquidity Shock; Learning-by-Doing, Exogenous Growth, and Log Utility.

tertemporal elasticity of substitution ψ, the research elasticity η, the exit rate δ_n, the capital depreciation rate in the steady state δ_k, the resalability of new equities θ, taxes on capital income τ_p and labor income τ_l, and the steady state utilization u. We use the same values in the extended model. The following parameters are estimated in the benchmark model: the elasticity of the capital depreciation rate in the steady state $\delta''(u)/\delta'(u)$, the liquidity shock in the steady state ϕ, the curvature of the investment adjustment costs $\bar{\Lambda}$, and the persistence and the volatility of structural shocks. We use those estimated values in the extended model. The rest of the parameters are calibrated in the benchmark model. We follow the same route in the extended model. Empirical targets are the growth rate of the economy $\gamma = 1.065$, the goverment consumption to GDP $Gov/\mathcal{Y}$ =0.2, the labor income share in GDP $\hat{W}(\sigma_w l)/\widehat{\mathcal{Y}} = 0.57$, the tangible investment to GDP $\sigma_e \hat{s}/\widehat{\mathcal{Y}} = 0.06$, the intangible investment to GDP $\sigma_e \hat{s}/\widehat{\mathcal{Y}}$, the fraction of liquid assets in the portfolio $p_b \gamma \hat{B}/\left(\widehat{Stock} + p_b \gamma \hat{B}\right) = 12\%$ in the steady state (Del Negro, Eggertsson, Ferrero, and Kiyotaki (2016) and Shi (2015)), and labor supply $l = 0.33$ in the steady state. The steady state detrended-gross output $\hat{Y}$ is set at $\hat{Y} = 1$ (normalization). We also assume that the home production function has the same elasticity of labor as the aggregate production function in the market sector, i.e., $\omega = 1 - \alpha$. The seven empirical targets together with one normalization and one parametric assumption pin down seven parameters: the steady state government consupmtion shock g, the scale parameter in the product development function ζ, the steady state preference shock φ, the curvature in the production function α, the paramter affecting the elasticity of substitution between intermediate goods ν, the fraction of entrepreneurs in the population σ_e, and the steady state govenment bonds $\hat{B}$. Finally, we set ψ_τ at $\psi_\tau = 0.66$ and check the robustness.

Figure 9 shows impulse response functions to a positive liquidity shock. Relative to the benchmark model, the shock has weaker effects on the economy in the model with government bonds. To see why, remember that tax revenues increase when the economy is booming. In the benchmark model, a part of the proceeds is rebated to entrepreneurs as transfers, who use the extra resources to innovate. In the extended model, however, this mechanism is absent because transfers are predetermined. Higher tax collection implies a decrease in bond issuances. Because government bonds provide liquidity, a reduction in bond issuances partially offsets the expansionary effect of a liquidity shock. Qualitatively speaking, the results are robust to the extension. A positive liquidity shock still causes simultaneous expansions in output, consumption, investment, R&D, and hours worked as well as stock market value in the extended model with government bonds.

Figure 10 shows impulse response functions to a government consumption shock. The observed responses are qualitatively different from those in the benchmark model. A sudden increase in government consumption is expansionary in the model with government bonds, except for private consumption, while it is contractionary in the model without. In the extended model, a sudden increase in government consumption is financed by debt, and newly issued bonds provide liquidity to the economy, favoring entrepreneurial activities and the economy.

Figure 11 shows the shocks' contribution to the financial crisis in the model with public debt.

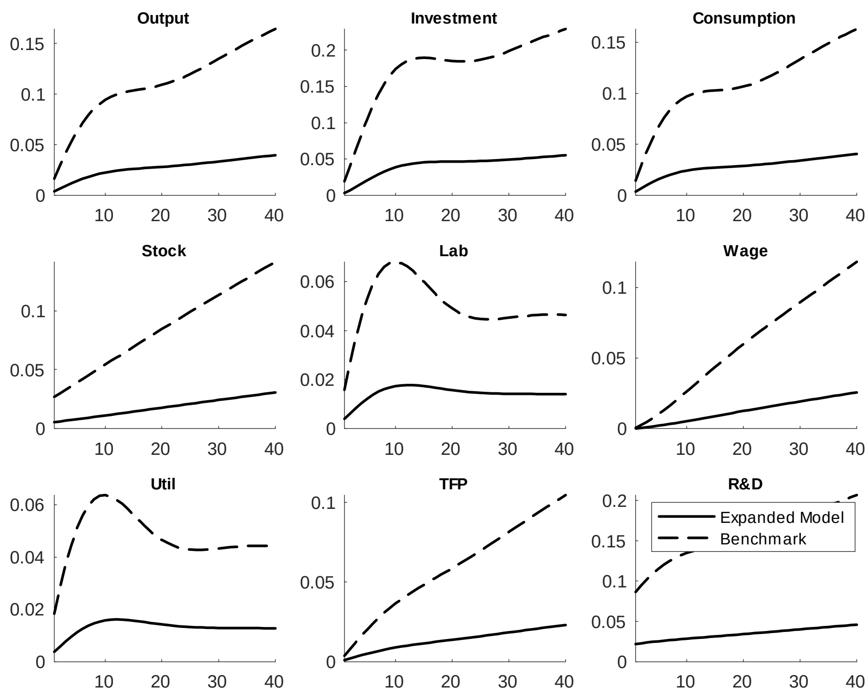

Figure 9: Responses to a Liquidity Shock; the Model with Government Bonds (Solid) and Benchmark (Dashed)

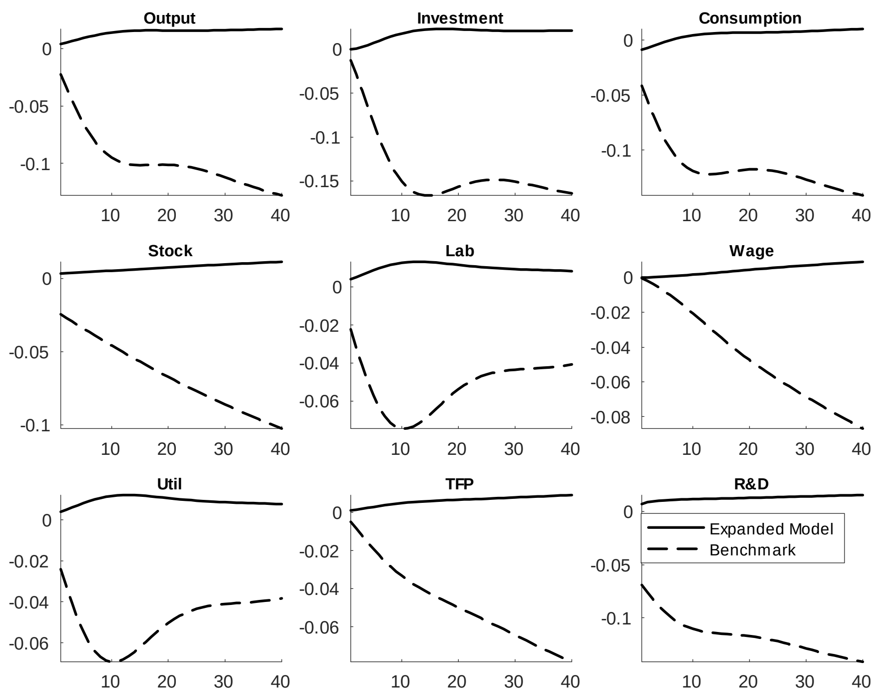

Figure 10: Responses to a Government Consumption Shock; the Model with Government Bonds (Solid) and Benchmark (Dashed)

