Introduction to LaTeX

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Abstract

The abstract text goes here.

1 Introduction

This is a simple application of Theorem 3. We have two series of coefficients.

$$\beta_{x,k}^{(j)} = \frac{\rho^{k2^j}\left(1-\rho_x^{2^{j-1}}\right)^2}{\sqrt{2^j}(1-\rho_x)}. \text{ for } j = 1, 2, \ldots, \tag{1}$$

These are the multiscale Wold coefficients for an AR(1,t,t+1).

We also consider the following series of coefficients:

$$\beta_{y,k}^{(j)} = \frac{\rho^{k2^{(j-j^*)}}\left(1-\rho^{2^{(j-j^*)-1}}\right)^2}{\sqrt{2^{(j-j^*)}}(1-\rho)}. \text{ for } j = j^*+1, \ldots, \tag{2}$$

$$\beta_{y,k}^{(j)} = 0 \text{ for } j \leq j^*$$

where $j^* = \log_2(8)$. These look like the coefficients of an AR(1,t,t+8) where the notation $AR(1, t, t+8)$ denotes an autoregressive process of order 1 on a grid $t, t+8, t+16, \ldots$. (TO BE FORMALIZED).

The idea is to generate a process x_t with wold coefficients given by

$$\beta_k^{(j)} = \lambda * \beta_{x,k}^{(j)} + \beta_{y,k}^{(j)} \quad j = 1, 2, \ldots,$$

where λ captures the relative contribution of each process from scale $j > j^*$.

Then

$$NEWPROCESS = \sum_{j=1}^{+\infty}\sum_{k=0}^{+\infty} \beta_k^{(j)} \varepsilon_{t-k2^j}^{(j)}$$

The simulation of the white noise is trivial.

1.1 Results

We have 3 parameters, fix $\lambda = 1$, $\rho_x = 0.9$, $\rho = 0.9$. Figure :simulation2Pers shows the autocorrelation function of the NEWPROCESS (blue line). In each panel we superimpose the autocorrelation function of an autoregressive process of order one with autoregressive parameter $= 0.95$ (Panel A) and $= 0.97$ (Panel B).

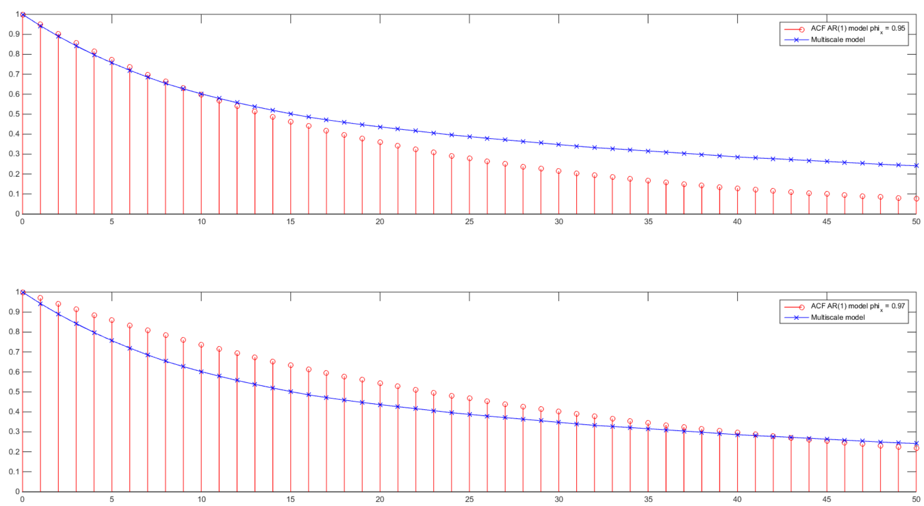

Figure 1: Simulation Results. 2 persistent processes

- The first part of the autocorrelation is well approximated by an AR(1) with $rho = 0.95$, but this process undershoots the long-lags;
- The long-lags part of the autocorrelation is well approximated by an AR(1) with $rho = 0.97$, but this process overshoots the short-lags.
- overall tension of an AR(1) model to focus more on short lags vs long lags
- If you fit an ARMA(1,1) to the multiscale process (so that now you have the same number of parameters), it gives the same results as in Panel A (red line), ie. it favors the short-lags. So it seems this process is difficult to replicate with ARMA(1,1) (but what about other processes?)

1.2 Comparison with OTT

In OTT we were assuming an autoregressive component at a specific level of persistence. Here we have that the sum of components at multiple scales is an autoregressive process.

1.3 Special cases

When $\rho_x = 0$ then

$$x_t = WN + AR(1, t, t+8)$$

WN is a white noise process with unit variance.

Figure 2 plots the autocorrelation of the process x_t (top panel, blue line with cross) and the autocorrelation of the aggregated process $x_t + \ldots + x_{t-7}$ (bottom panel, blue line with cross). We pick $\phi = 0.9$. We report the case where the coefficients $\beta_k^{(j)}$ have been rescaled so to deliver a variance equal to 1. Since the persistnet AR(1) is comparable in volatility to the white noise, we see that it is somehow hidden at fine scale.

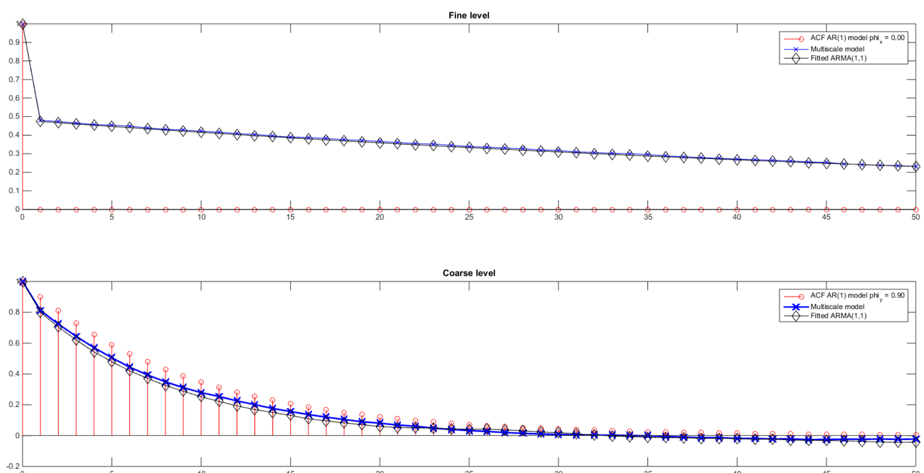

Figure 2: Simulation Results

So, in this case, it is possible to find parameters for an ARMA(1,1) to replicate the autocorrelation function of x_t and its aggregated version! See line with diamonds which overlaps with line with cross.

- Is this bad or good news (it means that the asset pricing implications of OTT are exactly the same as the one of LRR with ARMA11?)
- Is it possible to formally show this link between x_t and ARMA(1,1)?

2 Conclusion