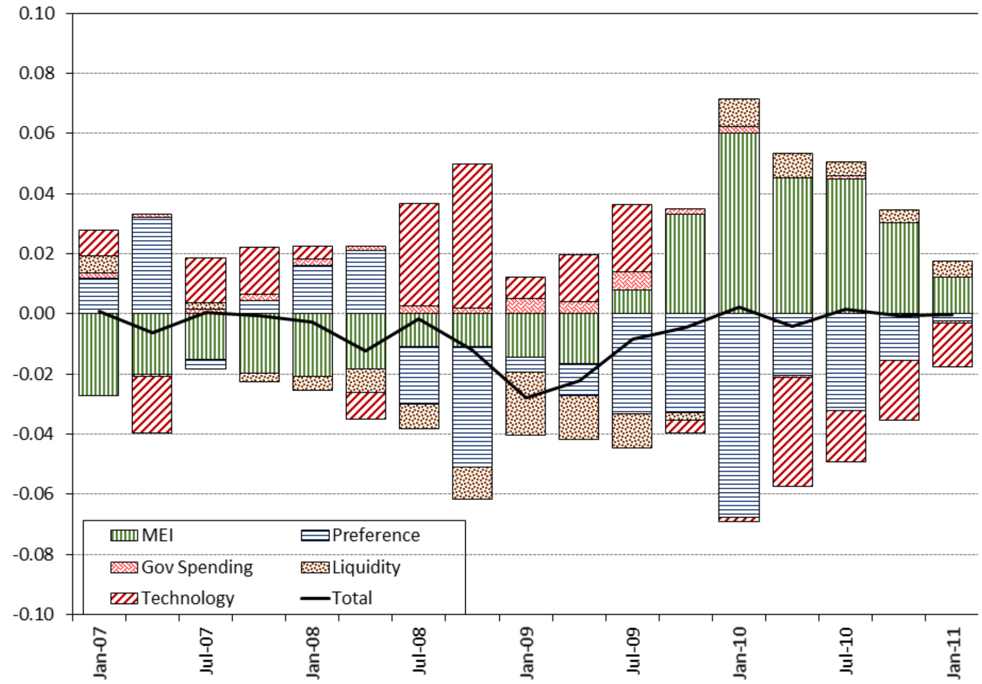

Figure 11: Shock Decomposition in the Model with Government Bonds

Not surprisingly, the government consumption shock does not play an important role in the Great Recession anymore. But the liquidity shock remains the most important headwind at the trough of the cycle. As with the benchmark model, liquidity is a minor contributor to the recovery, which explains in our framework the lackluster post-crisis years. Furthermore, this new scenario reveals that another type of financial innovation, those captured by preference shocks as in Smets and Wouters (2007), is also a contributor during the recession. In summary, our main results regarding the role of financial frictions and financial shocks are robust to the introduction of government bonds.

7 Exogenous growth model

This section presents an exogenous growth version of the benchmark model and shows results from it.

7.1 Model

A member of the household will be an investor with probability $\sigma_i \in [0,1]$ and a worker with probability $\sigma_w \in [0,1]$. They satisfy $\sigma_i + \sigma_w = 1$. Only investors can prepare investment goods.

The head of the household chooses the instructions to its members to maximize the value function defined as

$$v\left(q_t;\Gamma_t,\Theta_t\right)=\max\left\{\sigma_i\frac{\left(c_t^i\right)^{1-\frac{1}{\psi}}}{1-\frac{1}{\psi}}+\sigma_w\frac{\left[c_t^w+\frac{\varphi_t}{\omega}\left(\Psi_t\right)\left(1-l_t\right)^{\omega}\right]^{1-\frac{1}{\psi}}}{1-\frac{1}{\psi}}+\beta\mathbb{E}_t\left[v\left(q_{t+1};\Gamma_{t+1},\Theta_{t+1}\right)\right]\right\}$$

subject to

$$k_{t+1}=\sigma_i k_{t+1}^i+\sigma_w k_{t+1}^w+\left(1-\Lambda\left(\frac{i_t}{i_{t-1}}\right)\right)\sigma_i i_t,$$

$$c_t^i+\frac{i_t}{\chi_t}+p_{k,t}k_{t+1}^i=\left(1-\tau_p\right)\left(u_tR_tk_t\right)+p_{k,t}\left(1-\delta\left(u_t\right)\right)k_t+\tau_{tr,t},$$

$$c_t^w+p_{k,t}k_{t+1}^w=\left(1-\tau_p\right)u_tR_tk_t+p_{k,t}\left(1-\delta\left(u_t\right)\right)k_t+\left(1-\tau_l\right)W_tl_t+\tau_{tr,t},$$

and

$$k_{t+1}^i\geq\left(1-\phi_t\right)\left(1-\delta\left(u_t\right)\right)k_t. \tag{40}$$

Let μ_t^i and μ_t^w denote marginal utility of consumption for an investor and a worker, respectively,

$$\left(c_t^i\right)^{-\frac{1}{\psi}}=\mu_t^i$$

and

$$\left[c_t^w+\frac{\varphi_t}{\omega}\left(\Psi_t\right)\left(1-l_t\right)^{\omega}\right]^{-\frac{1}{\psi}}=\mu_t^w.$$

We will restrict our attention to the case in which $\mu_t^i>\mu_t^w$ always holds in the equilibrium. The liquidity constraint (40) must be binding at the optimum because otherwise, the household can increase the utility at the margin. Namely, the household can decrease an investor's capital holding k_{t+1}^i by $\Delta>0$, increase an investor's consumption c_t^i by $p_{k,t}\Delta$, increase a worker's capital holding k_{t+1}^w by $\left(\sigma_i/\sigma_w\right)\Delta$, and decrease a worker's consumption c_t^w by $p_{k,t}\left(\sigma_i/\sigma_w\right)\Delta$. These changes do not violate any constraints as long as Δ is sufficiently small, and in addition, are neutral to the household's asset position, but increase the utility approximately by $p_{k,t}\Delta\sigma_i\mu_t^i-p_{k,t}\left(\sigma_i/\sigma_w\right)\Delta\sigma_w\mu_t^w=\sigma_ip_{k,t}\Delta\left(\mu_t^i-\mu_t^w\right)>0$, which is a contradiction to the assumption that the initial allocation was optimal. Derivations of the first order optimality conditions are omitted.

A representative firm uses capital service KS_t and labor L_t to produce the final (consumption) goods according to the production technology

$$Y_t=\left(KS_t\right)^{\alpha}\left(\bar{A}A_tL_t\right)^{1-\alpha}.$$

$\bar{A}$ is a scale parameter, and A_t is the productivity shock following a non-stationary process:

$$\log\left(\frac{A_t}{A_{t-1}}\right)=\left(1-\rho_{\Delta a}\right)\log\left(\gamma\right)+\rho_{\Delta a}\log\left(\frac{A_{t-1}}{A_{t-2}}\right)+\varepsilon_{\Delta a,t}.$$

The firm maximizes profits defined as $Y_t - R_t(KS_t) - W_t L_t$.

The government consumption Gov_t is given by

$$\frac{Gov_t}{A_t} = g_t,$$

where g_t is the government consumption shock. The government keeps the balanced-budget:

$$Gov_t + \tau_{tr,t} = \tau_p u_t R_t K_t + \tau_l W_t \sigma_w l_t.$$

The competitive equilibrium is defined in a standard way. We define the aggregate stock market value, $Stock_t$, as the value of the tradable assets in the economy, which in the equilibrium is

$$Stock_t = p_{k,t}\left(1 - \delta\left(u_t\right)\right) K_t.$$

7.2 Model summary

The following equations summarize the model economy:

$$Y_t = \left(u_t K_t\right)^{\alpha} \left(\bar{A} A_t \sigma_w l_t\right)^{1-\alpha},$$

$$\varphi_t\left(\Psi_t\right)\left(1 - l_t\right)^{\omega-1} = \left(1 - \tau_l\right) W_t,$$

$$W_t = \left(1 - \alpha\right) \frac{Y_t}{\sigma_w l_t},$$

$$\left(c_t^i\right)^{-\frac{1}{\psi}} = \left(1 + \lambda_t\right)\left[c_t^w + \frac{\varphi_t}{\omega}\left(\Psi_t\right)\left(1 - l_t\right)^{\omega}\right]^{-\frac{1}{\psi}},$$

$$R_t = \alpha \frac{Y_t}{u_t K_t},$$

$$\begin{aligned} \frac{1}{\chi_t}\left(1 + \lambda_t\right) &= p_{k,t}\left(1 - \Lambda\left(\frac{i_t}{i_{t-1}}\right) - \Lambda'\left(\frac{i_t}{i_{t-1}}\right)\frac{i_t}{i_{t-1}}\right) \\ &\quad + \mathbb{E}_t\left[\beta\left(\frac{c_{t+1}^w + \frac{\varphi_{t+1}}{\omega}\left(\Psi_{t+1}\right)\left(1 - l_{t+1}\right)^{\omega}}{c_t^w + \frac{\varphi_t}{\omega}\left(\Psi_t\right)\left(1 - l_t\right)^{\omega}}\right)^{-\frac{1}{\psi}} p_{k,t+1}\Lambda'\left(\frac{i_{t+1}}{i_t}\right)\left(\frac{i_{t+1}}{i_t}\right)^2\right], \end{aligned}$$

$$p_{k,t} = \mathbb{E}_t\left[\begin{array}{c} \beta\left(\frac{c_{t+1}^w + \frac{\varphi_{t+1}}{\omega}\left(\Psi_{t+1}\right)\left(1 - l_{t+1}\right)^{\omega}}{c_t^w + \frac{\varphi_t}{\omega}\left(\Psi_t\right)\left(1 - l_t\right)^{\omega}}\right)^{-\frac{1}{\psi}} \\ \left(\begin{array}{c} \left(1 - \tau_p\right) u_{t+1} R_{t+1} + p_{k,t+1}\left(1 - \delta\left(u_{t+1}\right)\right) \\ +\sigma_i \lambda_{t+1}\left[\left(1 - \tau_p\right) u_{t+1} R_{t+1} + \phi_{t+1} p_{k,t+1}\left(1 - \delta\left(u_{t+1}\right)\right)\right] \end{array}\right) \end{array}\right],$$

$$\left(1 - \tau_p\right) R_t - p_{k,t}\delta'\left(u_t\right) + \sigma_i \lambda_t\left[\left(1 - \tau_p\right) R_t - \phi_t p_{k,t}\delta'\left(u_t\right)\right] = 0,$$

$$c_t^i + \frac{i_t}{\chi_t} = u_t R_t K_t + \phi_t p_{k,t}\left(1 - \delta\left(u_t\right)\right) K_t + \tau_l W_t \sigma_w l_t - Gov_t,$$

$$\frac{Gov_t}{A_t} = g_t,$$

$$K_{t+1} = (1 - \delta(u_t)) K_t + \left(1 - \Lambda\left(\frac{i_t}{i_{t-1}}\right)\right)(\sigma_i i_t),$$

$$Y_t = \sigma_i c_t^i + \sigma_w c_t^w + \sigma_i \frac{i_t}{\chi_t} + Gov_t,$$

$$\Psi_t = p_{k,t} K_t,$$

$$Stock_t = p_{k,t}(1 - \delta(u_t)) K_t,$$

$$\log\left(\frac{TFP_t}{TFP_{t-1}}\right) = \alpha \log\left(\frac{u_t}{u_{t-1}}\right) + (1-\alpha)\log\left(\frac{A_t}{A_{t-1}}\right),$$

$$\log\left(\frac{A_t}{A_{t-1}}\right) = (1 - \rho_{\Delta a})\log(\gamma) + \rho_{\Delta a}\log\left(\frac{A_{t-1}}{A_{t-2}}\right) + \varepsilon_{\Delta a,t}.$$

Detrended system is summarized as follows.

$$\hat{Y}_t = \left(u_t \frac{\hat{K}_t}{\gamma_t}\right)^{\alpha} \left(\bar{A}\sigma_w l_t\right)^{1-\alpha},$$

$$\varphi_t\left(\hat{\Psi}_t\right)(1 - l_t)^{\omega - 1} = (1 - \tau_l)\hat{W}_t,$$

$$\hat{W}_t = (1-\alpha)\frac{\hat{Y}_t}{\sigma_w l_t},$$

$$\left(\hat{c}_t^i\right)^{-\frac{1}{\psi}} = (1 + \lambda_t)\left[\hat{c}_t^w + \frac{\varphi_t}{\omega}\left(\hat{\Psi}_t\right)(1 - l_t)^{\omega}\right]^{-\frac{1}{\psi}},$$

$$R_t = \alpha \frac{\hat{Y}_t}{u_t \frac{\hat{K}_t}{\gamma_t}},$$

$$\begin{aligned}
\frac{1}{\chi_t}(1 + \lambda_t) \quad = \quad & p_{k,t}\left(1 - \Lambda\left(\gamma_t \frac{\hat{\imath}_t}{\hat{\imath}_{t-1}}\right) - \Lambda'\left(\gamma_t \frac{\hat{\imath}_t}{\hat{\imath}_{t-1}}\right)\gamma_t \frac{\hat{\imath}_t}{\hat{\imath}_{t-1}}\right) \\
& + \mathbb{E}_t\left[\beta\left(\gamma_{t+1}\frac{\hat{c}_{t+1}^w + \frac{\varphi_{t+1}}{\omega}\left(\hat{\Psi}_{t+1}\right)(1 - l_{t+1})^{\omega}}{\hat{c}_t^w + \frac{\varphi_t}{\omega}\left(\hat{\Psi}_t\right)(1 - l_t)^{\omega}}\right)^{-\frac{1}{\psi}} p_{k,t+1}\Lambda'\left(\gamma_{t+1}\frac{\hat{\imath}_{t+1}}{\hat{\imath}_t}\right)\left(\gamma_{t+1}\frac{\hat{\imath}_{t+1}}{\hat{\imath}_t}\right)^2\right],
\end{aligned}$$

$$p_{k,t} = \mathbb{E}_t\left[\begin{array}{c} \beta\left(\gamma_{t+1}\frac{\hat{c}_{t+1}^w + \frac{\varphi_{t+1}}{\omega}\left(\hat{\Psi}_{t+1}\right)(1 - l_{t+1})^{\omega}}{\hat{c}_t^w + \frac{\varphi_t}{\omega}\left(\hat{\Psi}_t\right)(1 - l_t)^{\omega}}\right)^{-\frac{1}{\psi}} \\ \left(\begin{array}{c} (1 - \tau_p)u_{t+1}R_{t+1} + p_{k,t+1}(1 - \delta(u_{t+1})) \\ +\sigma_i\lambda_{t+1}\left[(1 - \tau_p)u_{t+1}R_{t+1} + \phi_{t+1}p_{k,t+1}(1 - \delta(u_{t+1}))\right] \end{array}\right) \end{array}\right],$$

$$(1 - \tau_p)R_t - p_{k,t}\delta'(u_t) + \sigma_i\lambda_t\left[(1 - \tau_p)R_t - \phi_t p_{k,t}\delta'(u_t)\right] = 0,$$

$$\hat{c}_t^i + \frac{\hat{\imath}_t}{\chi_t} = u_t R_t \frac{\hat{K}_t}{\gamma_t} + \phi_t p_{k,t} \left(1 - \delta\left(u_t\right)\right) \frac{\hat{K}_t}{\gamma_t} + \tau_l \hat{W}_t \sigma_w l_t - g_t,$$

$$\hat{K}_{t+1} = \left(1 - \delta\left(u_t\right)\right) \frac{\hat{K}_t}{\gamma_t} + \left(1 - \Lambda\left(\gamma_t \frac{\hat{\imath}_t}{\hat{\imath}_{t-1}}\right)\right)\left(\sigma_i \hat{\imath}_t\right),$$

$$\hat{Y}_t = \sigma_i \hat{c}_t^i + \sigma_w \hat{c}_t^w + \sigma_i \frac{\hat{\imath}_t}{\chi_t} + g_t,$$

$$\hat{\Psi}_t = p_{k,t} \frac{\hat{K}_t}{\gamma_t},$$

$$\widehat{Stock}_t = p_{k,t} \left(1 - \delta\left(u_t\right)\right) \frac{\hat{K}_t}{\gamma_t},$$

$$\log\left(\frac{TFP_t}{TFP_{t-1}}\right) = \alpha \log\left(\frac{u_t}{u_{t-1}}\right) + (1 - \alpha) \log\left(\gamma_t\right),$$

$$\log\left(\gamma_t\right) = \left(1 - \rho_{\Delta a}\right) \log\left(\gamma\right) + \rho_{\Delta a} \log\left(\gamma_{t-1}\right) + \varepsilon_{\Delta a,t}.$$

Hat variables denote the original variables divided by A_t, i.e., $\hat{Y}_t = Y_t / A_t$, and so on, except for $\hat{K}_t$ which is defined as $\hat{K}_t = K_t / A_{t-1}$. γ_t is defined as $\gamma_t = A_t / A_{t-1}$.

7.3 Calibration strategy

We calibrate $\bar{A}$ and φ so that the steady state detrended gross output $\hat{Y}$ is set at $\hat{Y} = 1$, and the steady state labor supply l is set at $l = 1/3$. We also assume that $\omega = 1 - \alpha$, implying that the home production function has the same elasticity of labor as the aggregate production function in the market sector. The aggregate wage income is

$$\hat{W}\left(\sigma_w l\right) = 1 - \alpha.$$

The labor share is therefore

$$\frac{W \sigma_w l}{Y} = \frac{\hat{W} \sigma_w l}{\hat{Y}} = 1 - \alpha.$$

Using this relation, we calibrate α to match the steady state labor share.

Pricing equation for capital in the steady state is

$$p_k = \beta\left(\gamma\right)^{-\frac{1}{\psi}} \left(uR\left(1 - \tau_p\right) + p_k\left(1 - \delta_k\right) + \sigma_i \lambda\left(uR\left(1 - \tau_p\right) + \phi p_k\left(1 - \delta_k\right)\right)\right), \tag{41}$$

where

$$uR = \alpha \frac{\gamma}{\hat{K}}$$

and

$$\hat{K} = \frac{\gamma \sigma_i \hat{\imath}}{\gamma - 1 + \delta_k}.$$

We assume that the steady state value of the investment specific technology shock χ is $\chi = 1$. Optimality for investment in the steady state implies

$$1 + \lambda = p_k.$$

Investor's budget constraint in the steady state is

$$\hat{c}^i + \frac{\sigma_i \hat{\imath}}{\sigma_i} = \alpha + \phi p_k (1 - \delta_k) \frac{\hat{K}}{\gamma} + \tau_l (1 - \alpha) - g, \tag{42}$$

where g is calibrated to match the steady state government consumption share. Optimality for an intratemporal resource allocation is

$$\left(\hat{c}^i\right)^{-\frac{1}{\psi}} = (1 + \lambda) \left[\hat{c}^w + \frac{1 - \tau_l}{\omega} \frac{1 - \alpha}{1 - \sigma_i} \frac{1 - l}{l}\right]^{-\frac{1}{\psi}} \tag{43}$$

where

$$\omega = 1 - \alpha$$

by assumption. The resource constraint in the steady state is

$$1 = \sigma_e \hat{c}^e + (1 - \sigma_e) \hat{c}^w + \sigma_i \hat{\imath} + g. \tag{44}$$

Note that equations (41) to (44) are a four-equation, four-unknown system once we specify ψ, β, γ, ϕ, δ_k, τ_p, τ_l, $\sigma_i \hat{\imath}$, and g. Using it, we calibrate σ_i, p_k, $\hat{c}^i$, and $\hat{c}^w$. Other steady state values are backed out in a similar way as in the benchmark model.

7.4 Estimation

The estimation is conducted with the same strategy and the same data set. Table 1 reports the estimated parameters. Both the liquidity shock and the government consumption shock are less persistent and less volatile in the exogenous growth model than in the benchmark model. In contrast, the technology shock, the preference shock, and the M.E.I. shock are more volatile in the exogenous growth model than in the benchmark model.[8]

Shock decomposition in Figure 12 shows that liquidity plays a minor role during the crisis if we assume exogenous growth. Even at the trough of the cycle, liquidity decline output by about half of a percentage point. The Great Recession is instead attributed to the M.E.I. shock, the preference shock, and the technology shock. Specifically, the drop in investment is attributed to the M.E.I. shock, the drop in hours worked is attributed to the preference shock, and the drop in

[8]Notice that because the technology shock in the exogenous growth model is a shock to the growth rate of A_t (denoted by Δa in the tables), $\rho_{\Delta a} = 0.022$ in the exogenous growth model implies that the technology shock is nearly random walk.

Table 1: Parameter Values

Exogenous Growth			Benchmark		
Parameter	Median	90% Interval	Parameter	Median	90% Interval
$\rho_{\Delta a}$	0.022	[0.001,0.043]	ρ_A	0.901	[0.872,0.937]
ρ_ϕ	0.716	[0.619,0.809]	ρ_ϕ	0.996	[0.993,0.998]
ρ_g	0.904	[0.862,0.938]	ρ_g	0.987	[0.984,0.989]
ρ_φ	0.999	[0.999,0.999]	ρ_φ	0.879	[0.845,0.907]
ρ_χ	0.932	[0.904,0.953]	ρ_χ	0.642	[0.574,0.704]
$\sigma_{\Delta a}$	0.017	[0.015,0.019]	σ_A	0.015	[0.013,0.016]
σ_ϕ	0.017	[0.015,0.019]	σ_ϕ	0.033	[0.029,0.039]
σ_g	0.030	[0.027,0.033]	σ_g	0.046	[0.041,0.051]
σ_φ	0.011	[0.010,0.013]	σ_φ	0.012	[0.011,0.014]
σ_χ	0.048	[0.044,0.053]	σ_χ	0.023	[0.019,0.027]
σ_m	0.030	[0.029,0.030]	σ_m	0.030	[0.029,0.030]
σ_t	0.005	[0.005,0.006]	σ_t	0.007	[0.006,0.007]
δ''/δ'	0.119	[0.078,0.173]	δ''/δ'	0.268	[0.214,0.330]
ϕ	0.246	[0.219,0.246]	ϕ	0.121	[0.105,0.138]
$\bar{\Lambda}$	0.482	[0.295,0.710]	$\bar{\Lambda}$	0.590	[0.502,0.694]

TFP is attributed to the technology shock. The recovery phase is driven by preference shocks and to a lesser degree by liquidity and government consumption. It is also interesting that virtually none of the stock market volatilities is explained by structural shocks.

8 No Liquidity Friction

8.1 Discussion on the Household Problem

We assume that product development can be equity financed, i.e., $\theta = 1$. Liquidity constraints do not bind in the equilibrium in this case. We omit non-negativity constraints because they do not bind in the equilibrium either. The household is effectively facing the consolidated budget constraint defined by

$$\begin{aligned} & \sigma_e c_t^e + \sigma_w c_t^w + (1 - p_{n,t}\vartheta_t)\,\sigma_e s_t + \sigma_w \frac{i_t}{\chi_t} + p_{n,t} n_{t+1} + p_{k,t}\bar{k}_{t+1} \\ = \;& (1-\tau_p)\left(\Pi_t n_t + u_t R_t k_t\right) + p_{n,t}\left(1-\delta_n\right) n_t + p_{k,t}\left(1 - \delta\left(u_t\right)\right) k_t + \left(1-\tau_l\right) W_t \sigma_w l_t + \tau_{tr,t} \end{aligned}$$

where

$$\bar{k}_{t+1} = \sigma_e k_{t+1}^e + \sigma_w k_{t+1}^w$$

because funds can be freely transferred between members through asset markets. Namely, if the head of the household would like to shift resource from workers to entrepreneurs, he or she can instruct entrepreneurs to decrease the purchase of, say, n_{t+1}^e by Δ and at the same time,

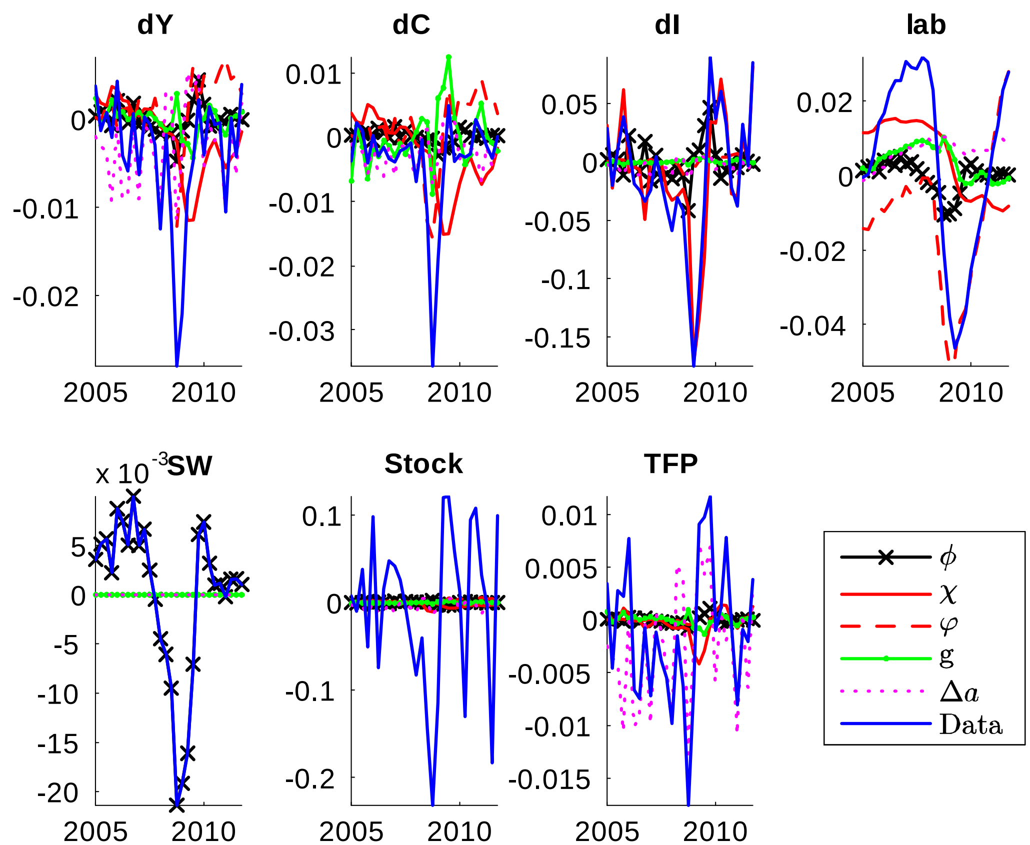

Figure 12: Shock Decomposition with Exogenous Growth: Main Shocks

instruct workers to increase the purchase of n_{t+1}^{w} by $(\sigma_e/\sigma_w)\,\Delta$. Resource is shifted from workers to entrepreneurs, and asset position at the end of the period does not change.

Note also that solving the intra-temporal resource allocation problem

$$\max \sigma_e \frac{(c_t^e)^{1-\frac{1}{\psi}}}{1-\frac{1}{\psi}} + \sigma_w \frac{\left[c_t^w + \frac{\varphi_t}{\omega}(\Psi_t)(1-l_t)^{\omega}\right]^{1-\frac{1}{\psi}}}{1-\frac{1}{\psi}}$$

subject to

$$\sigma_e c_t^e + \sigma_w c_t^w = c_t$$

and given level of l_t, we find that the optimal allocation is

$$c_t^e = c_t + \sigma_w \frac{\varphi_t}{\omega}(\Psi_t)(1-l_t)^{\omega}$$

and

$$c_t^w = c_t - \sigma_e \frac{\varphi_t}{\omega}(\Psi_t)(1-l_t)^{\omega}.$$

Both an entrepreneur and a worker have the same utility level under this consumption allocation, namely,

$$\frac{\left[c_t + \sigma_w \frac{\varphi_t}{\omega}(\Psi_t)(1-l_t)^{\omega}\right]^{1-\frac{1}{\psi}}}{1-\frac{1}{\psi}}.$$

The above discussions imply that we can rewrite the household's problem as follows;

$$v(q_t;\Gamma_t,\Theta_t) = \max \left\{ \frac{\left[c_t + \sigma_w \frac{\varphi_t}{\omega}(\Psi_t)(1-l_t)^{\omega}\right]^{1-\frac{1}{\psi}}}{1-\frac{1}{\psi}} + \beta \mathbb{E}_t\left[v(q_{t+1};\Gamma_{t+1},\Theta_{t+1})\right] \right\}$$

subject to

$$\begin{aligned} & c_t + (1-p_{n,t}\vartheta_t)\,\sigma_e s_t + \sigma_w \frac{i_t}{\chi_t} + p_{n,t} n_{t+1} + p_{k,t}\bar{k}_{t+1} \\ = \; & (1-\tau_p)(\Pi_t n_t + u_t R_t k_t) + p_{n,t}(1-\delta_n) n_t + p_{k,t}(1-\delta(u_t)) k_t + (1-\tau_l) W_t \sigma_w l_t + \tau_{tr,t} \end{aligned}$$

and

$$k_{t+1} = \bar{k}_{t+1} + \left(1 - \Lambda\left(\frac{i_t}{i_{t-1}}\right)\right)\sigma_w i_t.$$

Note that although there are heterogenous members in the household (entrepreneurs and workers), the household's problem is reduced to a completely standard form if there is no liquidity friction. Other parts of the model are the same as the benchmark model.

8.2 Model summary

The following equations summarize the model economy:

$$Y_t = (u_t K_t)^{\alpha} (Z_t \sigma_w l_t)^{1-\alpha},$$

$$Z_t = \left(\bar{A}\right)(A_t)(N_t),$$

$$\varphi_t (\Psi_t)(1-l_t)^{\omega-1} = (1-\tau_l) W_t,$$

$$W_t = (1-\xi)(1-\alpha)\frac{Y_t}{\sigma_w l_t},$$

$$p_{n,t} = \mathbb{E}_t \left[\beta \left(\frac{c_{t+1} + \sigma_w \frac{\varphi_{t+1}}{\omega}(\Psi_{t+1})(1-l_{t+1})^{\omega}}{c_t + \sigma_w \frac{\varphi_t}{\omega}(\Psi_t)(1-l_t)^{\omega}}\right)^{-\frac{1}{\psi}} \left((1-\tau_p)\Pi_{t+1} + p_{n,t+1}(1-\delta_n)\right)\right],$$

$$\Pi_t = \left(\frac{\nu-1}{\nu}\right)\xi \frac{Y_t}{N_t},$$

$$R_t = (1-\xi)\alpha \frac{Y_t}{u_t K_t},$$

$$p_{k,t} = \mathbb{E}_t \left[\beta \left(\frac{c_{t+1} + \sigma_w \frac{\varphi_{t+1}}{\omega}(\Psi_{t+1})(1-l_{t+1})^{\omega}}{c_t + \sigma_w \frac{\varphi_t}{\omega}(\Psi_t)(1-l_t)^{\omega}}\right)^{-\frac{1}{\psi}} \left((1-\tau_p) u_{t+1} R_{t+1} + p_{k,t+1}(1-\delta(u_{t+1}))\right)\right],$$

$$(1-\tau_p) R_t = p_{k,t}\delta'(u_t),$$

$$\begin{aligned} \frac{1}{\chi_t} &= p_{k,t}\left(1-\Lambda\left(\frac{i_t}{i_{t-1}}\right) - \Lambda'\left(\frac{i_t}{i_{t-1}}\right)\frac{i_t}{i_{t-1}}\right) \\ &\quad + \mathbb{E}_t \left[\beta \left(\frac{c_{t+1} + \sigma_w \frac{\varphi_{t+1}}{\omega}(\Psi_{t+1})(1-l_{t+1})^{\omega}}{c_t + \sigma_w \frac{\varphi_t}{\omega}(\Psi_t)(1-l_t)^{\omega}}\right)^{-\frac{1}{\psi}} p_{k,t+1}\Lambda'\left(\frac{i_{t+1}}{i_t}\right)\left(\frac{i_{t+1}}{i_t}\right)^2\right], \end{aligned}$$

$$1 = p_{n,t}\vartheta_t,$$

$$N_{t+1} = (1-\delta_n) N_t + \vartheta_t (\sigma_e s_t),$$

$$K_{t+1} = (1-\delta(u_t)) K_t + \left(1-\Lambda\left(\frac{i_t}{i_{t-1}}\right)\right)\sigma_w i_t,$$

$$\vartheta_t = \frac{\zeta N_t}{(\sigma_e s_t)^{1-\eta}(N_t)^{\eta}},$$

$$\left(1-\frac{\xi}{\nu}\right) Y_t = c_t + \sigma_w \frac{i_t}{\chi_t} + \sigma_e s_t + Gov_t,$$

$$\frac{Gov_t}{N_t} = g_t,$$

$$\Psi_t = p_{k,t} K_t,$$

$$Stock_t = p_{k,t}\left(1-\delta\left(u_t\right)\right)K_t + p_{n,t}N_{t+1},$$

$$\log\left(\frac{TFP_t}{TFP_{t-1}}\right) = \alpha\log\left(\frac{u_t}{u_{t-1}}\right) + (1-\alpha)\log\left(\frac{A_t}{A_{t-1}}\right) + (1-\alpha)\log\left(\frac{N_t}{N_{t-1}}\right).$$

Detrended system is summarized by the following equations:

$$\hat{Y}_t = \left(u_t\hat{K}_t\right)^{\alpha}\left(\left(\bar{A}\right)\left(A_t\right)\left[\sigma_w l_t\right]\right)^{1-\alpha},$$

$$\varphi_t\left(\hat{\Psi}_t\right)\left(1-l_t\right)^{\omega-1} = \left(1-\tau_l\right)\hat{W}_t,$$

$$\hat{W}_t = (1-\xi)(1-\alpha)\frac{\hat{Y}_t}{\sigma_w l_t},$$

$$p_{n,t} = \mathbb{E}_t\left[\beta\left(\gamma_{t+1}\frac{\hat{c}_{t+1}+\sigma_w\frac{\varphi_{t+1}}{\omega}\left(\hat{\Psi}_{t+1}\right)\left(1-l_{t+1}\right)^{\omega}}{\hat{c}_t+\sigma_w\frac{\varphi_t}{\omega}\left(\hat{\Psi}_t\right)\left(1-l_t\right)^{\omega}}\right)^{-\frac{1}{\psi}}\left(\left(1-\tau_p\right)\Pi_{t+1}+p_{n,t+1}\left(1-\delta_n\right)\right)\right],$$

$$\Pi_t = \left(\frac{\nu-1}{\nu}\right)\xi\hat{Y}_t,$$

$$R_t = (1-\xi)\alpha\frac{\hat{Y}_t}{u_t\hat{K}_t},$$

$$p_{k,t} = \mathbb{E}_t\left[\beta\left(\gamma_{t+1}\frac{\hat{c}_{t+1}+\sigma_w\frac{\varphi_{t+1}}{\omega}\left(\hat{\Psi}_{t+1}\right)\left(1-l_{t+1}\right)^{\omega}}{\hat{c}_t+\sigma_w\frac{\varphi_t}{\omega}\left(\hat{\Psi}_t\right)\left(1-l_t\right)^{\omega}}\right)^{-\frac{1}{\psi}}\left(\left(1-\tau_p\right)u_{t+1}R_{t+1}+p_{k,t+1}\left(1-\delta\left(u_{t+1}\right)\right)\right)\right],$$

$$\left(1-\tau_p\right)R_t - p_{k,t}\delta'\left(u_t\right) = 0,$$

$$\begin{aligned}\frac{1}{\chi_t} &= p_{k,t}\left(1-\Lambda\left(\gamma_t\frac{\hat{\imath}_t}{\hat{\imath}_{t-1}}\right)-\Lambda'\left(\gamma_t\frac{\hat{\imath}_t}{\hat{\imath}_{t-1}}\right)\left(\gamma_t\frac{\hat{\imath}_t}{\hat{\imath}_{t-1}}\right)\right)\\ &\quad+\mathbb{E}_t\left[\beta\left(\gamma_{t+1}\frac{\hat{c}_{t+1}+\sigma_w\frac{\varphi_{t+1}}{\omega}\left(\hat{\Psi}_{t+1}\right)\left(1-l_{t+1}\right)^{\omega}}{\hat{c}_t+\sigma_w\frac{\varphi_t}{\omega}\left(\hat{\Psi}_t\right)\left(1-l_t\right)^{\omega}}\right)^{-\frac{1}{\psi}}p_{k,t+1}\Lambda'\left(\gamma_{t+1}\frac{\hat{\imath}_{t+1}}{\hat{\imath}_t}\right)\left(\gamma_{t+1}\frac{\hat{\imath}_{t+1}}{\hat{\imath}_t}\right)^2\right],\end{aligned}$$

$$1 = p_{n,t}\vartheta_t,$$

$$\gamma_{t+1} = 1-\delta_n+\vartheta_t\left(\sigma_e\hat{s}_t\right),$$

$$\gamma_{t+1}\hat{K}_{t+1} = \left(1-\delta\left(u_t\right)\right)\hat{K}_t + \left(1-\Lambda\left(\gamma_t\frac{\hat{\imath}_t}{\hat{\imath}_{t-1}}\right)\right)\sigma_w\hat{\imath}_t,$$

$$\vartheta_t = \zeta\left(\sigma_e\hat{s}_t\right)^{\eta-1},$$

$$\left(1-\frac{\xi}{\nu}\right)\hat{Y}_t = \hat{c}_t+\sigma_w\frac{\hat{\imath}_t}{\chi_t}+\sigma_e\hat{s}_t+g_t,$$

$$\hat{\Psi}_t = p_{k,t}\hat{K}_t,$$

$$\widehat{Stock}_t = p_{k,t}\left(1-\delta\left(u_t\right)\right)\hat{K}_t + p_{n,t}\gamma_{t+1},$$

$$\log\left(\frac{TFP_t}{TFP_{t-1}}\right) = \alpha\log\left(\frac{u_t}{u_{t-1}}\right) + (1-\alpha)\log\left(\frac{A_t}{A_{t-1}}\right) + (1-\alpha)\log\left(\gamma_t\right).$$

Hat variables denote the original variables divided by N_t, i.e., $\hat{Y}_t = Y_t/N_t$, and so on. γ_{t+1} is defined as $\gamma_{t+1} = N_{t+1}/N_t$.

8.3 Calibration Strategy

We calibrate A and φ so that the steady state detrended gross output $\hat{Y}$ is set at $\hat{Y} = 1$, and the steady state labor supply l is set at $l = 1/3$. We also assume that the home production function has the same elasticity of labor as the aggregate production function in the market sector, i.e., $\omega = 1-\alpha$. We set σ_e at the same value as in the benchmark model.[9] The labor share is a function of α, ν, and ξ,

$$\frac{\hat{W}\left(\sigma_w l\right)}{\hat{\mathcal{Y}}} = \frac{(1-\xi)(1-\alpha)}{1-\xi/\nu} \tag{45}$$

where ξ is a function of α and ν because we impose a parameter restriction

$$\xi = \frac{1-\alpha}{\nu-\alpha}.$$

Pricing equation for capital in the steady state is

$$1 = \beta\left(\gamma\right)^{-\frac{1}{\psi}}\left(uR\left(1-\tau_p\right)+1-\delta_k\right) \tag{46}$$

where

$$uR = (1-\xi)\,\alpha\frac{1}{\hat{K}},$$

$$\hat{K} = \frac{\sigma_w\hat{\imath}}{\gamma-1+\delta_k},$$

and

$$\sigma_w\hat{\imath} = \left(1-\frac{\xi}{\nu}\right)\left(\frac{\sigma_w\hat{\imath}}{\hat{\mathcal{Y}}}\right).$$

Pricing equation for a product in the steady state is

$$p_n = \beta\left(\gamma\right)^{-\frac{1}{\psi}}\left(\Pi\left(1-\tau_p\right)+p_n\left(1-\delta_n\right)\right) \tag{47}$$

[9] This parameter becomes very difficult to pin down in a model without financial frictions because it does not have direct impact on investment to R&D. It instead is almost a scale parameter in the reduced-form utility function.

where

$$\Pi = \left(\frac{\nu - 1}{\nu}\right)\xi.$$

The products' law of motion in the steady state is

$$\gamma = 1 - \delta_n + \frac{1}{p_n}\sigma_e \hat{s} \tag{48}$$

where

$$\frac{\sigma_s \hat{s}}{\iota} = \left(1 - \frac{\xi}{\nu}\right)\left(\frac{\sigma_e \hat{s}}{\widehat{\mathcal{Y}}}\right).$$

We assume that the steady state value of the investment specific technology shock is $\chi = 1$. The resource constraint in the steady state is

$$1 = \left(1 - \frac{\xi}{\nu}\right)^{-1} \hat{c} + \frac{\sigma_w \hat{\imath}}{\widehat{\mathcal{Y}}} + \frac{(\sigma_e \hat{s})}{\widehat{\mathcal{Y}}} + \frac{g}{\widehat{\mathcal{Y}}}, \tag{49}$$

where g is the steady state value of the government consumption shock, which is calibrated to match the government consumption share in GDP, i.e.,

$$g = \left(\frac{Gov}{\mathcal{Y}}\right)\left(1 - \frac{\xi}{\nu}\right).$$

Equation (45) to (49) are five-equation, five-unknown system if we specify β, γ, δ_k, τ_p, $Gov/\mathcal{Y}$, $\hat{W}(\sigma_w l)/\widehat{\mathcal{Y}}$, $\sigma_w \hat{\imath}/\widehat{\mathcal{Y}}$, and $\sigma_e \hat{s}/\widehat{\mathcal{Y}}$. We solve it and find/calibrate p_n, $\hat{c}$, α, ν, and δ_n. Other steady state values are backed out in a similar way as in the benchmark model.

8.4 Estimation results

The estimation is conducted with the same strategy and the same data set. Table 2 reports the estimated parameters. The government consumption shock is less persistent and less volatile in the model without liquidity friction than in the benchmark model. Stochastic processes of other shocks are largely similar in the two models. It is interesting that the elasticity of $\delta'(u)$ in the steady state is estimated to be at a large value in the model without liquidity friction. In other words, the data prefer having a weak amplification mechanism in the model without liquidity friction.

Shock decomposition in Figure 13 shows that productivity is the driver of the crisis in the model without liquidity friction. The MEI shock also plays a crucial role dragging the economy to the recession; this is particularly so for investment. The recovery is initially driven by shocks to the efficiency of investment and later by shocks to labor supply.

The overall message in the two variants of the benchmark model (the exogenous growth model and the model without finanical frictions) is that productivity played a key role during the crisis.

Table 2: Parameter Values

No Liquidity Friction			Benchmark		
Parameter	Median	90% Interval	Parameter	Median	90% Interval
ρ_A	0.929	[0.893,0.961]	ρ_A	0.901	[0.872,0.937]
ρ_ϕ	-	-	ρ_ϕ	0.996	[0.993,0.998]
ρ_g	0.866	[0.793,0.927]	ρ_g	0.987	[0.984,0.989]
ρ_φ	0.900	[0.858,0.933]	ρ_φ	0.879	[0.845,0.907]
ρ_χ	0.555	[0.472,0.634]	ρ_χ	0.642	[0.574,0.704]
σ_A	0.014	[0.012,0.015]	σ_A	0.015	[0.013,0.016]
σ_ϕ	-	-	σ_ϕ	0.033	[0.029,0.039]
σ_g	0.022	[0.020,0.025]	σ_g	0.046	[0.041,0.051]
σ_φ	0.013	[0.012,0.015]	σ_φ	0.012	[0.011,0.014]
σ_χ	0.024	[0.020,0.029]	σ_χ	0.023	[0.019,0.027]
σ_m	0.030	[0.029,0.030]	σ_m	0.030	[0.029,0.030]
σ_t	0.006	[0.005,0.007]	σ_t	0.007	[0.006,0.007]
δ''/δ'	4.322	[3.092,6.000]	δ''/δ'	0.268	[0.214,0.330]
ϕ	-	-	ϕ	0.121	[0.105,0.138]
$\bar{\Lambda}$	0.455	[0.365,0.562]	$\bar{\Lambda}$	0.590	[0.502,0.694]

This is because liquidity has no longer an impact on productivity, and therefore, technology shock is the only shock that has enough strength to shift the economy's trend. Furthermore, government consumption (or more generally, shocks to households' wealth) has almost no impact on the economy during the Great Recession and its recovery.

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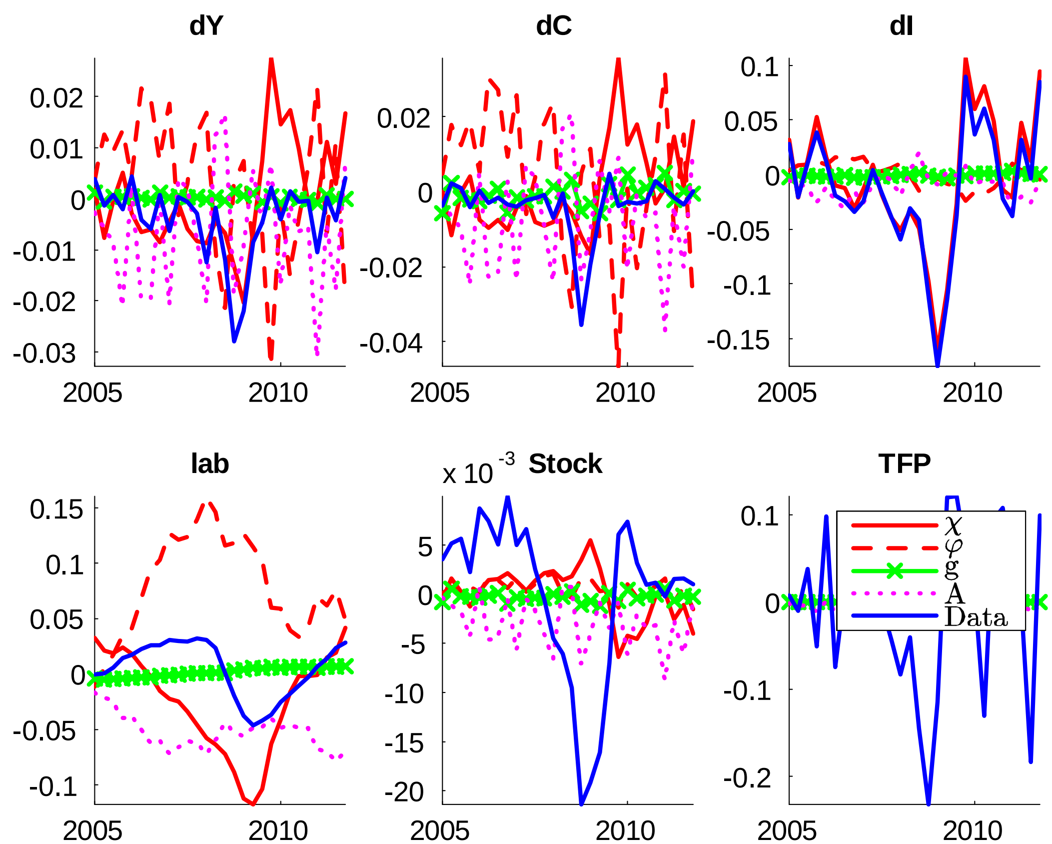

Figure 13: Shock Contribution: No-Liquidity Model

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